Revising A Classical Game Theory Paradox with Quantum Walks

Vikash Mittal

National Tsing Hua University, Hsinchu, TAIWAN

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Overview

- Quantum Walks and Why?

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- Quantum Walks and Why?
- Parrondo's Paradox

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- Quantum Walk and Parrondo's Paradox

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- Quantum Walk and Parrondo's Paradox
- Conclusion and Outlook

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Parrondo's paradox in quantum walks with inhomogeneous coins

Vikash Mittal [1,*] and Yi-Ping Huang [1,2,3,†]
[1]*Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan*
[2]*Physics Division, National Center for Theoretical Sciences, Taipei 10617, Taiwan*
[3]*Institute of Physics, Academia Sinica, Taipei 115, Taiwan*

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Parrondo's paradox, a counterintuitive phenomenon in which two losing strategies combine to produce a winning outcome, has been a subject of interest across various scientific fields, including quantum mechanics. In this study, we investigate the manifestation of Parrondo's paradox in discrete-time quantum walks. We demonstrate the existence of Parrondo's paradox using site- and time-dependent coins without the need for a higher-dimensional coin or adding decoherence to the system. Our results enhance the feasibility of practical implementations and provide deeper insights into the underlying quantum dynamics, specifically the propagation constrained by the interference pattern of quantum walks. The implications of our results suggest the potential for more accessible and efficient designs in quantum transport, broadening the scope and application of Parrondo's paradox beyond conventional frameworks.

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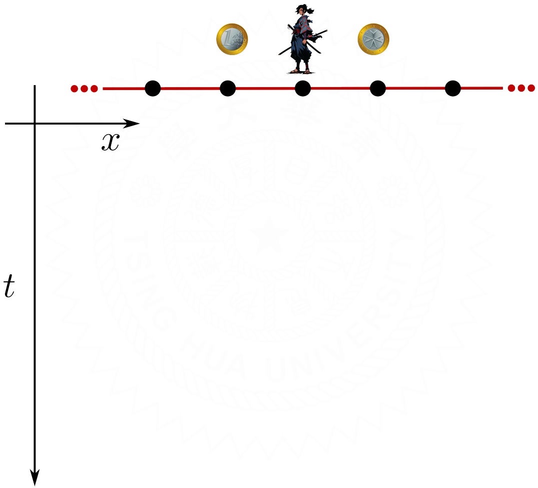
Classical Random Walk



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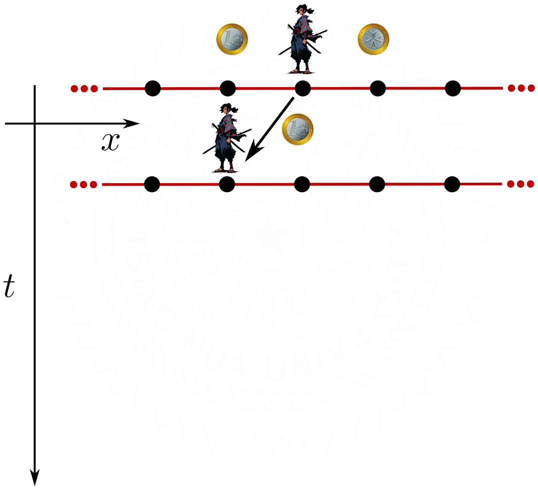

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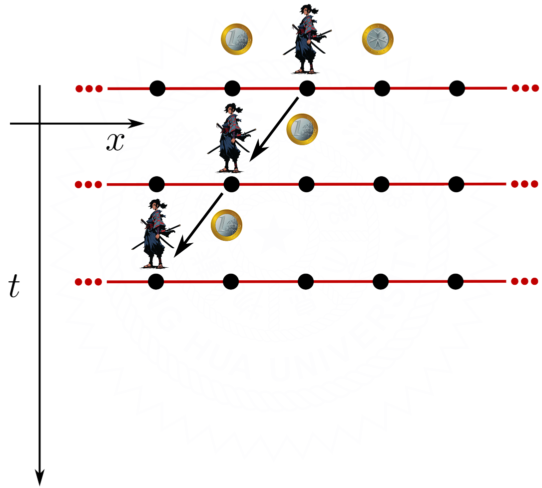

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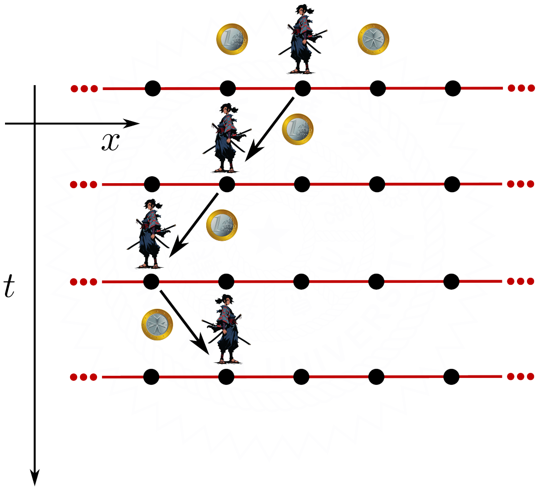

