# Neutrino Mass Mechanisms: Dirac and Majorana

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**Understanding the Universe Through Neutrinos**
**ICTS Bangalore**
**03rd May 2024**

# Outline

- Lepton Number symmetry and its Breaking Pattern
  - : Dirac and Majorana Neutrinos
- Effective Operators for Neutrino Mass
  - Weinberg Operator
  - Generalized Weinberg Operators
- Ultra Violet Completions: Dirac and Majorana
  - Tree Level Models
  - Loop Models
- Some Interesting Connections

# Symmetries of SM

- **Symmetries in SM:** Spacetime and Internal
    - **Internal Symmetries:** Gauge Symmetries
        - SM based on $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry
- However only $SU(3)_c \otimes U(1)_{em}$ remains conserved
- **Accidental Symmetries:** Lepton number $U(1)_L$ and Baryon number $U(1)_B$ are automatically conserved in SM
- $U(1)_B$ and $U(1)_L$ conservation have important consequences
    - **Baryon number conservation: Proton stability**
    - **Lepton number conservation: Dirac neutrinos**

# Neutrinos: Dirac or Majorana

**What experiments tell?**

- Debate about neutrino nature: As old as neutrinos themselves
- Discerning their nature from experiments: A difficult task
  - V-A nature of Standard Model: All observables sensitive to nature of neutrinos suppressed by powers of $m\nu$
- Still some potentially feasible processes:
  - **Neutrinoless Double Beta Decay ($0\nu2\beta$)**
  - **LHC signatures of lepton number violation**
  - **KATRIN measures $m\nu$ + no $0\nu2\beta$**
- Current Status: No experimental or observational evidence/hint in favor of either Dirac or Majorana neutrinos

# Are Majorana Neutrinos Natural?

**Spacetime Symmetry: Transformation under Poincaré group**

- Majorana neutrinos more natural: In what sense?
- **Current understanding: Under Poincaré group**
  - **Majorana fermions:** Two-component fundamental irreducible spinorial representations
  - **Dirac fermions:** Four component reducible spinorial representations
- From Poincaré symmetry point of view: Majorana fermions are more fundamental
- Dirac fermions: Can be thought of as two Majorana fermions degenerate in mass
- All fermions should be Majorana and all scalars should be real scalars
- Spacetime symmetry: Not the only symmetry conserved in nature

# Are Majorana Neutrinos Natural?

## Conserved Internal Symmetries

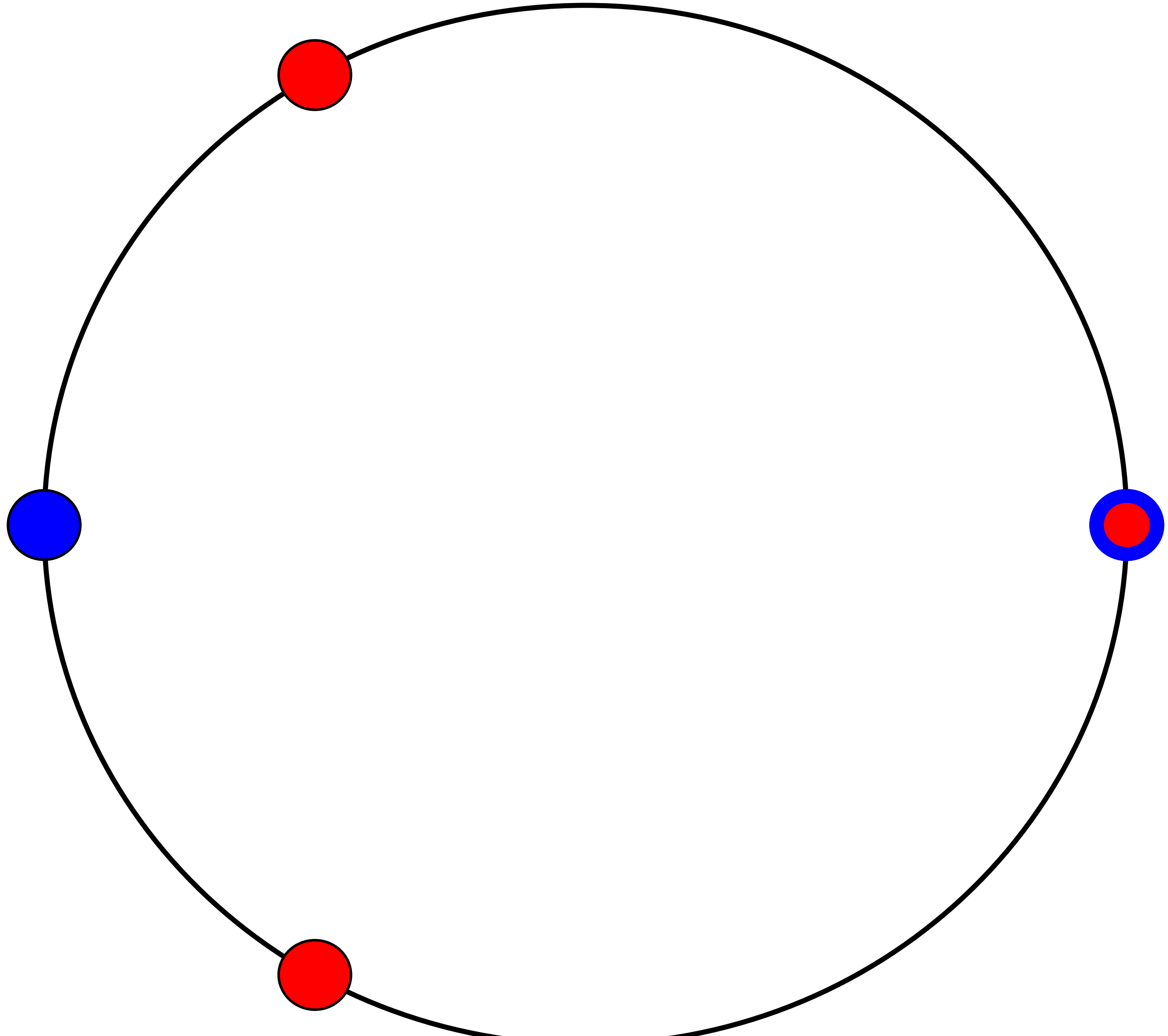
- Spacetime symmetry: Not the only symmetry conserved in nature
- Electromagnetism $U(1)_{EM}$ and Color $SU(3)_C$ Symmetries seem to be conserved
  - Majorana mass term: Violates both $U(1)_{EM}$ and $SU(3)_C$
  - Conserved Internal Symmetries: Charged leptons and quarks are forced to be Dirac particles
- Dirac/Majorana nature: Take into account all conserved symmetries
- In absence of any other hitherto unknown conserved symmetries:
  - Dirac/Majorana nature depends on the $U(1)_L$ breaking pattern

# Nature of Neutrinos

**Lepton Number Breaking Pattern** *[Hirsch, RS, Valle '17]*

- If $U(1)_L$ is conserved: Neutrinos are Dirac
  - Accidental Symmetry of SM: New physics beyond SM need not conserve it
- If $U(1)_L$ is broken: Symmetry breaking pattern will determine the nature of neutrinos
  - $U(1)_L$ symmetry only admits $Z_M$ subgroups i.e. cyclic groups of m elements
  - If x is a non-identity group element of $Z_M$, then $x^{M+1} \equiv x$
  - The $Z_M$ groups only admit one-dimensional irreducible representations
  - Conveniently represented by using the n-th roots of unity, $\omega = Exp[2\pi I/M]$ where $\omega^M = 1$

# Residual Subgroups



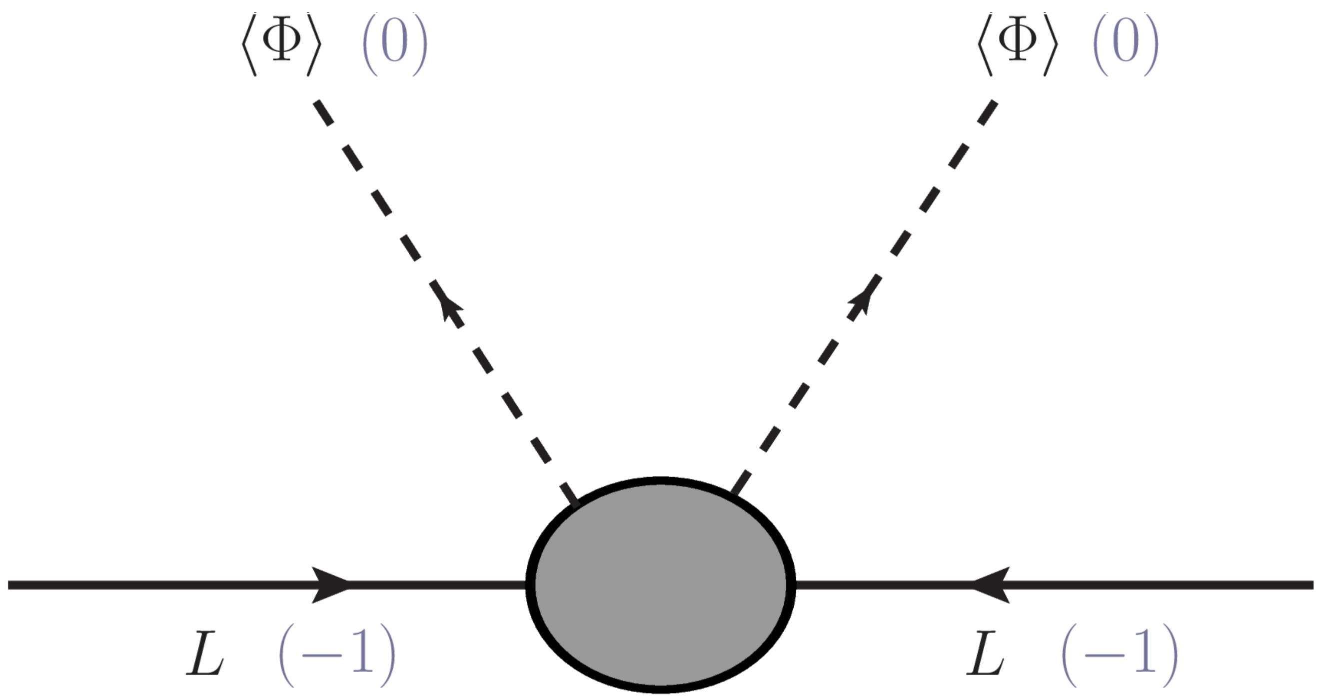
# Nature of Neutrinos

**Lepton Number breaking pattern** *[Hirsch, RS, Valle , PLB '18]*

- $U(1)_L \longrightarrow Z_M$ subgroup with neutrinos transforming non-trivially under $Z_M$
  - $U(1)_L \longrightarrow Z_M \equiv Z_{2N+1}$ where $N \geq 1$
    - **Neutrinos are always Dirac!!!**
  - $U(1)_L \longrightarrow Z_M \equiv Z_{2N}$ where $N \geq 1$
    - **Neutrinos can be either Dirac or Majorana**
- For $U(1)_L \longrightarrow Z_{2N}$ case one can make further broad classification
  - If $L_i \nsim \omega^N$ under $Z_{2N} \longrightarrow$ **Neutrinos are Dirac!!!**
  - If $L_i \sim \omega^N$ under $Z_{2N} \longrightarrow$ **They are Majorana**
- From symmetry point of view: **Dirac neutrinos are more natural !!!**

