Understanding the Born Rule in Weak Quantum Measurements

Apoorva Patel

Centre for High Energy Physics, IISc, Bangalore
and ICTS-TIFR, Bangalore

Quantum Trajectories, ICTS-TIFR, Bangalore
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N. Gisin, Phys. Rev. Lett. 52 (1984) 1657
A. Patel and P. Kumar, Phys Rev. A96 (2017) 022108 [arXiv:1509.08253]
P. Kumar, S. Kundu, M. Chand, R. Vijay and A. Patel [arXiv:1804.03413]

Axioms of Quantum Dynamics

(1) Unitary evolution (Schrödinger):

$$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle \ , \quad i\frac{d}{dt}\rho = [H,\rho] \ .$$

Continuous, Reversible, Deterministic.

Pure state evolves to pure state.

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(2) Projective measurement (von Neumann):

$$|\psi\rangle \longrightarrow P_i|\psi\rangle/|P_i|\psi\rangle|, \ P_i = P_i^\dagger, \ P_iP_j = P_i\delta_{ij}, \ \sum_i P_i = I.$$

Discontinuous, Irreversible, Probabilistic choice of "i".

Pure state evolves to pure state. Consistent on repetition.

$\{P_i\}$ is fixed by the measurement apparatus eigenstates. But there is no prediction for which "i" will occur in a particular experimental run.

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Instead, with Born rule and ensemble interpretation,

$$prob(i) = \langle\psi|P_i|\psi\rangle = Tr(P_i\rho) \ , \quad \rho \longrightarrow \sum_i P_i\rho P_i \ .$$

Pure state evolves to mixed state. Predicted expectation values are averages over many experimental runs with the same initial state.

Weak Measurements

Information about the measured observable is extracted from the system at a slow rate (e.g. by weak coupling). Stretching out the time scale can allow one to monitor collapse of the system to a measurement eigenstate.

Note: A measurement interaction is the one where the apparatus does not, for whatever reasons, remain in a superposition of pointer states.

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The answers are important for increasing accuracy of quantum control and feedback. Knowledge of what happens in a particular experimental run (and not just the ensemble average) can improve efficiency and stability.

The projective measurement axiom needs to be replaced by a different continuous stochastic dynamics.

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Unraveling of quantum collapse:

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Such a dynamical process exists! Gisin (1984)

Salient Features

A precise ratio of evolution towards the measurement eigenstates and unbiased white noise is needed to reproduce the Born rule as a constant of evolution.

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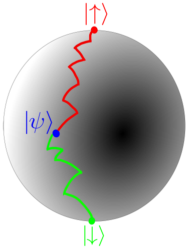
Technological advances allow us to monitor the quantum evolution during weak measurements. That can test the validity of the stochastic measurement formalism, and then help us figure out what may lie beyond.



Measurement $\equiv$ An effective process of a more fundamental theory.

Quantum Geodesic Trajectory

Leave out $i[\rho, H]$ from the evolution description for simplicity.
Unitary interpolation between ρ and P_i gives the geodesic evolution:

$$\frac{d}{dt}\rho = g[\rho P_i + P_i\rho - 2\rho \; Tr(P_i\rho)] \; .$$

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- In a bipartite setting, $\{P_i\} = \{P_{i_1} \otimes P_{i_2}\}$ and $\sum_i P_i = I$ imply that partial trace over the unobserved degrees of freedom (and projections) gives the same equation for the reduced density matrix for the system.

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- For pure states, the equation can be written as:

$$\frac{d}{dt}\rho = -2g\mathcal{L}[\rho]P_i$$

This structure (involving the Lindblad operator) hints at an action-reaction relation between the dynamics of the system and the apparatus.

Ensemble of Quantum Geodesic Trajectories

The pointer basis $\{P_i\}$ is fixed by the system-apparatus interaction.
A criterion is needed to determine which of the many fixed points P_i will be approached in a particular experimental run.