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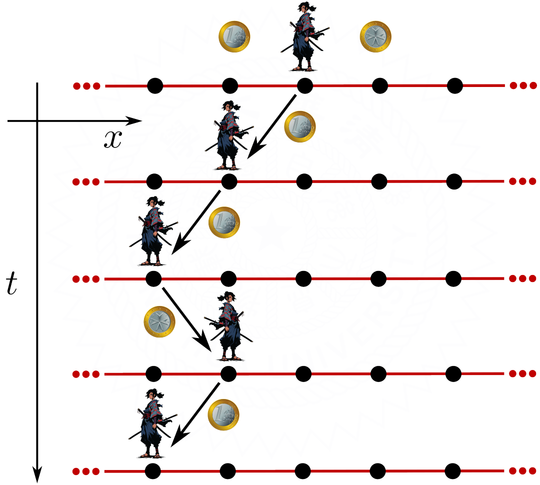

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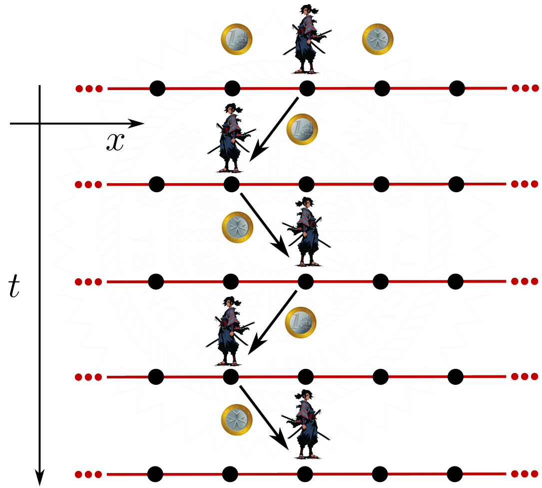

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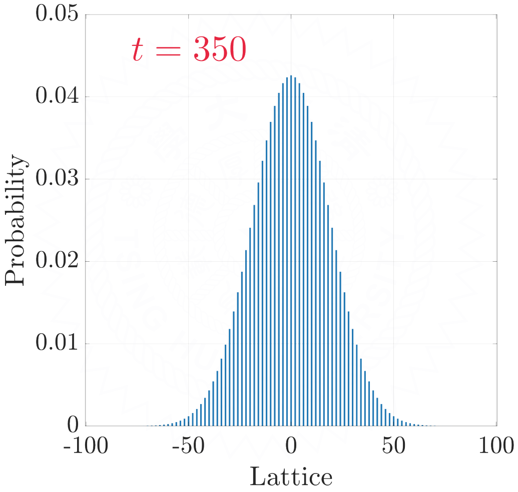
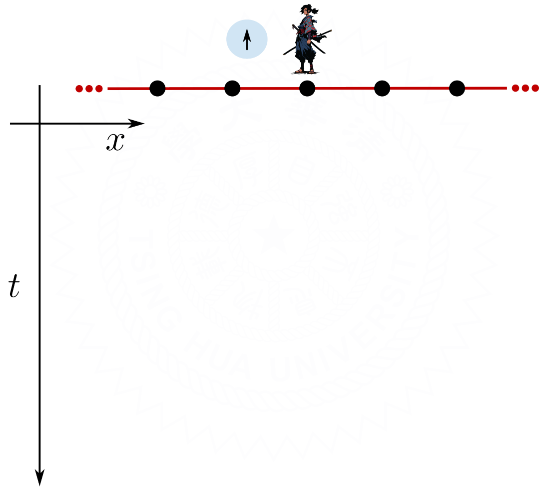
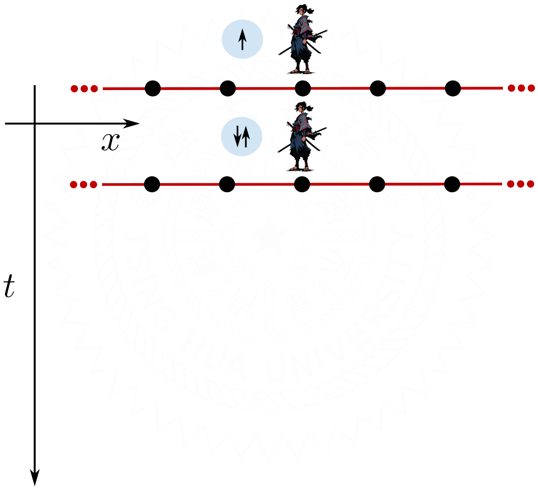


Quantum ~~Random~~ Walk



Quantum ~~Random~~ Walk



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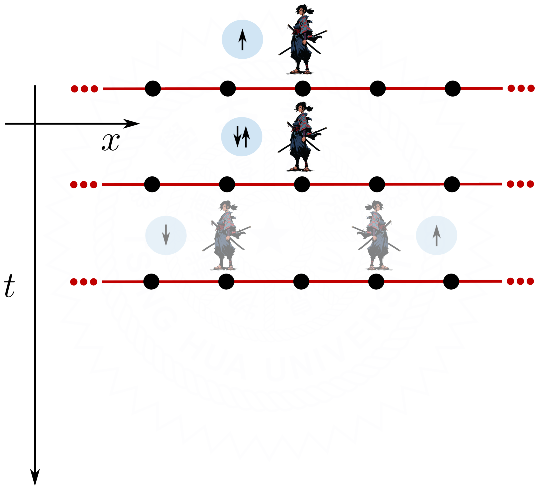