# Majorana Neutrinos: Weinberg Operator

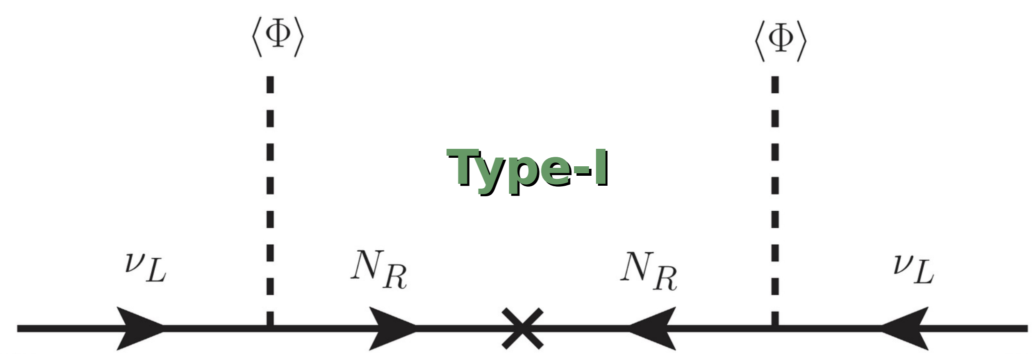
- Weinberg Operator: Provides "effective" description of how Majorana neutrino mass can be generated **[S. Weinberg '79]**

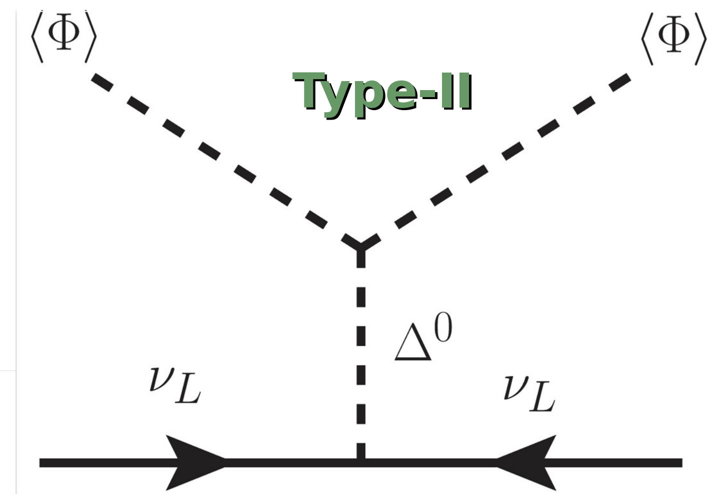


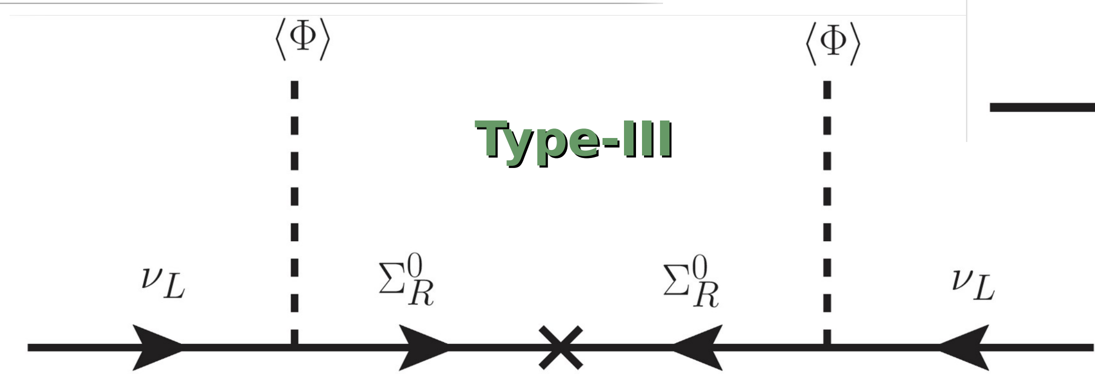

- Breaks $U(1)_L \to \mathcal{Z}_2$
  - Both reps of $\mathcal{Z}_2$ satisfy the Majorana condition
  - **All UV completions of Weinberg operator will always lead to Majorana neutrinos**

# Majorana Neutrino Mass Mechanisms

- Majorana neutrinos: Many elegant mechanism to explain smallness of neutrino mass
  - Seesaw Mechanisms: **[Minkowski '77, Yanagida '79, Mohapatra-Senjanovic '80, Schechter-Valle '80]**




  - Loop Mechanisms **[Zee '80, Babu '88, Ma '06]**

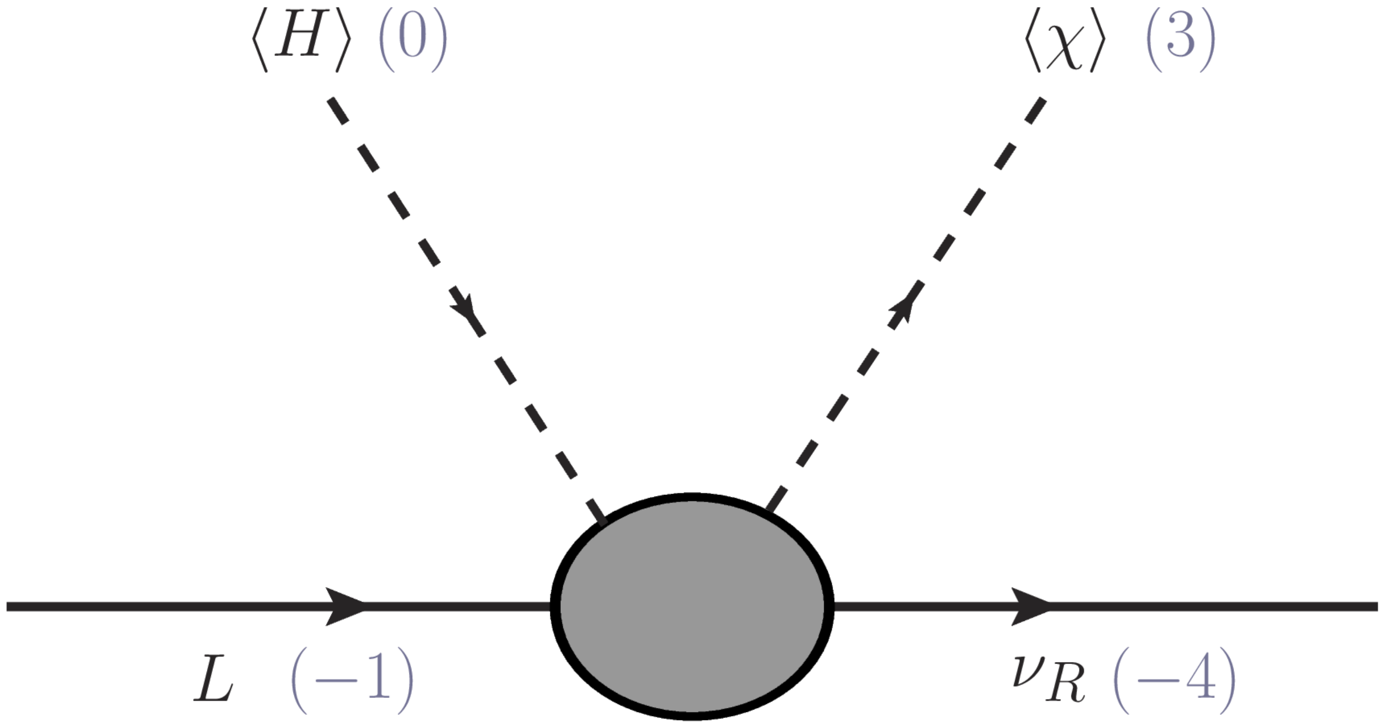
# Lepton Number of Right Handed Neutrinos

- Baryon and Lepton number of all SM particles are fixed
- What is the Lepton number of Right Handed Neutrinos?
  - B and L symmetries are anomalous
  - Only B – L combination can be anomaly free if lepton number carrying right handed neutrinos are added to SM
- Vector solution : Add three right handed neutrinos with B–L charges of (-1,-1,-1)
- New Chiral Solution: Right handed neutrinos with B – L charges of (-4,-4,5) **[Ma, RS PLB '14, Pollard, Ma, RS, Reza PLB '15]**
  - Yukawa term $L\tilde{\Phi}\nu_R$ automatically forbidden
  - Paves way for "naturally small" Dirac neutrino masses: **Dirac neutrino mass mechanisms** **[Several Works]**

# Generalized Weinberg Operator

- Neutrino Mass can be generated at dim-5 level

  **[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,RS '18,'19]**



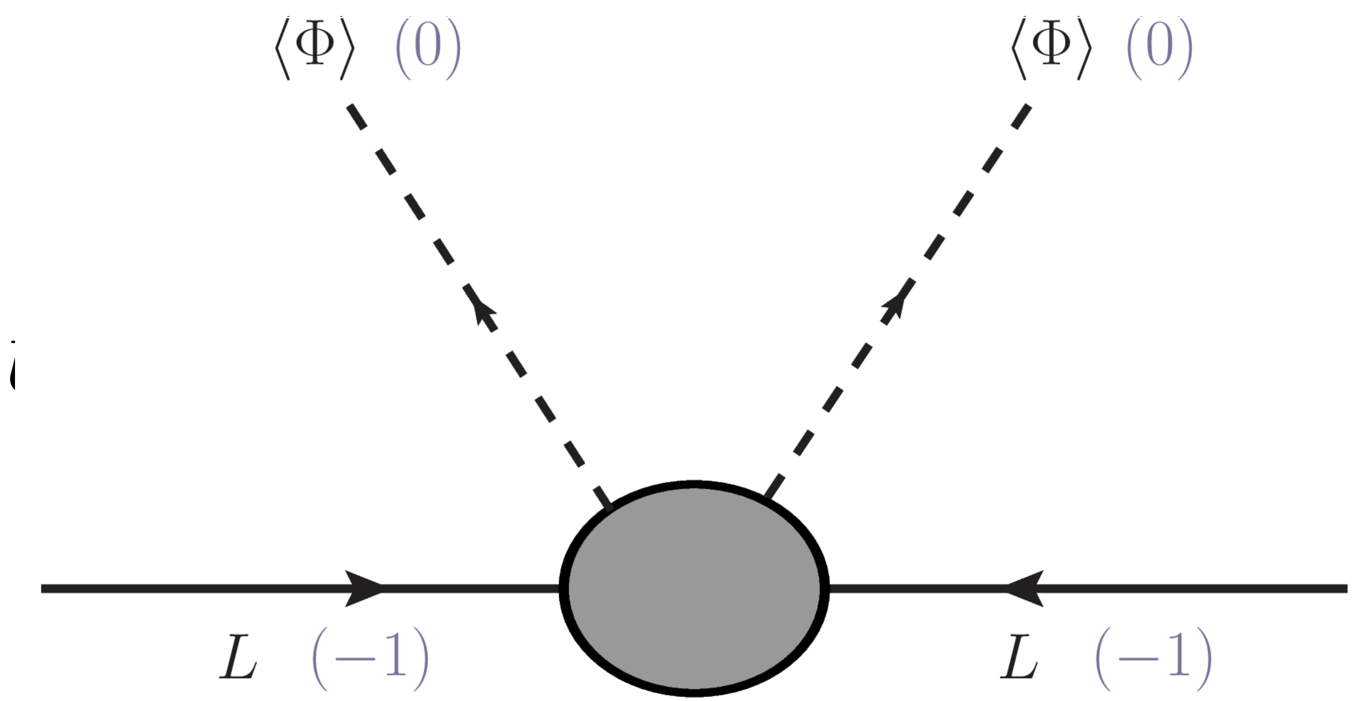

- Since $\chi \sim 3$, its vev breaks $U(1)_{B-L} \to \mathcal{Z}_{3m}; m \in \mathbb{Z}^+$
  - The exact residual subgroup depends on UV completion