Assign time-dependent real weights $w_i(t)$ to the evolution trajectory for P_i.

$$\frac{d}{dt}\rho = \sum_i w_i \; g[\rho P_i + P_i\rho - 2\rho Tr(P_i\rho)] \; , \quad \sum_i w_i = 1 \; .$$

Evolution still preserves $\rho^2 = \rho$. Every $\rho = P_i$ becomes a fixed point.
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The weighted trajectory evolution is:

$$\frac{d}{dt}(P_j \rho P_k) = P_j \rho P_k \; g[w_j + w_k - 2\sum_i w_i Tr(P_i \rho)] \; .$$

Diagonal projections of ρ fully determine the evolution:

$$\frac{2}{P_j \rho P_k} \frac{d}{dt}(P_j \rho P_k) = \frac{1}{P_j \rho P_j} \frac{d}{dt}(P_j \rho P_j) + \frac{1}{P_k \rho P_k} \frac{d}{dt}(P_k \rho P_k)$$

The evolution is totally decoupled from the decoherence process.

There are $n-1$ independent variables (diagonal projections $Tr(P_i \rho)$).

Choice of Trajectory Weights

The diagonal projections evolve according to:

$$\frac{d}{dt} d_j = 2g\ d_j (w_j - w_{\rm av})\ , \quad w_{\rm av} \equiv \sum_i w_i d_i\ .$$

Diagonal elements with $w_j > w_{\rm av}$ grow; those with $w_j < w_{\rm av}$ decay.

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Naive guess (instantaneous Born rule): $w_j = w_j^{IB} \equiv Tr(\rho(t)P_j)$

The evolution converges towards the subspace specified by the dominant diagonal projections of $\rho(t=0)$, i.e. the closest fixed points.

Though this result is consistent on repetition, it conflicts with experiments, because it is (i) deterministic and (ii) does not obey the Born rule.

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A way out: Instead of heading towards the nearest fixed point, the trajectories can be made to wander around the state space and explore other fixed points, by adding noise to the geodesic dynamics.

Properties of such a noise have to be found, while retaining $\sum_i w_i = 1$.

The type of the noise is not universal. It depends on the choice of the apparatus.

Quantum Diffusion: Single Qubit Measurement

The evolution equations simplify considerably for a qubit.
Let $|0\rangle$ and $|1\rangle$ be the measurement eigenstates.

$$\frac{d}{dt}\rho_{00} = 2g\ (w_0 - w_1)\rho_{00}\rho_{11}\ ,$$

$$\rho_{01}(t) = \rho_{01}(0)\left[\frac{\rho_{00}(t)\rho_{11}(t)}{\rho_{00}(0)\rho_{11}(0)}\right]^{1/2}.$$

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Evolution obeys Langevin dynamics, when unbiased white noise with spectral density S_ξ is added to w_i^{IB}. The trajectory weights become:

$$w_0 - w_1 = \rho_{00} - \rho_{11} + \sqrt{S_\xi}\ \xi\ .$$

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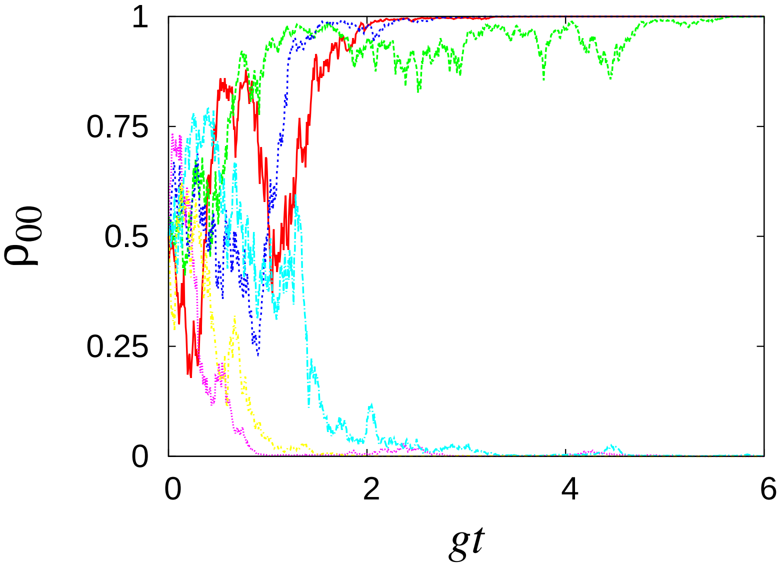
This is a stochastic differential process on the interval $[0, 1]$.
The fixed points at $\rho_{00} = 0, 1$ are perfectly absorbing boundaries.
A quantum trajectory would zig-zag through the interval before ending at one of the two boundary points.