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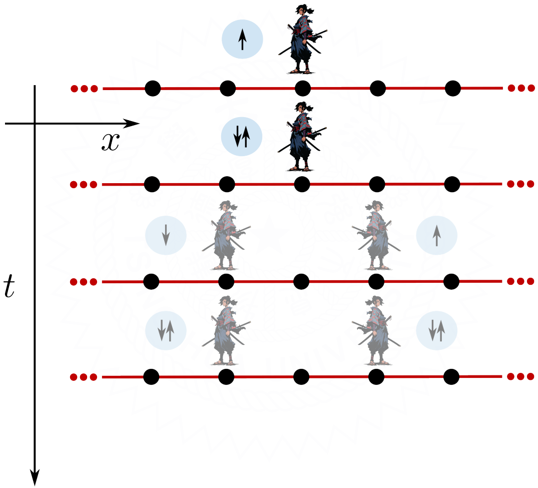

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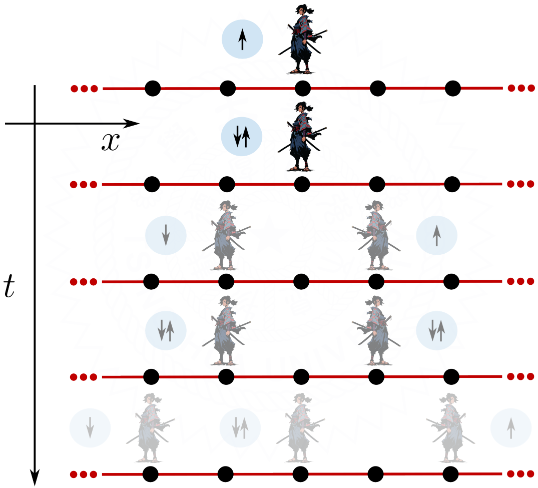

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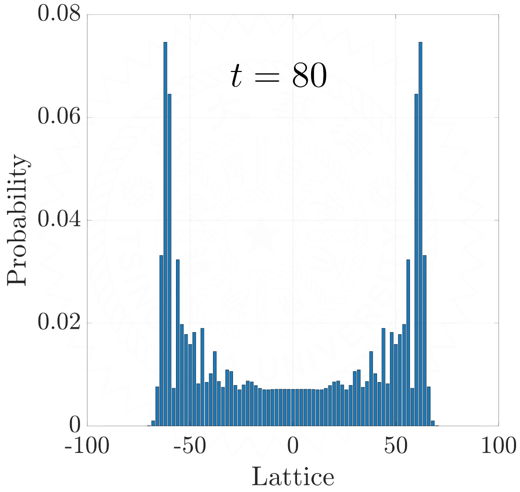

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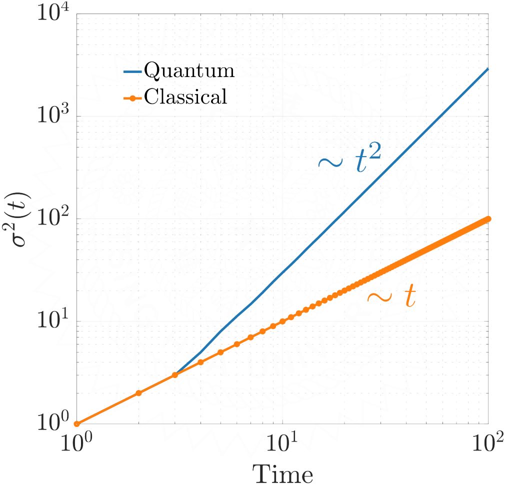

$$T = \sum_x |\uparrow\rangle\langle\uparrow| \otimes |x+1\rangle\langle x| + |\downarrow\rangle\langle\downarrow| \otimes |x-1\rangle\langle x|\,,$$

$$R(\theta) = e^{-i\theta(\cancel{x},\cancel{t})\sigma_y/2} \otimes \mathbb{1}_N,$$

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such that

$$U(\theta) = TR(\theta).$$

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$$|\psi(t)\rangle = (U(\theta))^t \, |\psi(0)\rangle = \sum_j \psi_{\uparrow,x}(t) \, |{\uparrow}\rangle \otimes |x\rangle + \psi_{\downarrow,x}(t) \, |{\downarrow}\rangle \otimes |x\rangle \, ,$$

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$$P(x,t) = |\langle\uparrow| \otimes \langle x|\psi(t)\rangle|^2 + |\langle\downarrow| \otimes \langle x|\psi(t)\rangle|^2 = |\psi_{\uparrow,x}(t)|^2 + |\psi_{\downarrow,x}(t)|^2.$$

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- generation of hybrid-entangled states (Mittal *et al*, 2023),
- generation of robust entanglement (Work in Progress)