# Majorana Neutrinos: Tree Level UV Completion of Weinberg Operator

- The Weinberg Operator $\frac{1}{\Lambda}\bar{L}^c \otimes \Phi \otimes \Phi \otimes L$

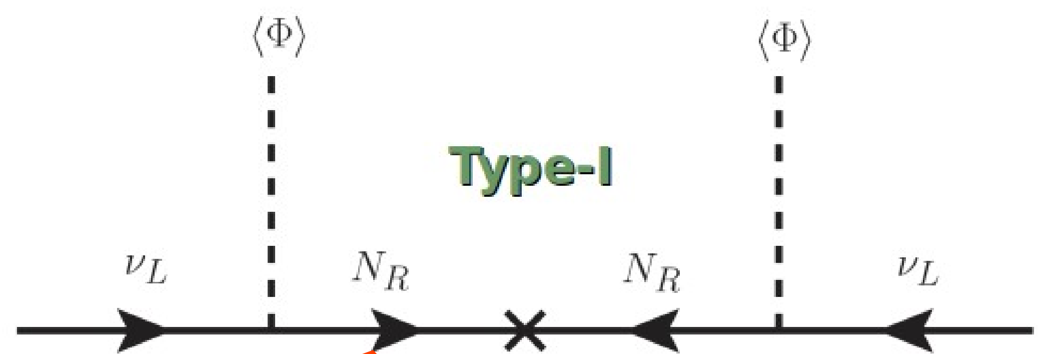
**[S. Weinberg '79]**

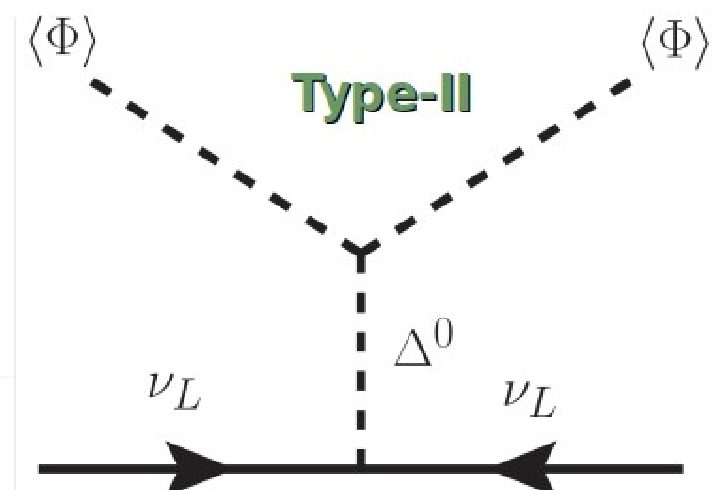
- Its a Dim-5 operator and hence nonrenomalizable
- Many Possible Ultra Violet completions
- Lead to renormalizable theories with naturally small Majorana neutrino masses



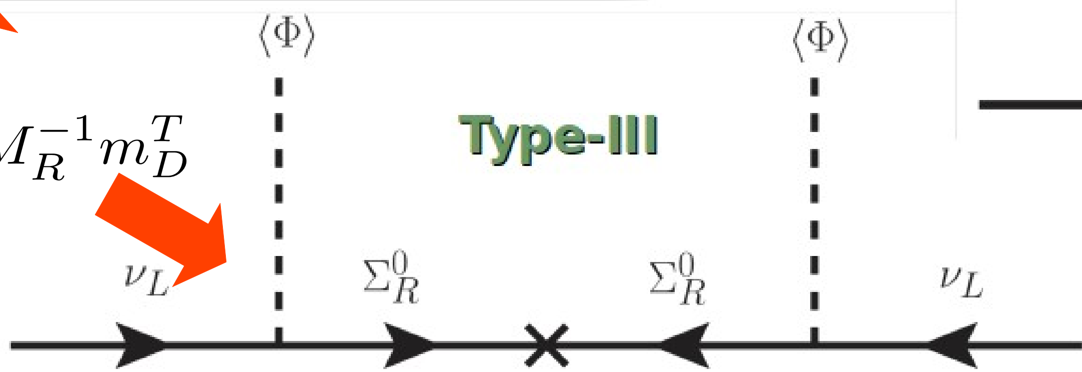
# Tree Level UV Completions: Seesaw Mechanisms

$$\underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{1} \otimes \underbrace{\Phi \otimes L}_{1}}_{\text{Type I}} \, , \quad \underbrace{\underbrace{\bar{L}^c \otimes L}_{3} \otimes \underbrace{\Phi \otimes \Phi}_{3}}_{\text{Type II}} \, , \quad \underbrace{\underbrace{\bar{L}^c \otimes \Phi}_{3} \otimes \underbrace{\Phi \otimes L}_{3}}_{\text{Type III}}$$




$$(m_\nu)_{ij} = m_D M_R^{-1} m_D^T$$
$$m_D = y_{ij} v$$




$$(m_\nu)_{ij} = (Y_\Delta)_{ij} \frac{v_\Delta}{2}$$
$$v_\Delta = \frac{\mu v^2}{\sqrt{2} M_\Delta^2}$$

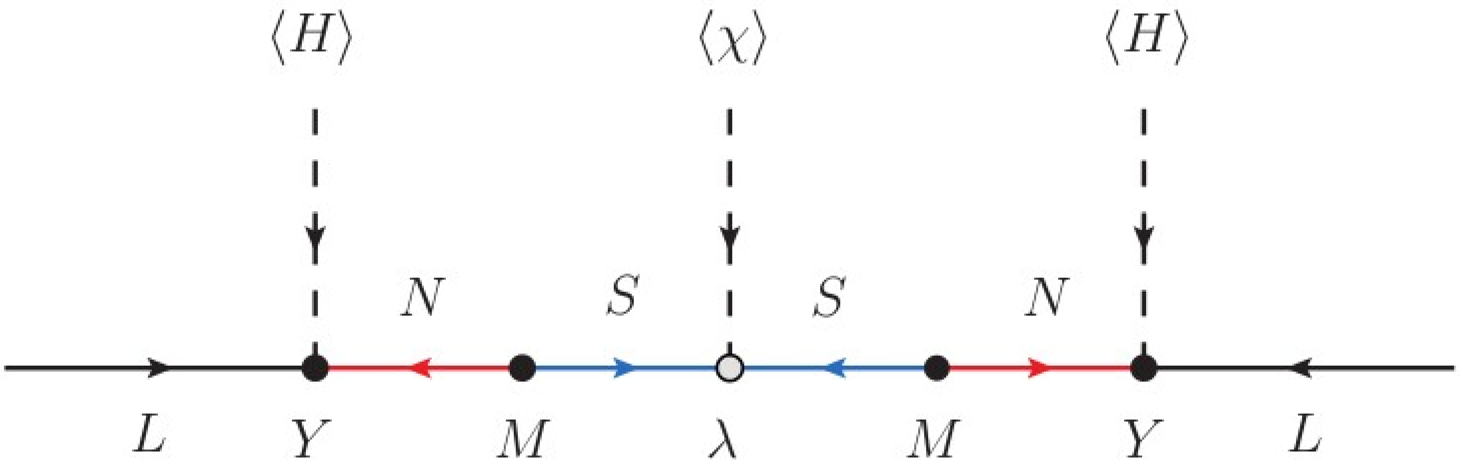


**Ma PRL '98; S.S.Chulia, RS, J.W.F.Valle PLB '18**

# Tree Level UV Completions: Low Scale Seesaw Mechanisms

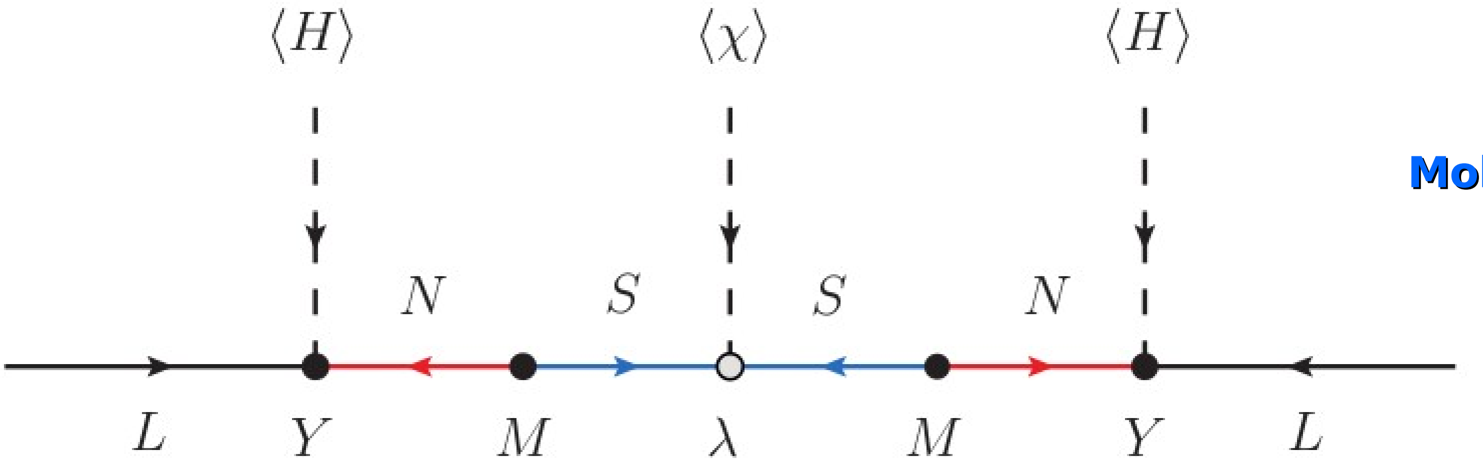
- Low scale seesaws with mediators within TeV range are also possible
- Inverse Seesaw