Individual quantum evolution trajectories for the initial state $\rho_{00} = 0.5$, with measurement eigenstates $\rho_{00} = 0, 1$, and in presence of measurement noise satisfying $gS_\xi = 1$.

Let $P(x)$ be the probability that the initial state with $\rho_{00} = x$ evolves to the fixed point at $\rho_{00} = 1$. Then by symmetry,

$$P(0) = 0,\ P(0.5) = 0.5,\ P(1) = 1\ .$$

No noise : $S_\xi = 0 \implies P(x) = \theta(x - 0.5)$.

Only noise : $S_\xi \to \infty \implies P(x) = 0.5$.

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It is instructive to convert the stochastic evolution equation from the differential Stratonovich form to the Itô form that specifies forward evolutionary increments:

$$d\rho_{00} = 2g \ \rho_{00}\rho_{11}(\rho_{00} - \rho_{11})(1 - gS_\xi)dt + 2g\sqrt{S_\xi} \ \rho_{00}\rho_{11} \ dW \ ,$$

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The first term produces drift in the evolution, while the second gives rise to diffusion. The evolution with no drift, i.e. the pure Wiener process with $gS_\xi = 1$, is rather special:

$$\langle\langle d\rho_{00} \rangle\rangle = 0 \iff \text{Born rule is a constant of evolution.}$$

Ensemble Evolution Dynamics

During measurement, the probability distribution $p(\rho_{00}, t)$ of the set of quantum trajectories evolves according to the Fokker-Planck equation:

$$\frac{\partial p(\rho_{00},t)}{\partial t} = 2g \frac{\partial^2}{\partial^2 \rho_{00}} \left(\rho_{00}^2 (1-\rho_{00})^2 p(\rho_{00}, t)\right) \ , \quad \text{with } gS_\xi = 1 \ .$$

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Its exact solution corresponding to initial $p(\rho_{00}, 0) = \delta(x)$ has two non-interfering components with areas x and $1-x$, monotonically travelling to the boundaries at $\rho_{00} = 1$ and 0 respectively.

Let $\tanh(z) = \rho_{00} - \rho_{11}$ map $\rho_{00} \in [0,1]$ to $z \in (-\infty, \infty)$. Then the two components are Gaussians centred at $z_\pm = z_0 \pm gt$, $z_0 = \tanh^{-1}(2x-1)$:

$$p(z,t) = \frac{1}{\sqrt{2\pi gt}} \left(x \exp\left[-\frac{(z-z_+)^2}{2gt} \right] + (1-x) \exp\left[-\frac{(z-z_-)^2}{2gt} \right] \right) .$$

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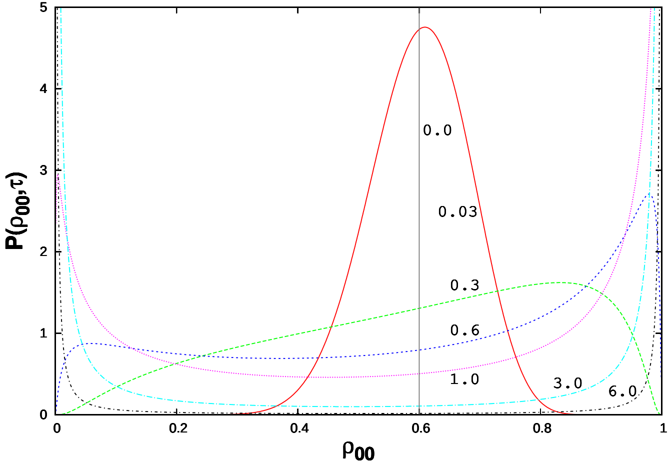
Parametric freedom: With the Born rule as a constant of evolution, g can be time-dependent, and gt is replaced by $\tau \equiv \int_0^t g(t')dt'$.
The white noise distribution remains unspecified beyond the mean and the variance. Suitable choice can be made, e.g. Gaussian noise or Z_2 noise.



Distribution of the quantum measurement trajectories for quantum diffusion evolution of a qubit. The initial state is $\rho_{00}(\tau = 0) = 0.6$, and the curves are labeled by the values of the dimensionless evolution parameter $\tau \equiv \int_0^t g(t')dt'$. The narrow initial distribution splits into two non-interfering components that travel to the measurement eigenstates at $\rho_{00} = 1, 0$ as $\tau \to \infty$.