Game A

Parrondo's Paradox

Game A

Coin 1

Game A

Coin 1

p_1 $1 - p_1$

Parrondo's Paradox

Game A

Game B

Coin 1

p_1

$1 - p_1$

Parrondo's Paradox

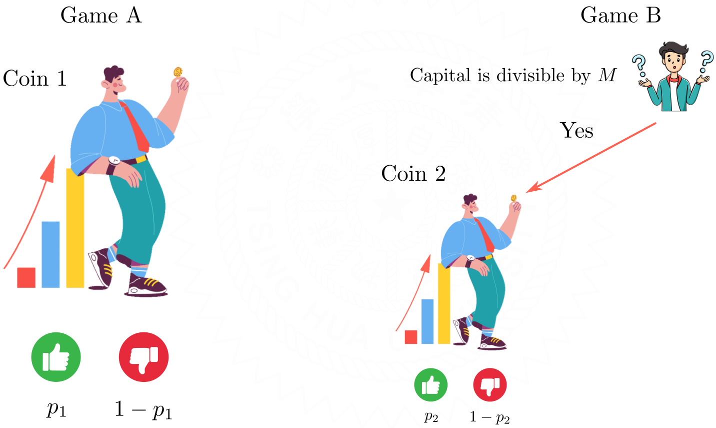
Game A

Coin 1

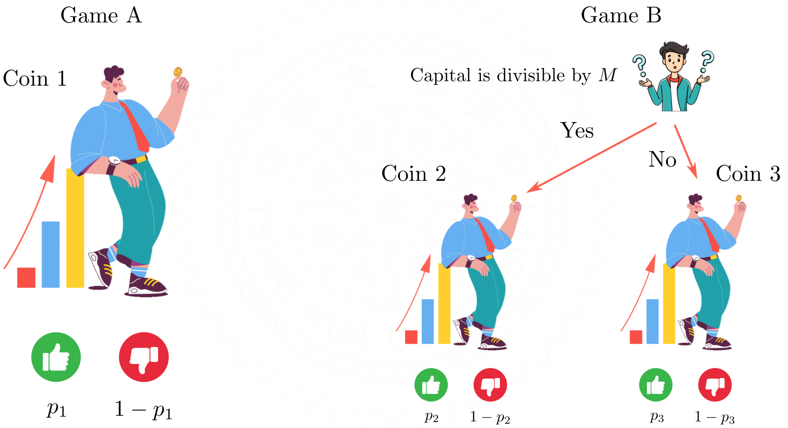
p_1 $1 - p_1$

Game B

Capital is divisible by M


Game A
Coin 1
p_1
$1 - p_1$
Game B
Capital is divisible by M
Yes
Coin 2
p_2
$1 - p_2$

Parrondo's Paradox



$$p_1 = 1/2 - \epsilon, \; p_2 = 1/10 - \epsilon, \quad p_3 = 3/4 - \epsilon, \text{ with } \quad \epsilon = 0.005$$

$$M = 3$$

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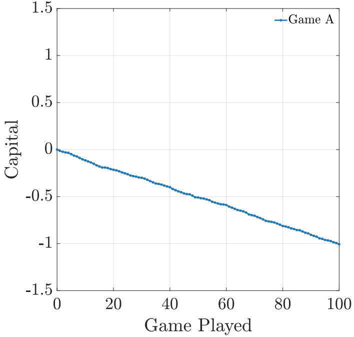

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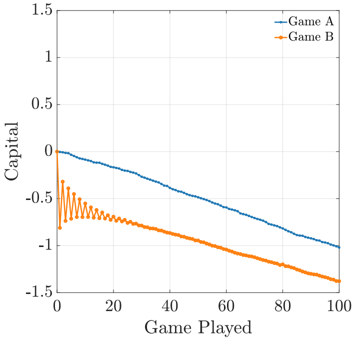
$$M = 3$$



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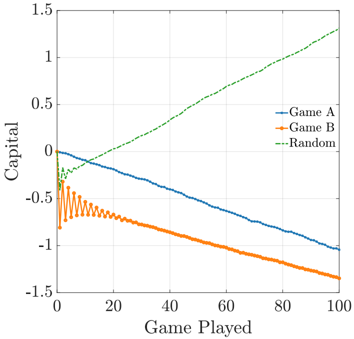

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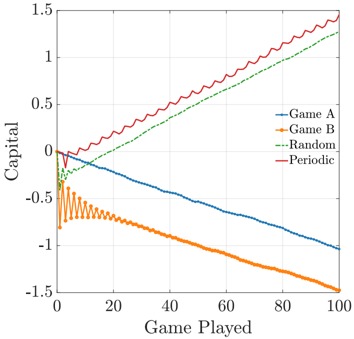
$$M = 3$$

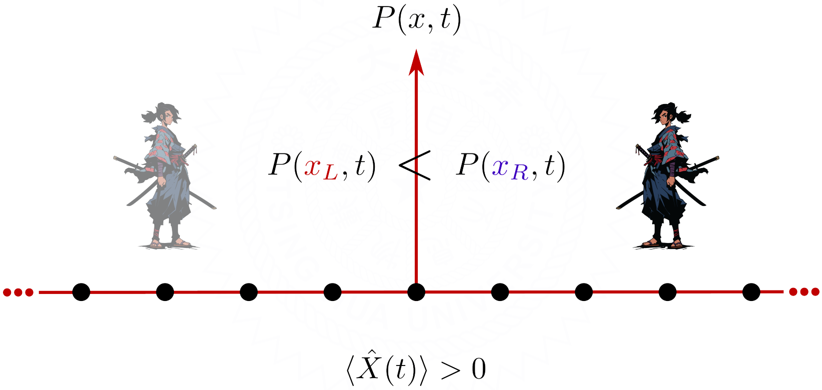


$p_1 = 1/2 - \epsilon, \;\; p_2 = 1/10 - \epsilon, \;\; p_3 = 3/4 - \epsilon, \;$ with $\;\epsilon = 0.005$