Mohapatra, Valle '86




| Fields | $SU(2)_L \otimes U(1)_Y$ | $U(1)_{B-L} \to \mathbb{Z}_2$ | Fields | $SU(2)_L \otimes U(1)_Y$ | $U(1)_{B-L} \to \mathbb{Z}_2$ |
|---|---|---|---|---|---|
| $L$ | $(\mathbf{2}, -1/2)$ | $-1 \to -1$ | | | |
| $N$ | $(\mathbf{1}, 0)$ | $1 \to -1$ | $S$ | $(\mathbf{1}, 0)$ | $-1 \to -1$ |
| $H$ | $(\mathbf{2}, 1/2)$ | $0 \to 1$ | $\chi$ | $(\mathbf{1}, 0)$ | $2 \to 1$ |

# Inverse Seesaw Mechanism



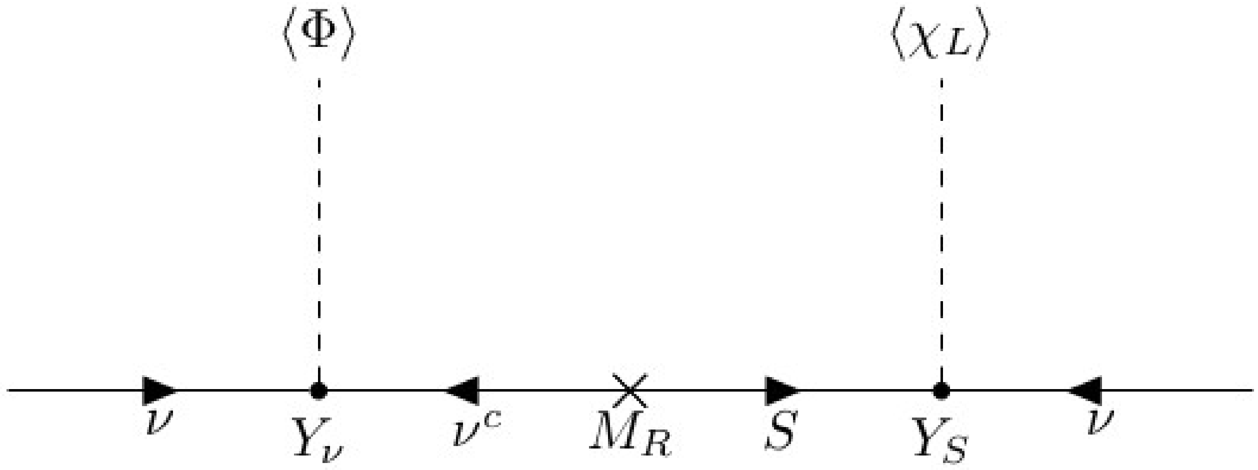

**Mohapatra, Valle '86**

$$\mathcal{L}_{\text{Maj}} = Y\,\bar{L}^c\,\widetilde{H}\,N + \lambda\,\bar{S}^c\,\chi\,S + M\,\bar{S}^c\,N + \text{h.c.}$$

$$\mathcal{L}_m = \begin{pmatrix}\bar{\nu}^c & \bar{N}^c & \bar{S}^c\end{pmatrix} \begin{pmatrix} 0 & Y\,v & 0 \\ Y^T\,v & \mu' & M^T \\ 0 & M & \mu \end{pmatrix} \begin{pmatrix} \nu \\ N \\ S \end{pmatrix} .$$

$$m_\nu = Y^2\,\frac{v^2\mu}{\mu\,\mu' - M^2}$$

# Linear Seesaw Mechanism




| | $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $\nu^c$ | $S$ | $\Phi$ | $\chi_L$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{SU(3)_C}$ | $\mathbf{3}$ | $\mathbf{3}^*$ | $\mathbf{3}^*$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathrm{SU(2)_L}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{2}$ |
| $\mathrm{U(1)_Y}$ | $\frac{1}{6}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $1$ | $0$ | $0$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $\mathrm{U(1)_L}$ | $0$ | $0$ | $0$ | $1$ | $-1$ | $-1$ | $1$ | $0$ | $-2$ |

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D & M_L \\ m_D^T & 0 & M_R \\ M_L^T & M_R^T & 0 \end{pmatrix}$$

$$M_L = \frac{Y_S^{ij} v_\chi}{\sqrt{2}} \qquad m_D = \frac{Y_\nu^{ij} v_\Phi}{\sqrt{2}}$$

$$m_{\text{light}} = m_D (M_L M_R^{-1})^T + (M_L M_R^{-1}) m_D{}^T$$

$$-\mathcal{L}_{\text{Yuk}} = Y_\nu^{ij} L_i^T C \nu_j^c \Phi + M_R^{ij} \nu_i^c C S_j + Y_S^{ij} L_i^T C S_j \chi_L + \text{h.c.}$$



**Akhmedov, Lindner, Schnapka, J.W.F. Valle PLB '95**

# Majorana Neutrinos: Generalized Weinberg Operators

- The Weinberg Operator is merely the lowest dimensional effective operator for Majorana neutrino masses.
    - Involves only SM fields
- Generalized Weinberg operators **S.S.Chulia, RS, J.W.F.Valle PLB '18**
    - Involving only SM fields

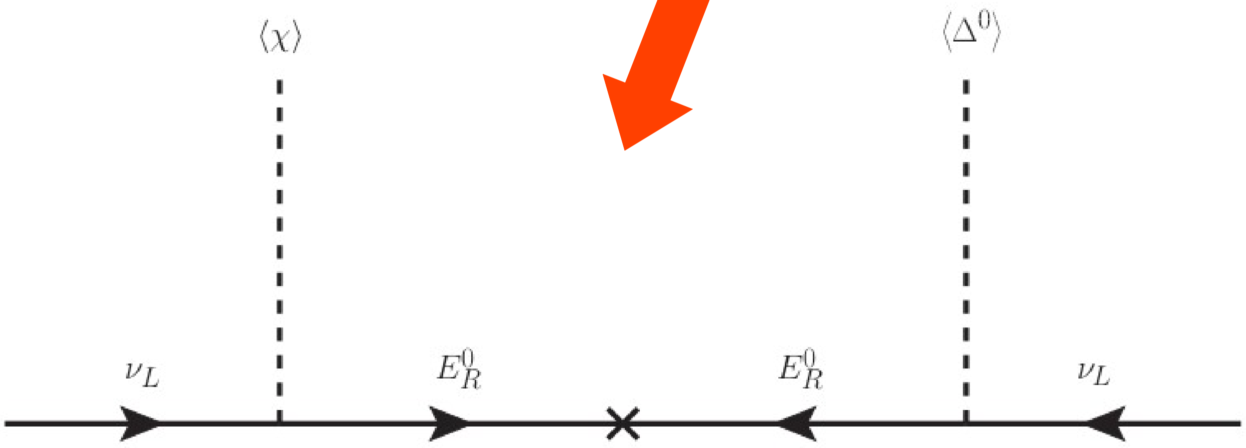
$$\frac{1}{\Lambda^{2n+1}}\,\bar{L}^c\Phi^2\left(\Phi^\dagger\Phi\right)^n L, \quad n\in\{0,1,2,3...\}$$

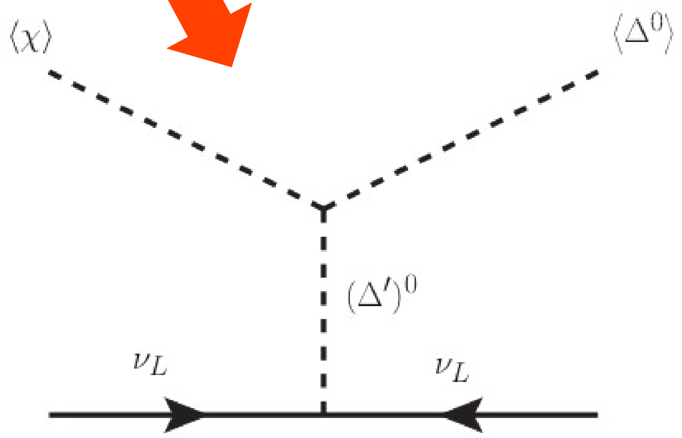
- Also involving beyond SM scalars

$$\mathcal{L}_M = \frac{\mathcal{C}_M}{\Lambda^{m+n-1}}\,\bar{L}^c\,L\,\Phi^{(m)}\,\sigma^{(n)} + \mathrm{h.c.}$$

# UV completion of Generalized Weinberg Operators

$$\underbrace{\underbrace{\bar{L}^c \otimes \chi}_{2} \otimes \underbrace{\Delta \otimes L}_{2}}_{\text{Type I like}} \;, \qquad \underbrace{\underbrace{\bar{L}^c \otimes L}_{3} \otimes \underbrace{\chi \otimes \Delta}_{3}}_{\text{Type II like}}$$