For $\tau > 10$, 99% of the probability is within 1% of the two fixed points.

Experimental Setup

The system is a superconducting 3D transmon qubit (nonlinear oscillator).

It consists of two Josephson junctions in a closed loop (SQUID) shunted by a capacitor.

It possesses good coherence and is insensitive to charge noise.

Decoherence time $\sim 50 - 100 \mu s$. Individual operation time: fraction of μs.

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It is kept in a microwave resonator cavity dispersively coupled to it.

The cavity frequency depends on the qubit state, whether $|0\rangle$ or $|1\rangle$.

The cavity is probed by a microwave pulse. The scattered wave is amplified by a near-quantum-limited Josephson parametric amplifier.

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One-way isolator and circulator help extract the scattered wave.
Interference of the amplified wave with the reference wave yields the quantum state signal, as a scattering phase-shift.

Both the cavity and the amplifier are bandwidth limited, with high frequencies suppressed.

Experimental Setup

The system is a superconducting 3D transmon qubit (nonlinear oscillator).

It consists of two Josephson junctions in a closed loop (SQUID) shunted by a capacitor.

It possesses good coherence and is insensitive to charge noise.

Decoherence time $\sim 50 - 100\mu s$. Individual operation time: fraction of μs.

It is kept in a microwave resonator cavity dispersively coupled to it.

The cavity frequency depends on the qubit state, whether $|0\rangle$ or $|1\rangle$.

The cavity is probed by a microwave pulse. The scattered wave is amplified by a near-quantum-limited Josephson parametric amplifier.

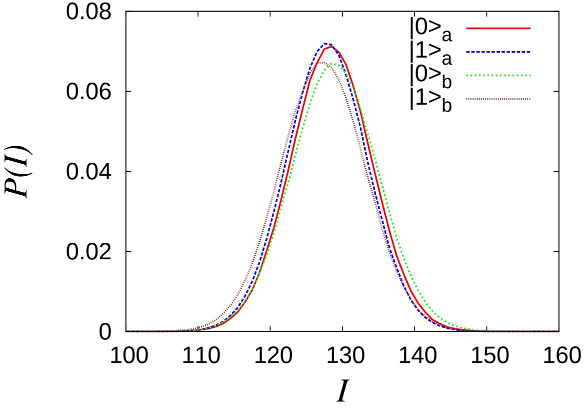
One quadrature of the signal is extracted with high gain and high accuracy.

One-way isolator and circulator help extract the scattered wave. Interference of the amplified wave with the reference wave yields the quantum state signal, as a scattering phase-shift.

Both the cavity and the amplifier are bandwidth limited, with high frequencies suppressed.

With a phase-sensitive amplifier, the scattering phase-shifts are Gaussians peaked at the two eigenvalues. Weak measurements result when the probe magnitude is small, making the two Gaussians closely overlap.

Weak Measurement Stern-Gerlach Signal



Measurement current distributions for the qubit eigenstates after evolution for $0.5\mu s$, for an ensemble of 10^6 trajectories. They are Gaussians to high accuracy. Weak measurement needs $|I_0 - I_1| \ll \sigma$. The system-apparatus coupling increases mostly by an increase in ΔI without much change in σ. The taller curves (a) correspond to a weaker system-apparatus coupling than the shorter curves (b). Gaussian fits give the parameters: (a) $I_0 = 128.443(2)$, $I_1 = 127.856(2)$, $\sigma = 5.56(3)$, and (b) $I_0 = 128.919(2)$, $I_1 = 127.286(2)$, $\sigma = 5.93(2)$.

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A quantum state initially polarised in XZ-plane is measured in the Z-basis.
Even though the weak measurement extracts only partial information, its back-action on the qubit is completely known, and the qubit evolution from a known starting state can be precisely constructed.