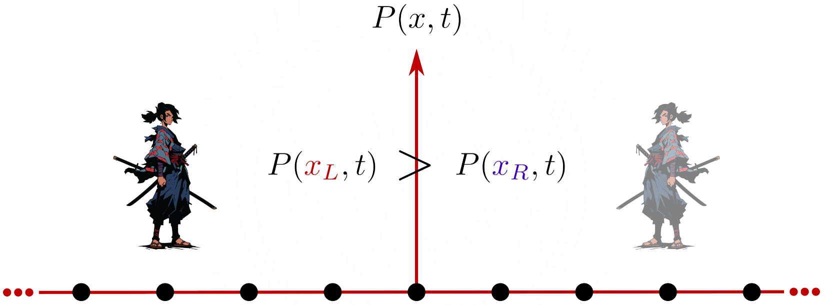
$M = 3$

A combination of losing strategies becomes a winning strategy.

Parrondo's Paradox in QW



Parrondo's Paradox in QW



$$\langle \hat{X}(t) \rangle < 0$$

Site-Dependent Coin

$$\mathcal{C}_A = e^{-i\theta_A \sigma_y/2} \otimes \mathbb{1}_N, \qquad \mathcal{C}_B = \sum_x e^{-i\theta_B(\theta_{B_+},\theta_{B_-};x)\sigma_y/2} \otimes |x\rangle\langle x|$$

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where

$$\theta_B(\theta_{B_+},\theta_{B_-};x) = \frac{\theta_{B_+}(1+\tanh(x)) + \theta_{B_-}(1-\tanh(x))}{2}.$$

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$$\theta_A = \pi/2, \;\; \theta_{B-} = -\pi/8, \;\; \theta_{B+} = \pi/4$$

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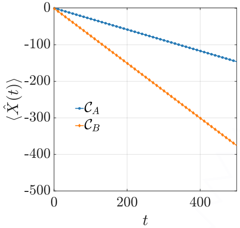
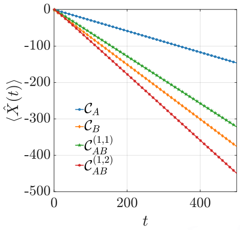
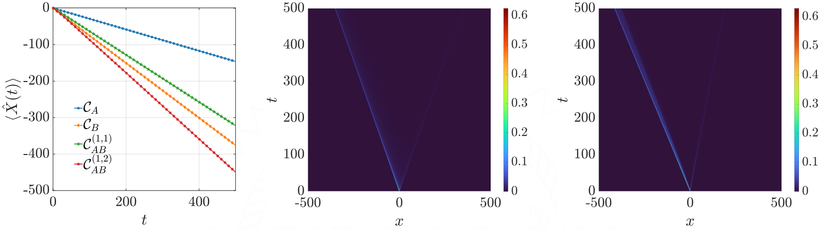
With the initial state

$$|\Psi(0)\rangle = |\downarrow\rangle \otimes |0\rangle \, .$$

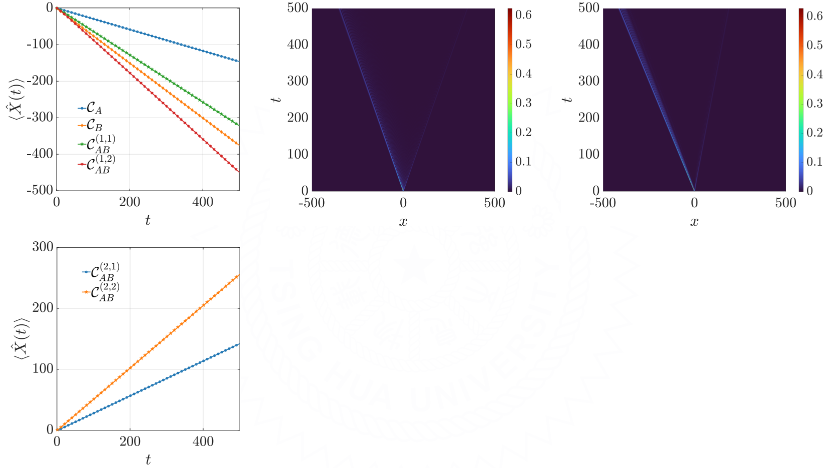

0
-100
-200
-300
-400
-500
0
200
400
t
$\langle \hat{X}(t) \rangle$
$\mathcal{C}_A$
$\mathcal{C}_B$



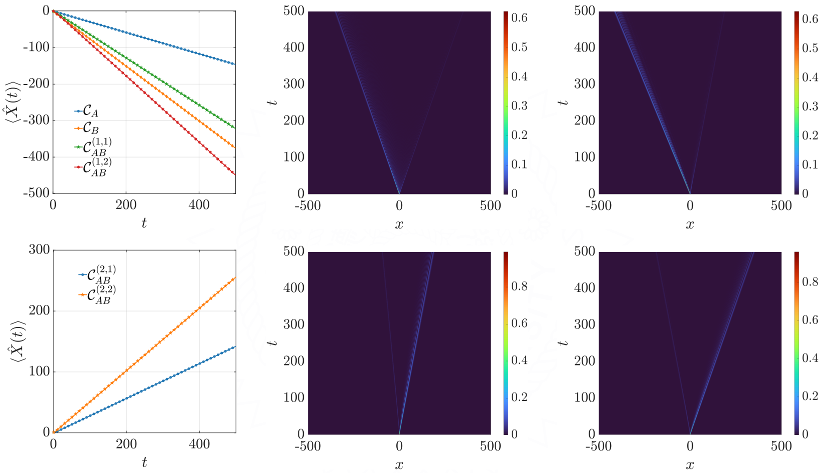
$$\mathcal{C}_{AB}^{(m,n)} \equiv \mathcal{C}_A^m \mathcal{C}_B^n$$