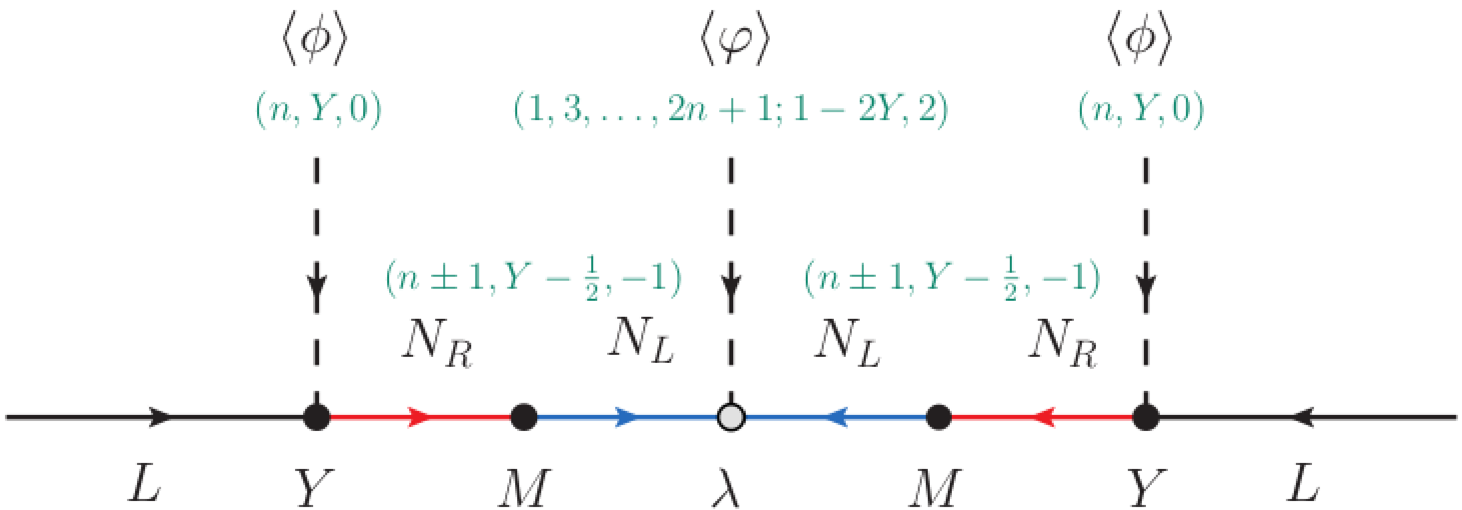


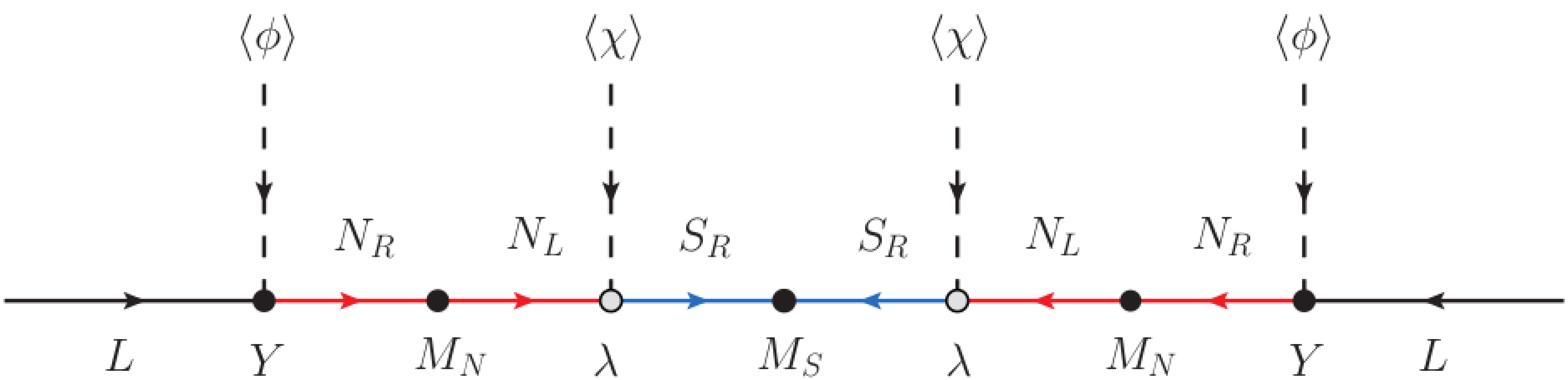






S.S.Chulia, RS, J.W.F.Valle PLB '18

# UV completion of Generalized Weinberg Operators



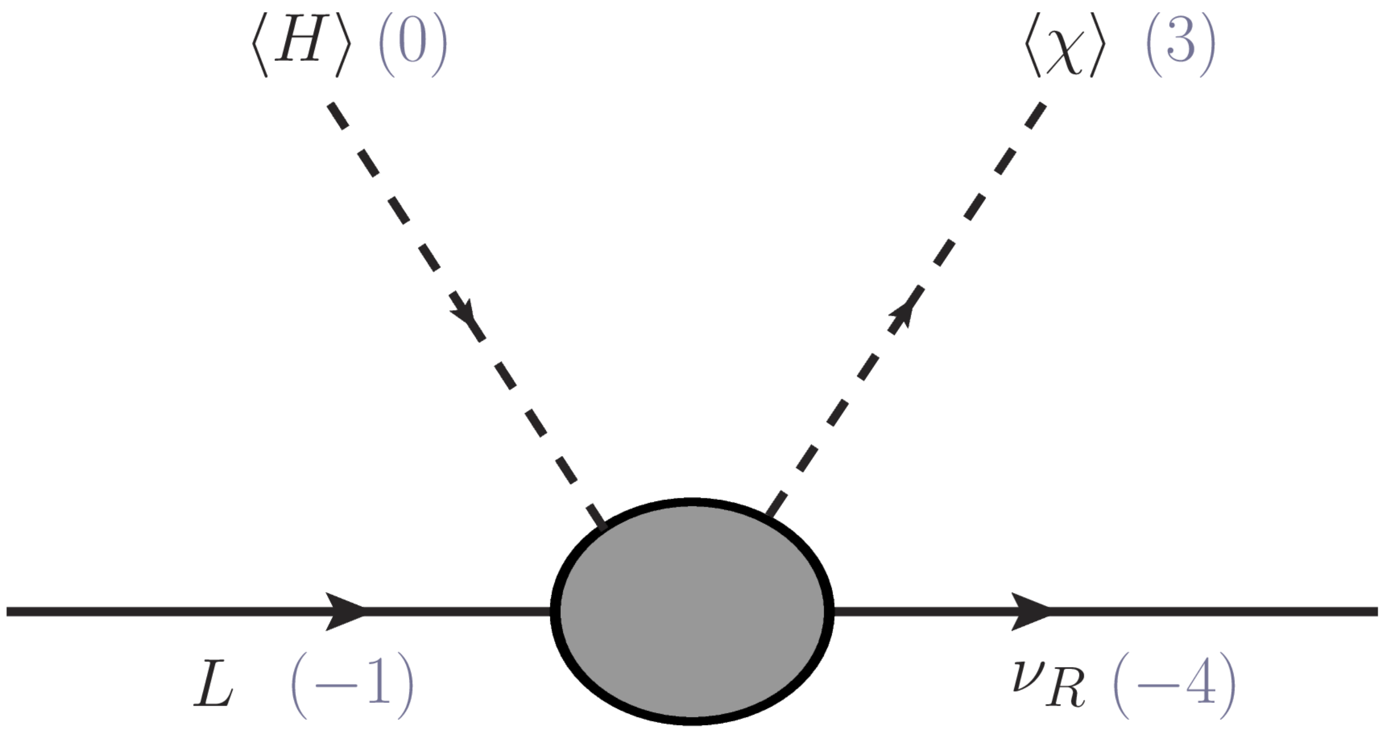






**S.S.Chulia, RS, A. Vicente JHEP ‘21**

# Generalized Weinberg Operator: Dirac Neutrinos

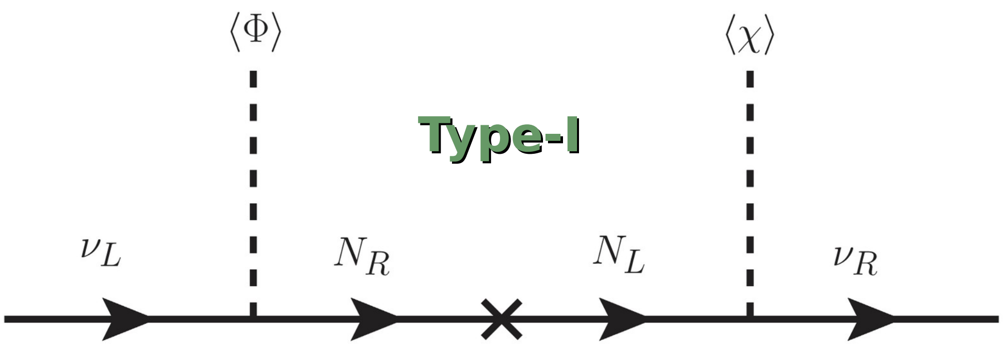
- Generalized Weinberg operators: Naturally small Dirac neutrino masses as well **[Bonilla,Chulia,Cepedello,Peinado,RS '18,'19]**

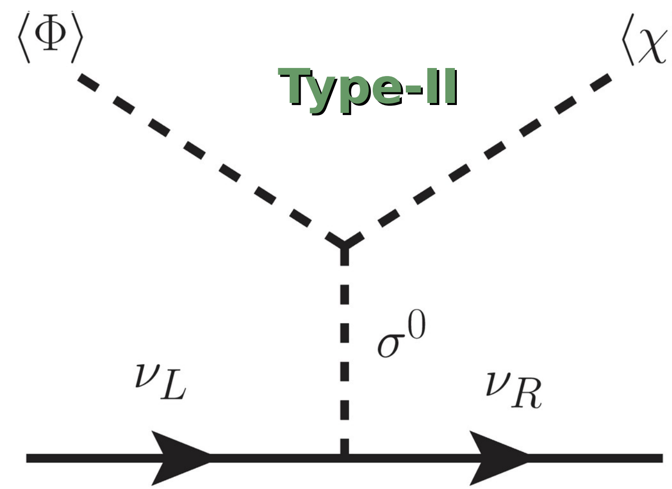


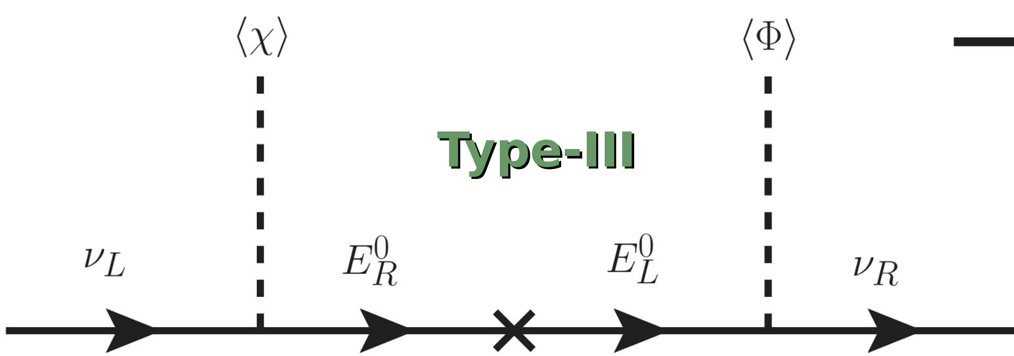

- Since $\chi \sim 3$ , its vev breaks $U(1)_{B-L} \rightarrow \mathcal{Z}_{3m}; m \in \mathbb{Z}^+$
  - The exact residual subgroup depends on UV completion

# Dirac Neutrino Mass Mechanisms

- Till recently Dirac neutrinos were thought to be “unnatural”
  - Elegant mass mechanism were not known
- A series of recent works by several groups shows that equally elegant mass mechanisms exist for Dirac neutrinos **[Ma, RS ‘14] [Polard, Ma, RS, Reza ‘15] [Chulia, Ma, RS, Valle ‘16] [Chulia, RS, Valle ‘16, ‘17, ‘18, ‘18]**