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The quantum state is infered from the integrated signal measurement, according to the Bayesian formalism (I_0, I_1, σ are known):

$$\frac{\rho_{00}(t+\Delta t)}{\rho_{11}(t+\Delta t)} = \frac{\rho_{00}(t)}{\rho_{11}(t)} \frac{\exp[-(I_m(\Delta t)-I_0)^2/2\sigma^2]}{\exp[-(I_m(\Delta t)-I_1)^2/2\sigma^2]} \ , \quad I_m(\Delta t) = \frac{1}{\Delta t}\int_0^{\Delta t} I(t')\ dt' \ .$$

Simultaneous relaxation of the excited state is accounted for, by

$$\rho_{11}(t+\Delta t) = \rho_{11}(t)\ \exp(-\Delta t/T_1) \ .$$

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Quantum diffusion is not monotonic in time (unlike spontaneous collapse).
Quantum trajectories stochastically diffuse along the meridians of the Bloch sphere (the phase of ρ_{01} remains unchanged).

Experimental Results (contd.)

Relaxation time T_1 is determined from the decay rate of the ensemble averaged current, after preparing the qubit in the excited state.

The experimentally observed trajectory distribution fits the quantum diffusion prediction very well, in terms of the single dimensionless evolution parameter $\tau \equiv \int_0^t g(t')dt'$, and excited state relaxation T_1.

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- With a large ensemble of trajectories, systematic errors dominate over statistical ones.
- For $\tau < 2$, best fits have $\chi^2 <$ few hundred, for 100 data points and one parameter.
- τ is independent of the initial state. It is almost linear in t, with a slower initial build-up.
- The mismatch between theory and experiment grows with increasing τ, quite likely due to magnification of small initial uncertainties due to the iterative evolution.

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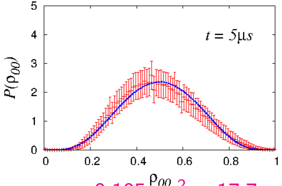
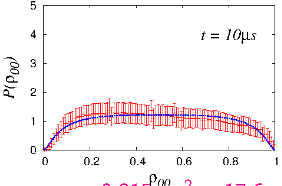
Systematic errors:

- Uncertainty in the initial state $\rho_{00}(0)$.
- Uncertainties in I_0, I_1.
- ◦ Leftover heralding photons, after the initial state preparation pulse.
- ◦ Thermal mixing with the higher excited transmon states.

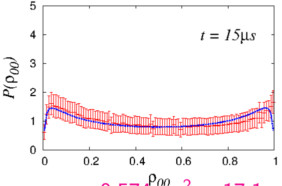
Detector inefficiency is absorbed in the value of τ. (Formally, $g\Delta t = (\Delta I)^2/(4\sigma^2)$.)