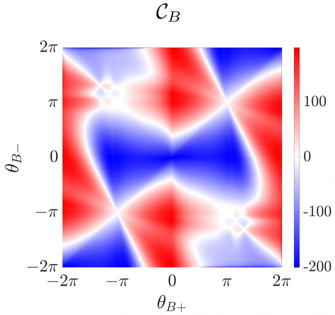
$\langle \hat{X}(t) \rangle$
t
$\mathcal{C}_A$
$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
$\mathcal{C}_{AB}^{(1,2)}$
t
x
t
x

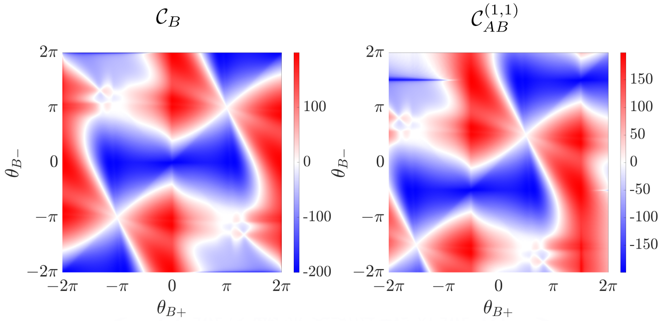


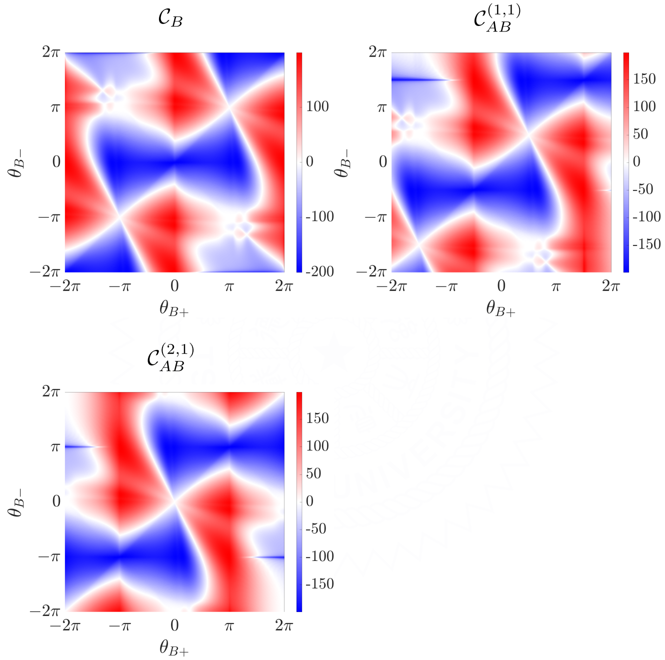
$\langle \hat{X}(t) \rangle$
t
$\mathcal{C}_A$
$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
$\mathcal{C}_{AB}^{(1,2)}$
$\mathcal{C}_{AB}^{(2,1)}$
$\mathcal{C}_{AB}^{(2,2)}$
x

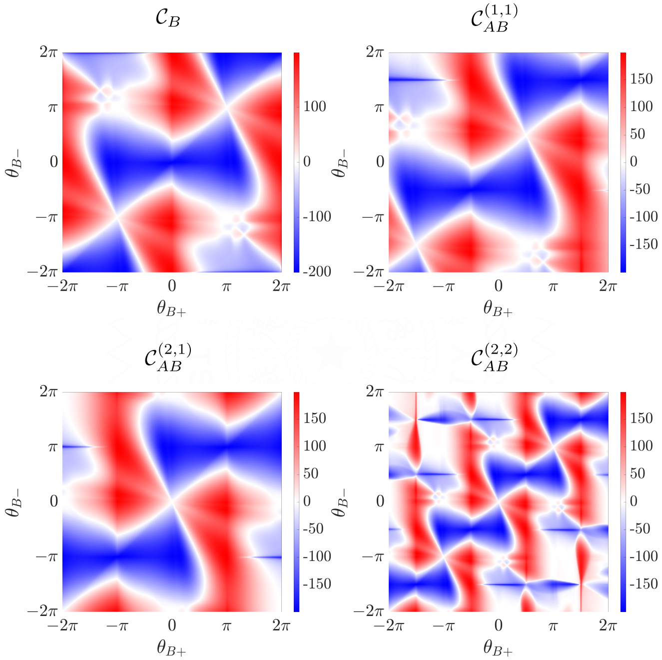


$\langle \hat{X}(t) \rangle$
t
$\mathcal{C}_A$
$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
$\mathcal{C}_{AB}^{(1,2)}$
$\mathcal{C}_{AB}^{(2,1)}$
$\mathcal{C}_{AB}^{(2,2)}$
x