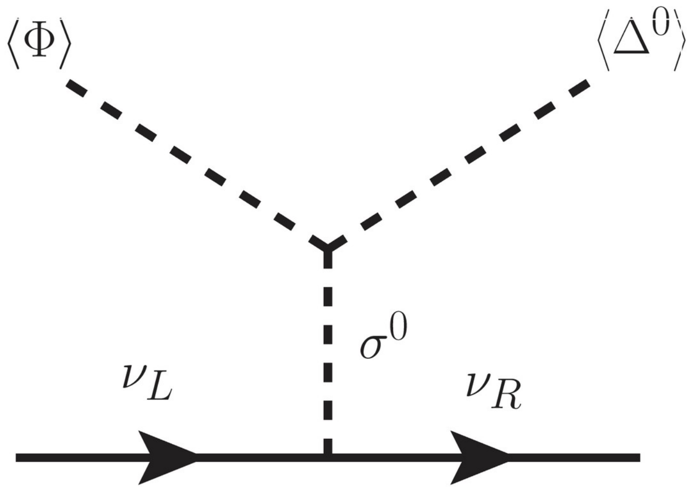


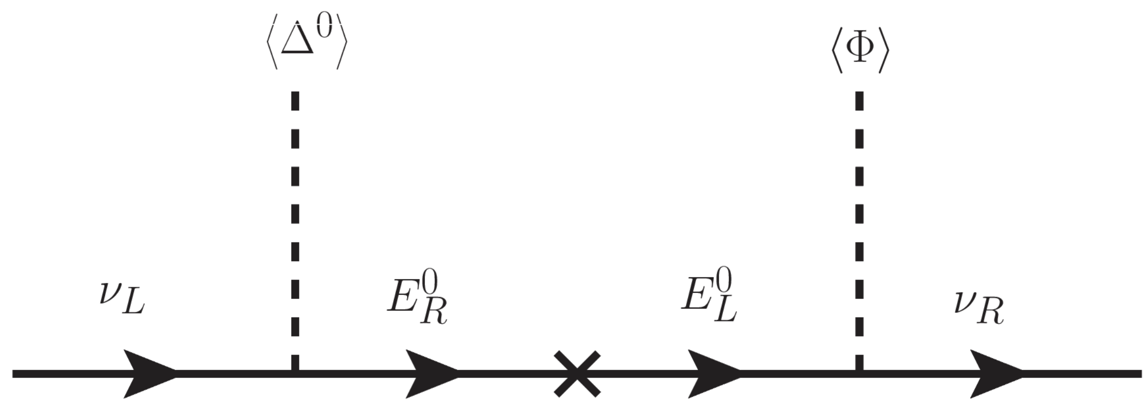



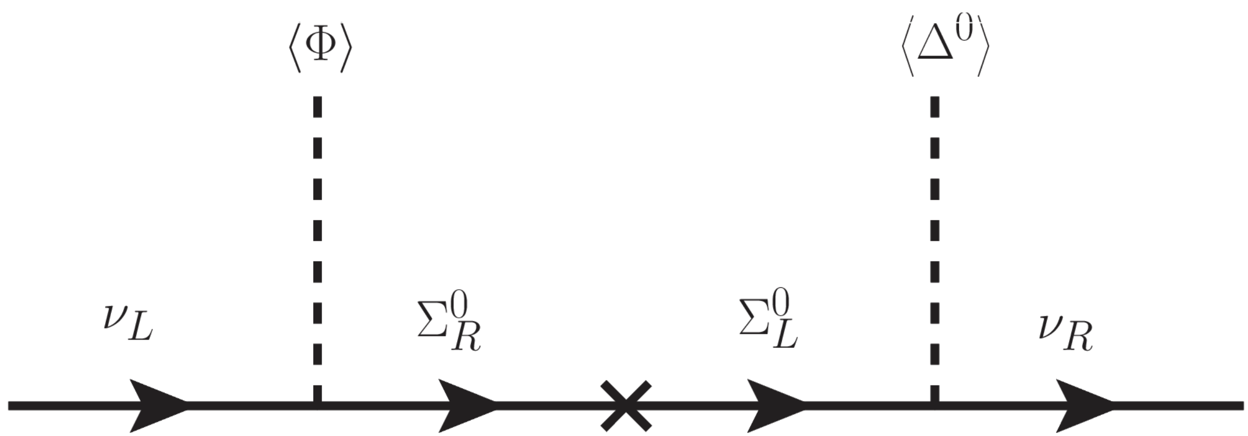




- In fact there are more possibilities for Dirac neutrinos !!!