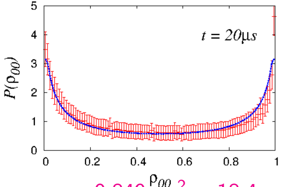
$\tau = 0.105, \chi^2 = 17.7$



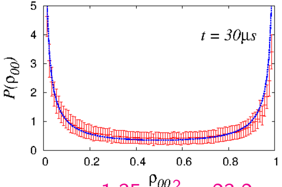
$\tau = 0.315, \chi^2 = 17.6$



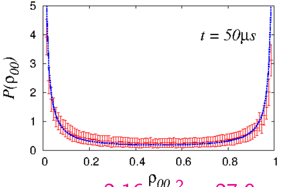
$\tau = 0.574, \chi^2 = 17.1$



$\tau = 0.849, \chi^2 = 18.4$



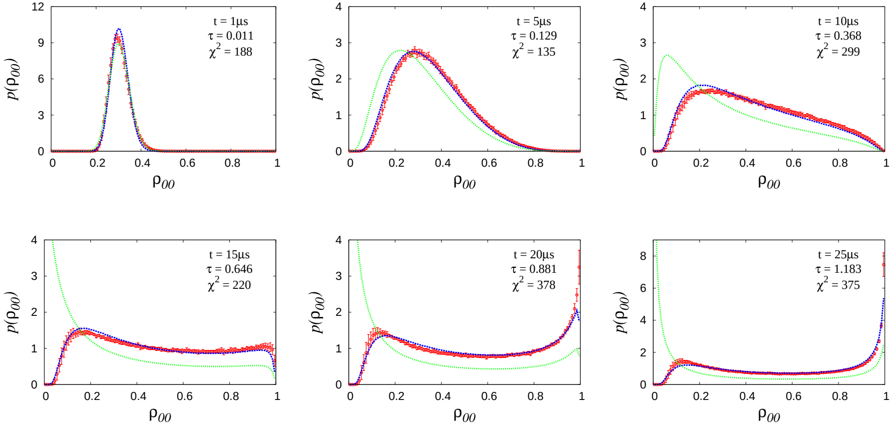
$\tau = 1.35, \chi^2 = 23.2$



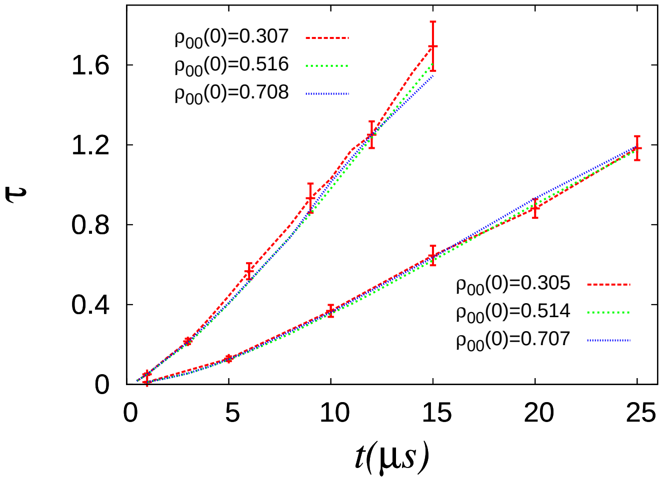
$\tau = 2.16, \chi^2 = 27.9$

Time integrated coupling: $\tau \approx 4.7 \times 10^4 t - 0.1$

Evolution of the quantum trajectory distribution for weak Z-measurement of a transmon qubit initially polarised along the X-axis. The histograms (red) represent the experimental data for an ensemble of 4×10^5 trajectories. The curves (blue) are the best fits to the quantum diffusion model distribution, with the single dimensionless evolution parameter $\tau \in [0, 2.2]$.



Evolution of the quantum trajectory distribution for weak Z-measurement of a transmon with the initial state $\rho_{00} = 0.305(3)$. The histograms with bin width 0.01 (red) represent the experimental data for an ensemble of 10^6 trajectories. The trajectory parameters (with errors) were $T_1 = 45(4)\mu s$, $\Delta t = 0.5\mu s$, $I_0 = 128.44(2)$, $I_1 = 127.68(3)$, $\sigma = 5.50(1)$. The blue curves are the best fits to the quantum diffusion model including relaxation, with the evolution parameter $\tau \in [0, 1.2]$; the green curves show the theoretical distributions with the same evolution parameters but with T_1 set to infinity.



The best fit values of the time integrated measurement coupling τ for two values of the system-apparatus coupling, when experimental data for weak Z-measurement of a transmon with different initial states $\rho_{00}(0)$ are compared to the theoretical predictions. It is obvious that τ is essentially independent of the initial state, and varies almost linearly with time after a slower initial build-up. The error bars correspond to changes in τ that would change the χ^2-values for the trajectory distribution fits by 100.

Fluctuation-Dissipation Relation

The geodesic parameter is $\rho_{00} - \rho_{11}$, with fixed points at ± 1.
The size of the fluctuations is, dropping the subleading $o(dt)$ terms:

$$\langle\!\langle (d\rho_{00} - d\rho_{11})^2 \rangle\!\rangle = 16 g^2 S_\xi \ \rho_{00}^2 \rho_{11}^2 \ dt \ .$$

The geodesic evolution term is:

$$(d\rho_{00} - d\rho_{11})_{\rm geo} = 4g(\rho_{00} - \rho_{11})\rho_{00}\rho_{11} \ dt \ .$$

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The constraint $gS_\xi = 1$ gives the coupling-free relation:

$$\langle\langle (d\rho_{00} - d\rho_{11})^2 \rangle\rangle = 4\rho_{00}\rho_{11} \frac{(d\rho_{00} - d\rho_{11})_{\rm geo}}{\rho_{00} - \rho_{11}} \ .$$

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In general stochastic processes, vanishing drift and fluctuation-dissipation relation are quite unrelated properties, involving first and second moments of the distribution respectively. The fact that both lead to the Born rule is an exceptional feature of quantum trajectory dynamics.

Implication: The environment can influence the measurement process only via the apparatus.

Non-classical Nature of Noise

Classical weights (probabilities) must satisfy $w_i \in [0, 1]$.
Non-classical weights correspond to $w_0 - w_1$ outside $[-1, 1]$.