$\mathcal{C}_B$
2π
π
0
$-\pi$
-2π
θ_{B-}
-2π
$-\pi$
0
π
2π
θ_{B+}
100
0
-100
-200


$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
θ_{B-}
θ_{B+}
2π
π
0
$-\pi$
-2π
100
0
-100
-200
150
50
-50
-150


$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
$\mathcal{C}_{AB}^{(2,1)}$
θ_{B-}
θ_{B+}
2π
π
0
$-\pi$
-2π
100
0
-100
-200
150
50
-50
-150



$\mathcal{C}_B$
$\mathcal{C}_{AB}^{(1,1)}$
$\mathcal{C}_{AB}^{(2,1)}$
$\mathcal{C}_{AB}^{(2,2)}$
θ_{B-}
θ_{B+}
-2π
$-\pi$
0
π
2π
-200
-150
-100
-50
50
100
150

Time-Dependent Coin

$$\mathcal{C}(q(t), \alpha(t), \beta(t)) = \begin{pmatrix} \sqrt{q(t)} & \sqrt{1-q(t)}e^{i\alpha(t)} \\ \sqrt{q(t)}e^{i\beta(t)} & -\sqrt{q(t)}e^{i(\alpha(t)+\beta(t))} \end{pmatrix}$$

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$$\mathcal{C}_A(t) = \mathcal{C}(1/2,\alpha(t),0) \quad \mathcal{C}_B(t) = \mathcal{C}(1/2,0,\beta(t)).$$

with

$$\alpha(t) = \frac{2\pi}{T}t = \beta(t)$$

Time-Dependent Coin

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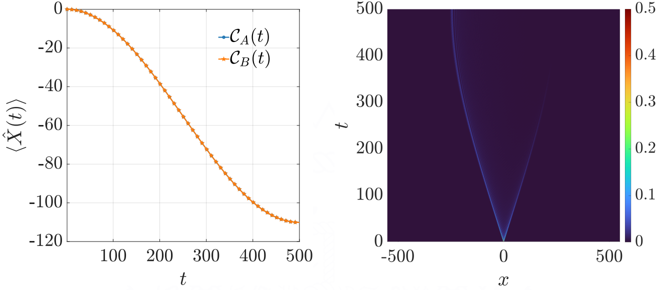
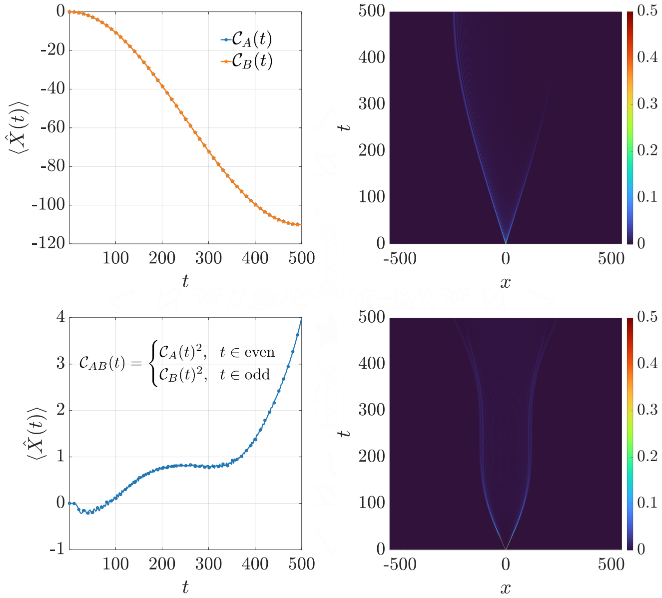
with

$$\alpha(t) = \frac{2\pi}{T}t = \beta(t)$$

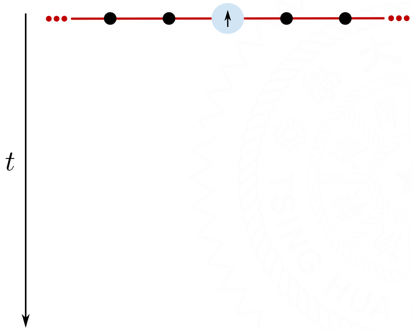
$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i\,|\downarrow\rangle)\otimes|0\rangle\,.$$



0
-20
-40
-60
-80
-100
-120
100
200
300
400
500
t
$\langle \hat{X}(t) \rangle$
$\mathcal{C}_A(t)$
$\mathcal{C}_B(t)$
500
400
300
200
100
0
-500
0
500
x
t
0.5
0.4
0.3
0.2
0.1
0


$\mathcal{C}_A(t)$
$\mathcal{C}_B(t)$
$\langle \hat{X}(t) \rangle$
t
x
$\mathcal{C}_{AB}(t) = \begin{cases} \mathcal{C}_A(t)^2, & t \in \text{even} \\ \mathcal{C}_B(t)^2, & t \in \text{odd} \end{cases}$
0
-20
-40
-60
-80
-100
-120
100
200
300
400
500
-500
0.5
0.4
0.3
0.2
0.1
4
3
2
1
-1

Motivation from SSQW (Kitagawa (2012), Asboth (2012))