# Dirac Neutrino Mass Mechanisms

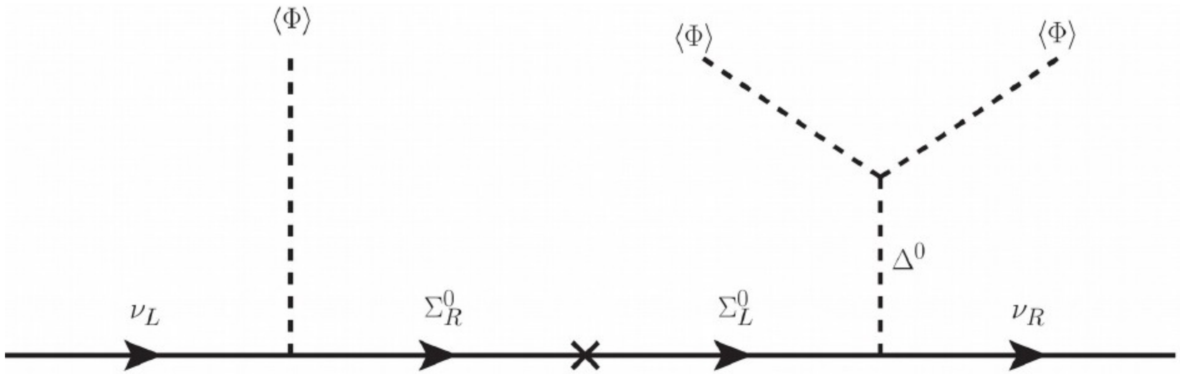


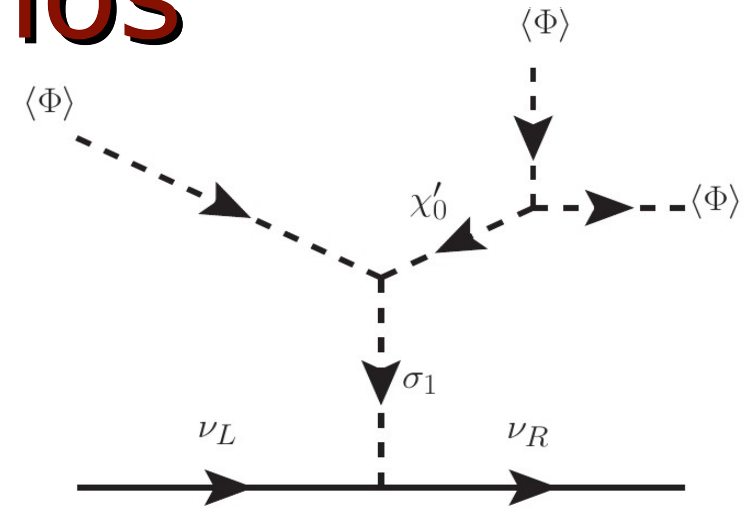



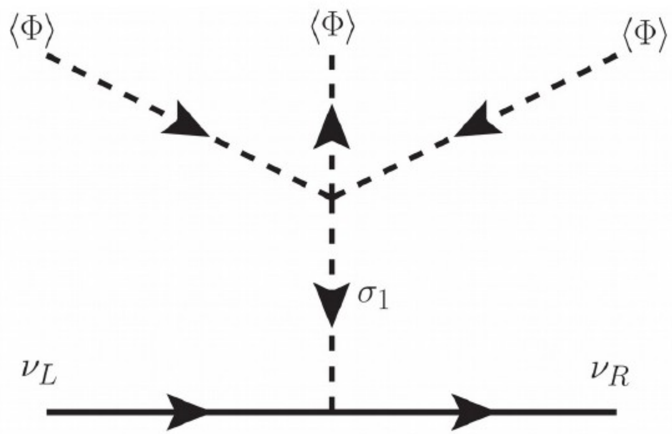



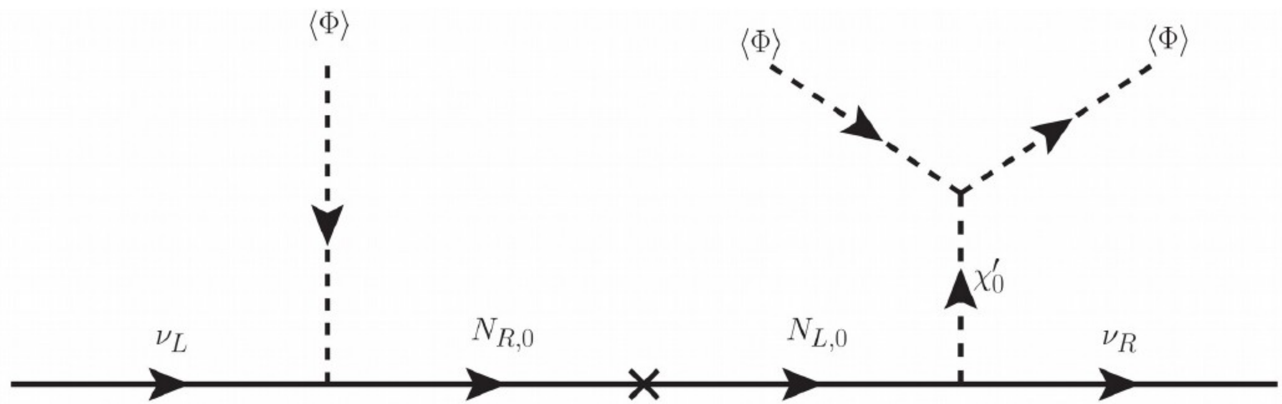

**[S.S.Chulia, RS, J.W.F.Valle PLB '18]**

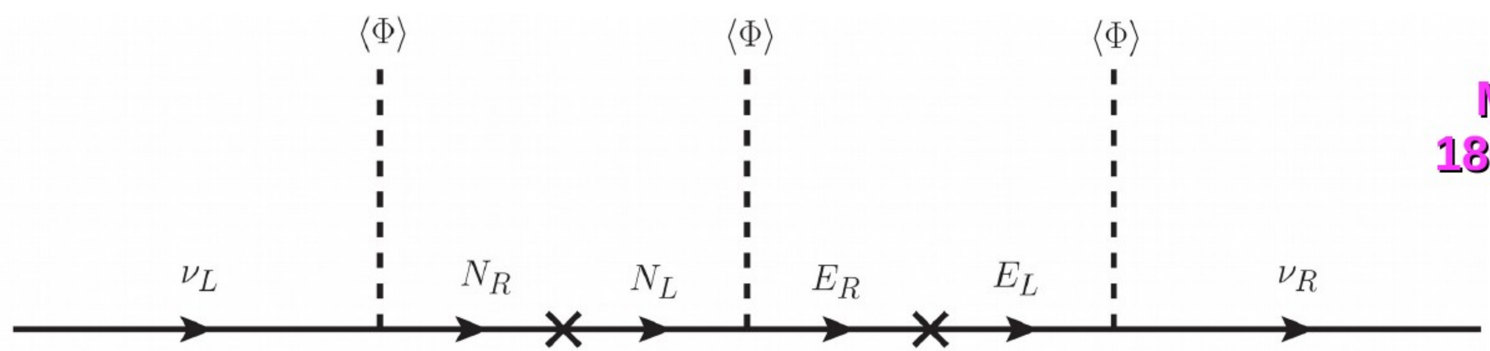
# Dirac Neutrinos



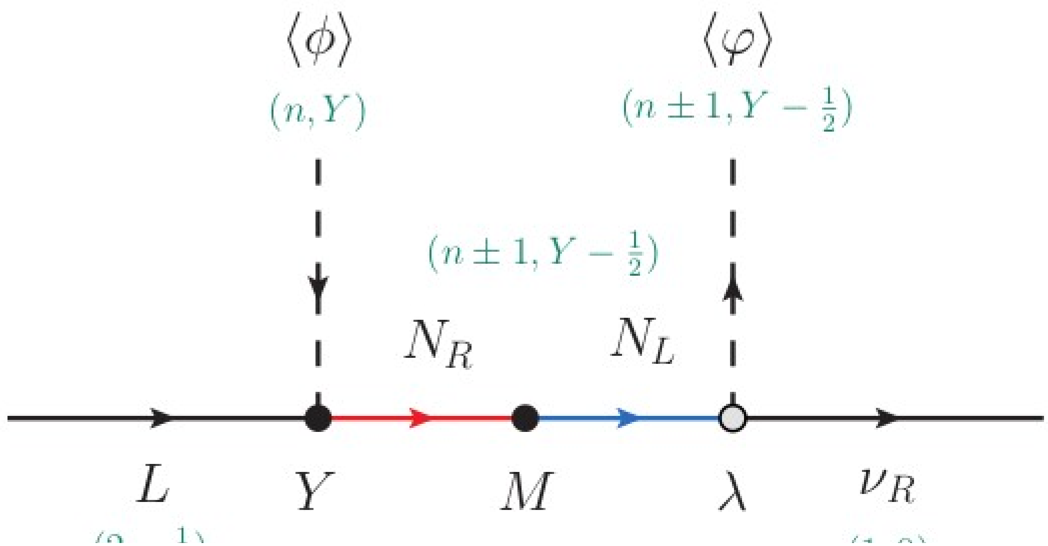



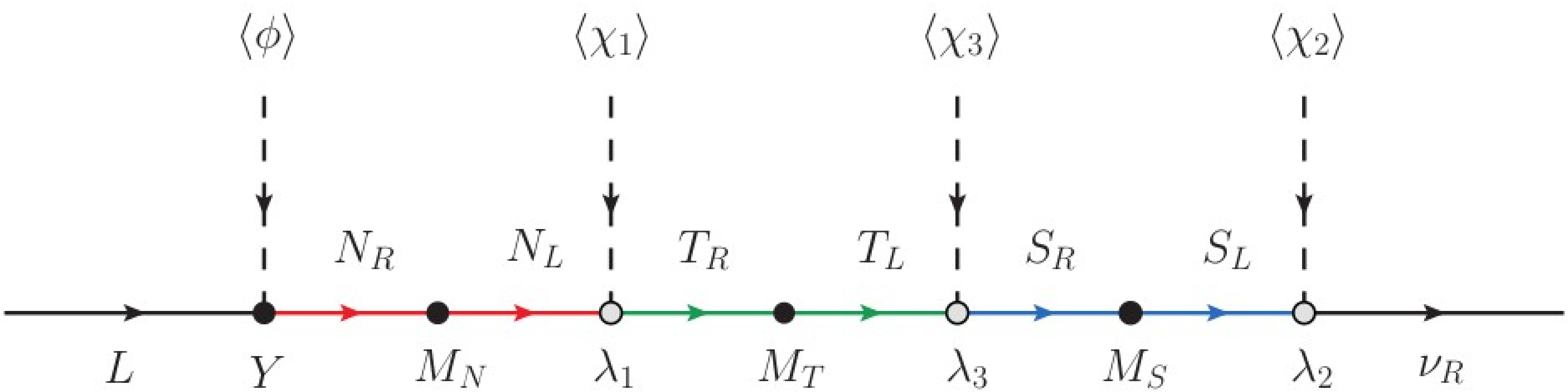




**More than 180 diagrams!**

*[RS et.al. '18,'18,'19]*

# Dirac Neutrinos: Low scale Seesaws



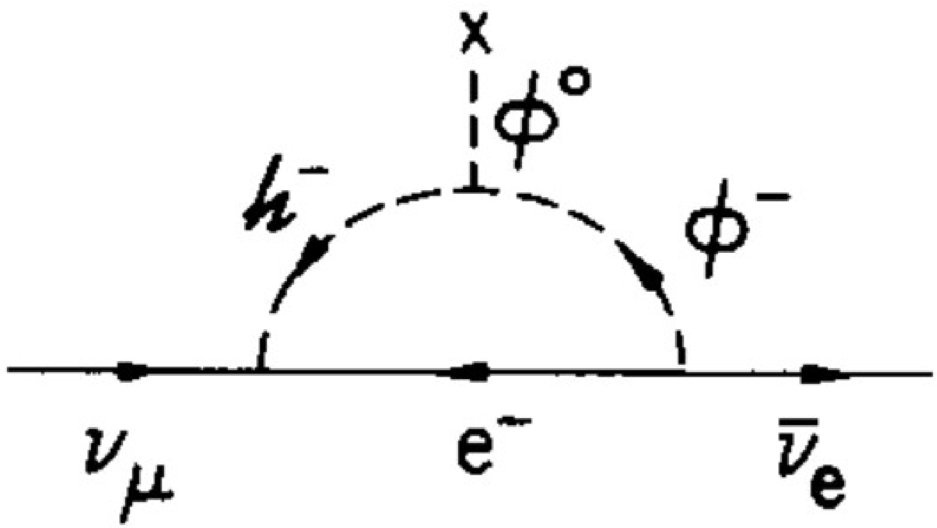




*Chulia, RS, Vicente JHEP'21*

# Loop Level Mass Models

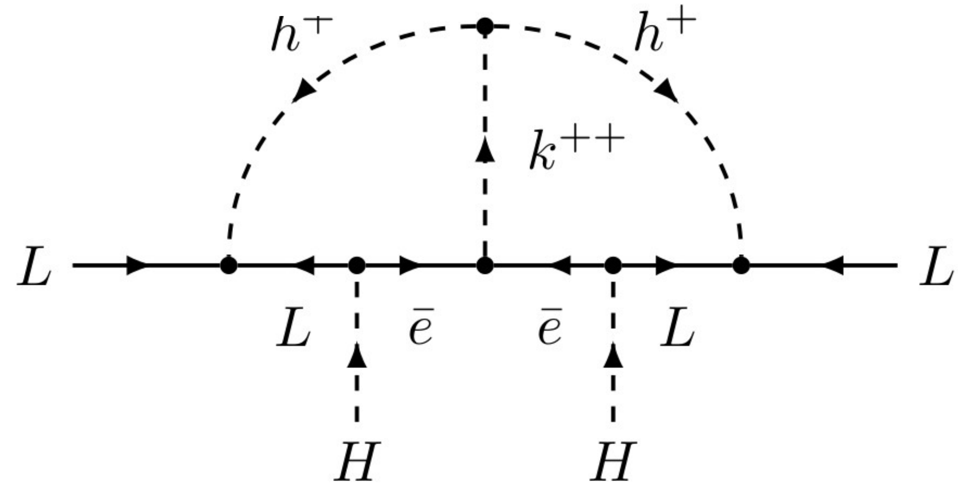
- It is not necessary that neutrino mass should be generated via tree level UV completion
  - Many loop level models are know in literature
- Zee Model

**A. Zee PLB '80**




  - Zee – Babu Model

**K. S. Babu PLB '88**



# Neutrinos and Dark Matter

- Traditionally they are treated separately
  - Different symmetries and particles required to understand them
  - Natural to wonder if they are related intimately

# Dark Matter

- No candidate in SM for Dark Matter
  - Need a massive particle which is either completely stable or has lifetime at least longer than age of Universe
    - Typically requires new symmetries for stability
  - Should gravitationally interact but should have no electromagnetic interaction
  - Should satisfy the observed “relic density” i.e. observed abundance of ~ 27%
- Many candidates
  - Very light particles like axion or Majoron
  - Weakly Interacting Massive Particle (WIMP)
  - Bound state of strongly interacting particles

  ….. etc

# Neutrinos and Dark Matter

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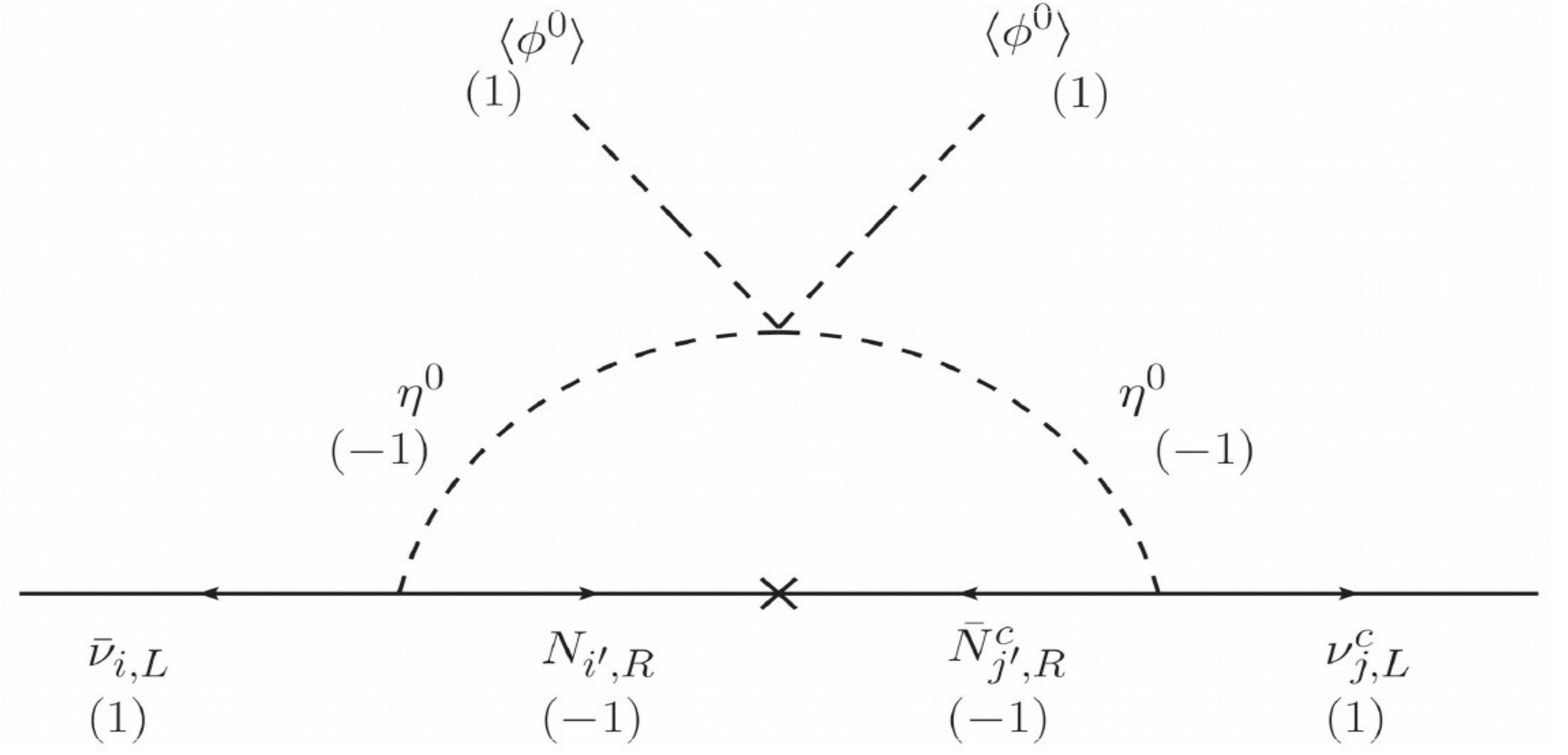
  - Different symmetries and particles required to understand them
  - Natural to wonder if they are related intimately