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It is non-classical when it exceeds its mean by $2\rho_{11}$ or falls below by $2\rho_{00}$.

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For Gaussian distributions, the probability of the noise being non-classical is, therefore, $\frac{1}{2}(\mathrm{erfc}(\sqrt{2\Delta t/S_\xi}\ \rho_{11}) + \mathrm{erfc}(\sqrt{2\Delta t/S_\xi}\ \rho_{00}))$.
It is larger for smaller Δt, and remains non-zero throughout the evolution.

With Gaussian current distributions, the multiplicative factor for ρ_{00}/ρ_{11} is $e^{-(I_m - \overline{I})\Delta I/\sigma^2}$.
The relation $(\Delta I)^2 = 4\Delta t\ \sigma^2/S_\xi$ then implies that the non-classical noise steps occur when $(I_m - \overline{I})/\Delta I$ is either larger than ρ_{00} or smaller than $-\rho_{11}$.

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Among all noise distributions, the Z_2 noise with values $\pm\sigma$ has the shortest tail. Even then, $\sqrt{S_\xi/\Delta t}$ will asymptotically always exceed either $2\rho_{00}$ or $2\rho_{11}$ or both.

Origin of Noise

The quadratically nonlinear quantum measurement equation for state collapse supplements the Schrödinger evolution:

$$d\rho = i[\rho, H]dt + \sum_i w_i \; g[\rho P_i + P_i\rho - 2\rho Tr(\rho P_i)] \; dt + noise \; .$$

The underlying dynamics is the non-universal system-apparatus measurement interaction, and the nature of the noise depends on it.

What mechanism can simultaneously produce attraction towards the measurement eigenstates (geodesic evolution) and irreducible noise (stochastic fluctuations), with precisely related magnitudes?

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The measurement problem, i.e. the location of the "Heisenberg Cut" separating the quantum and the classical behaviour, is thus shifted higher up in the dynamics of the apparatus-dependent amplification.

The Born rule is separated from this problem.

Dynamics of Apparatus

The observed signal is amplified from the quantum to the classical regime.
Coherent states in the Fock space, which continuously interpolate between these regimes, are a convenient choice for the apparatus pointer states:

$$|\alpha\rangle \equiv e^{\alpha a^\dagger - \alpha^* a}|0\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle \ , \quad a|\alpha\rangle = \alpha|\alpha\rangle \ .$$

e.g. the harmonic oscillator ground state is a stationary Gaussian (eigenstate of Hamiltonian), but a pendulum is an oscillating Gaussian coherent state (eigenstate of annihilation operator).

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$$H_{\rm int} = ig\ |1\rangle\langle 1| \otimes (a^\dagger - a) \ ,$$

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This interaction produces a macroscopic entangled state:

$$(c_0|0\rangle + c_1|1\rangle)_S|0\rangle_A \longrightarrow c_0|0\rangle_S|0\rangle_A + c_1|1\rangle_S|\alpha\rangle_A \ .$$

A qubit can get entangled with at most two states of a bosonic mode (Schmidt decomposition).
Amplification is a driven process. Irreversibility has to be added, possibly as a boundary condition, to convert entanglement into measurement.

Implications for State Collapse

Coherent pointer states have inherent uncertainty (equal to the zero-point fluctuations). Still unitary amplification cannot create a "Heisenberg Cut".

Actually, the dividing line between the system and the apparatus separates the reversible and the irreversible parts of the dynamics.

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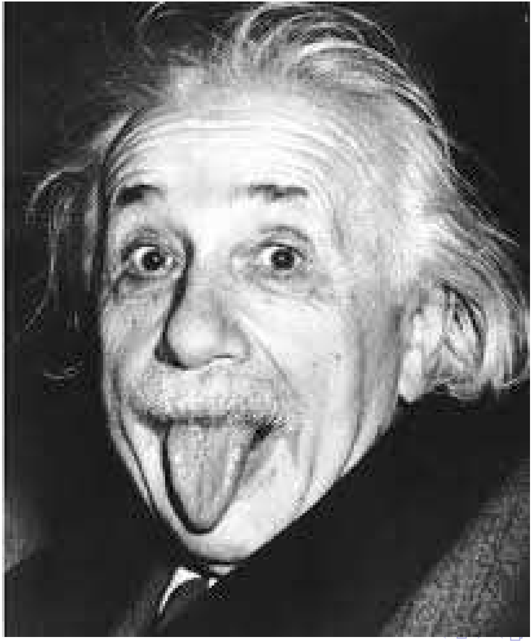
Does the coherent state uncertainty provide the quantum noise for the trajectories through back-action (influence of apparatus on system)?
If gravity is responsible for the non-unitary and irreversible collapse, it has to act only through the coherent pointer state mode.
(Sufficiently amplified coherent states have large energy compared to the quantum scale; they may also be spatially spread.)