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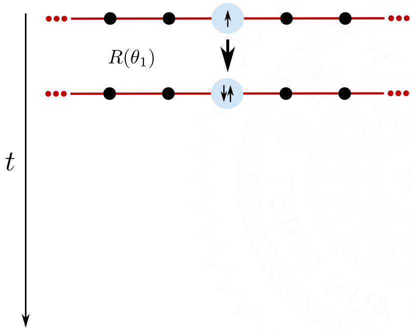
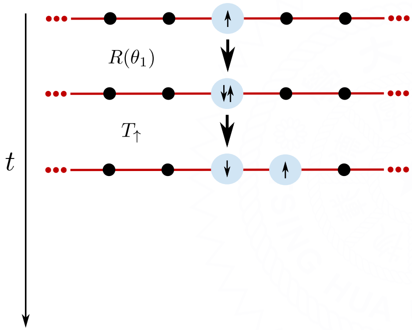


Motivation from SSQW (Kitagawa (2012), Asboth (2012))



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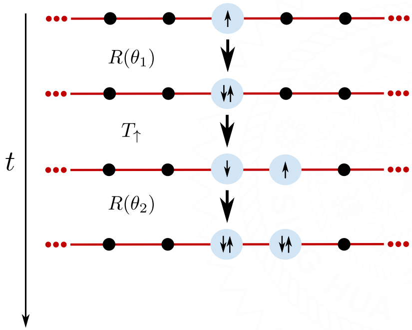

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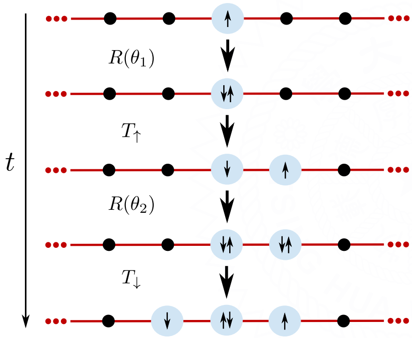

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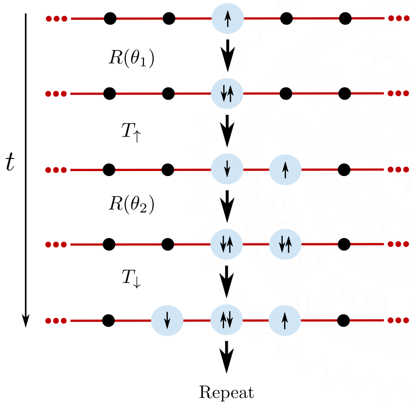
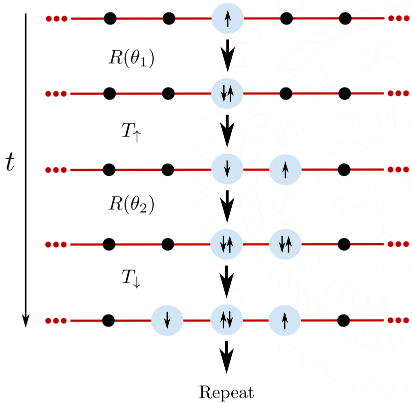
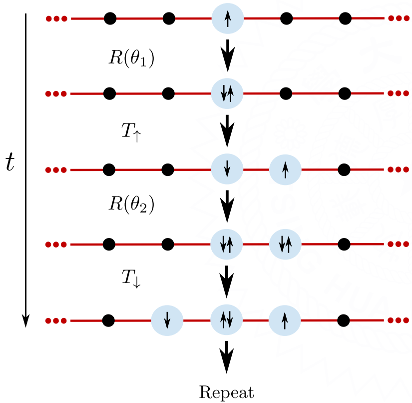


Motivation from SSQW (Kitagawa (2012), Asboth (2012))



$$U_{\mathrm{SS}}(\theta_1, \theta_2) = T_\downarrow R(\theta_2) T_\uparrow R(\theta_1)$$

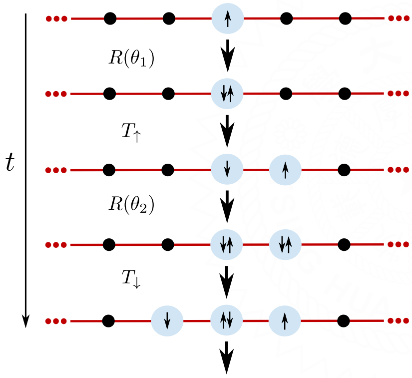
Motivation from SSQW (Kitagawa (2012), Asboth (2012))



$$U_{\text{SS}}(\theta_1, \theta_2) = T_\downarrow R(\theta_2) T_\uparrow R(\theta_1)$$

$$U_{\text{SS}} = e^{-iHt} \longrightarrow H_k(\theta_1, \theta_2)$$

Motivation from SSQW (Kitagawa (2012), Asboth (2012))



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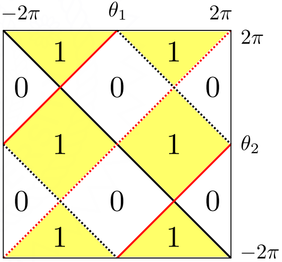

Summary and Outlook

Conclusion

- quantum version of Parrondo's paradox,

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- without the use of higher-dimensional coins, or decoherence in the system,

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Outlook

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- more generalized strategies
- different criterion for the game to win and realization of Parrondo's paradox

THANK YOU SO MUCH

Thanks for listening.

Please feel free to contact me if you have any questions/doubts.

vikashmittal.iiser@gmail.com