Einstein strikes back!



References

1. J.A. Wheeler and W.H. Zurek (Eds.), Quantum Theory and Measurement (Princeton University Press, 1983).
2. V.B. Braginsky and F.Ya. Khalili, Quantum Measurement (Cambridge University Press, 1992).
3. G. Lindblad, On the generators of quantum dynamical subgroups. Comm. Math. Phys. **48**, 119-130 (1976).
4. V. Gorini, A. Kossakowski and E.C.G. Sudarshan, Completely positive semigroups of N-level systems. J. Math. Phys. **17**, 821-825 (1976).
5. D. Giulini, E. Joos, C. Kiefer, J. Kuptsch, I.-O. Stamatescu and H.D. Zeh, Decoherence and the Appearance of a Classical World in Quantum Theory (Springer, 1996).
6. H.M. Wiseman and G.J. Milburn, Quantum Measurement and Control (Cambridge University Press, 2010).
7. Y. Aharonov, D.Z. Albert and L. Vaidman, How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. Phys. Rev. Lett. **60**, 1351-1354 (1988).
8. N. Gisin, Quantum measurements and stochastic processes. Phys. Rev. Lett. **52** 1657-1660 (1984); Stochastic quantum dynamics and relativity. Helvetica Physica Acta **62** 363-371 (1989).
9. T.A. Brun, A simple model of quantum trajectories. Am. J. Phys. **70** 719-737 (2002).
10. K. Jacobs and D.A. Steck, A straightforward introduction to continuous quantum measurement. Contemporary Physics **47** 279-303 (2006).
11. G. Ghirardi, Collapse theories, The Stanford Encyclopedia of Philosophy, E.N. Zalta (ed.) (2011). `http://plato.stanford.edu/archives/win2011/entries/qm-collapse/`
12. A. Bassi, K. Lochan, S. Satin, T.P. Singh and H. Ulbricht, Models of wave-function collapse, underlying theories and experimental tests. Rev. Mod. Phys. **85**, 471-527 (2013).
13. A.N. Korotkov, Quantum Bayesian approach to circuit QED measurement. arXiv:1111.4016 (2011).
14. J. Koch, T.M. Yu, J. Gambetta, A.A. Houck, D.I. Schuster, J. Majer, A. Blais, M.H. Devoret, S.M. Girvin and R.J. Schoelkopf, Charge-insensitive qubit design derived from the Cooper pair box. Phys. Rev. A **76**, 042319 (2007).
15. K.W. Murch, R. Vijay and I. Siddiqi, Weak measurement and feedback in superconducting quantum circuits. Superconducting Devices in Quantum Optics (Springer, 2016), pp.163-185 [arXiv:1507.04617].
16. M. Hatridge, S. Shankar, M. Mirrahimi, F. Schackert, K. Geerlings, T. Brecht, K.M. Sliwa, B. Abdo, L. Frunzio, S.M. Girvin, R.J. Schoelkopf and M.H. Devoret, Quantum back-action of an individual variable-strength measurement. Science **339**, 178-181 (2013).
17. K.W. Murch, S.J. Weber, C. Macklin and I. Siddiqi, Observing single quantum trajectories of a superconducting quantum bit. Nature **502**, 211-214 (2013).
18. S.J. Weber, A. Chantasri, J. Dressel, A.N. Jordan, K.W. Murch and I. Siddiqi, Mapping the optimal route between two quantum states. Nature **511**, 570-573 (2014).
19. A.A. Clerk, M.H. Devoret, S.M. Girvin, F. Marquardt and R.J. Schoelkopf, Introduction to quantum noise, measurement and amplification. Rev. Mod. Phys. **82**, 1155-1208 (2010).