**Scotogenic Model** [E. Ma '06]



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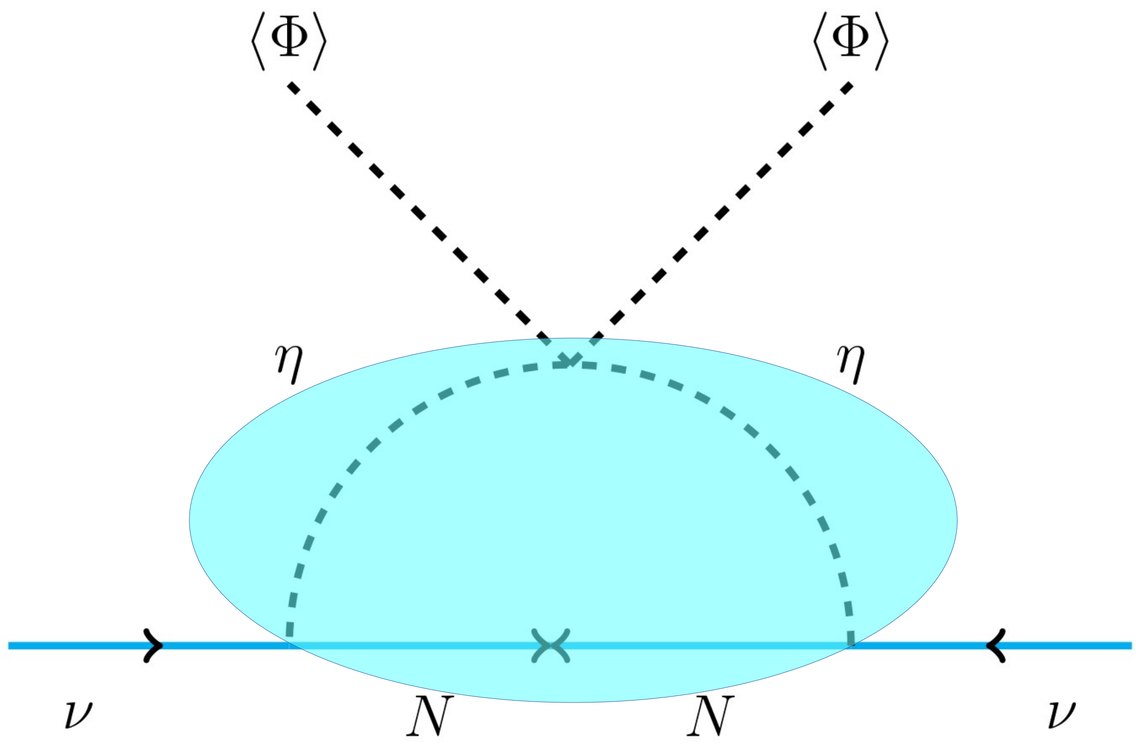
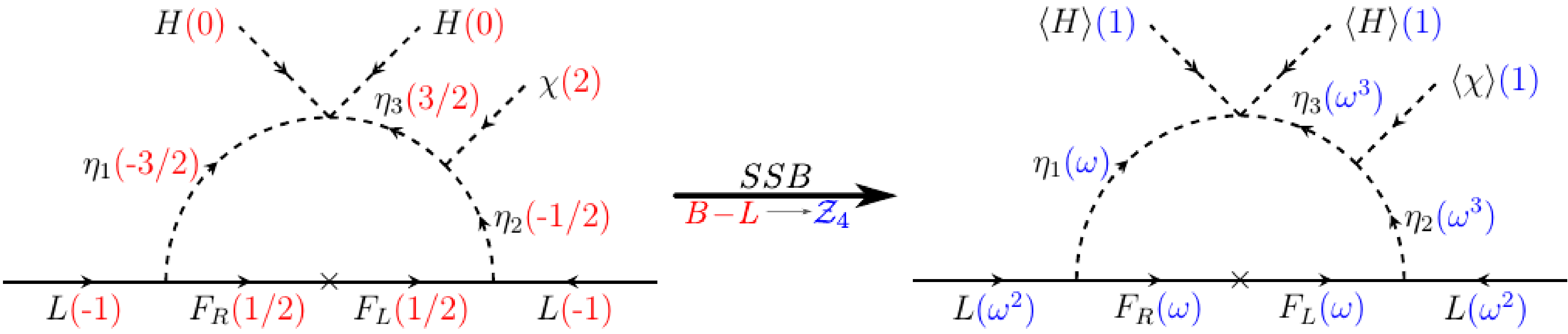
**Scotogenic Model** [E. Ma '06]



# B-L Symmetry as Scotogenic Symmetry: Majorana Neurinos




*[S.C.Chulia,R.Cepedello,E.Peinado,R.S '19]*

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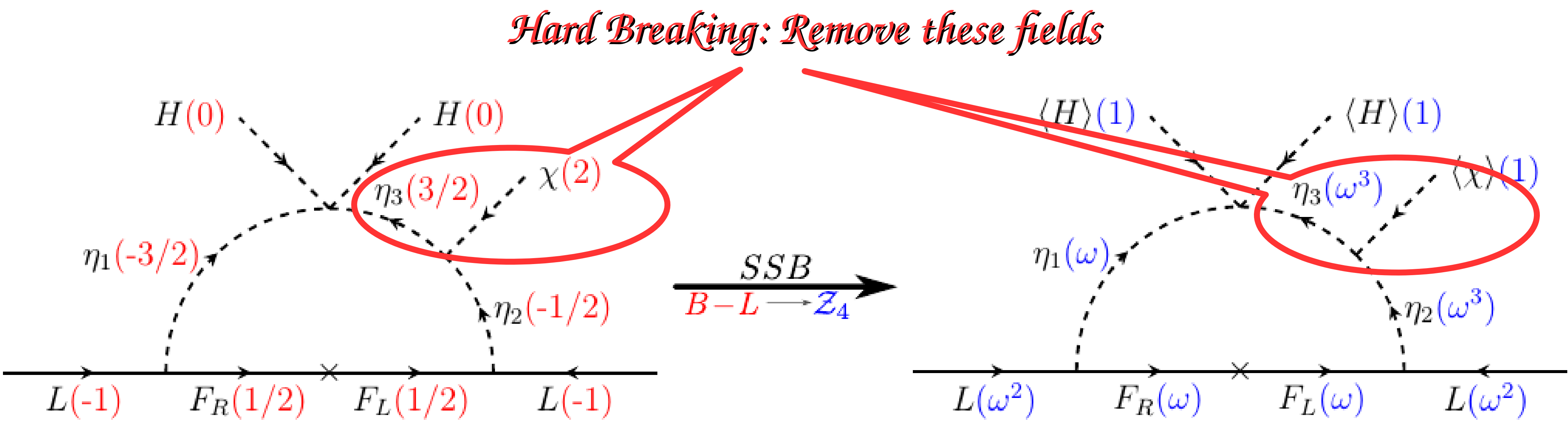
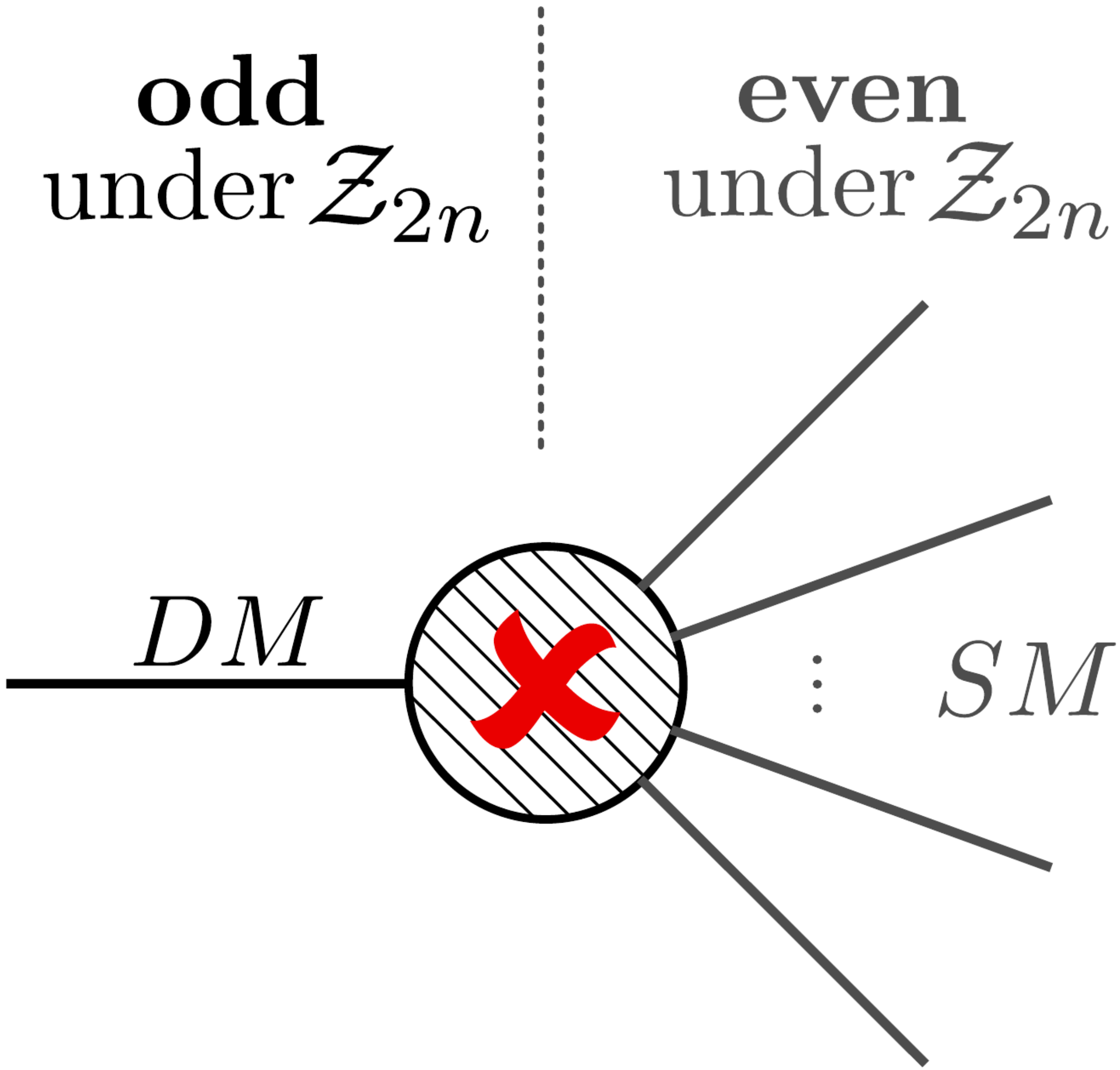


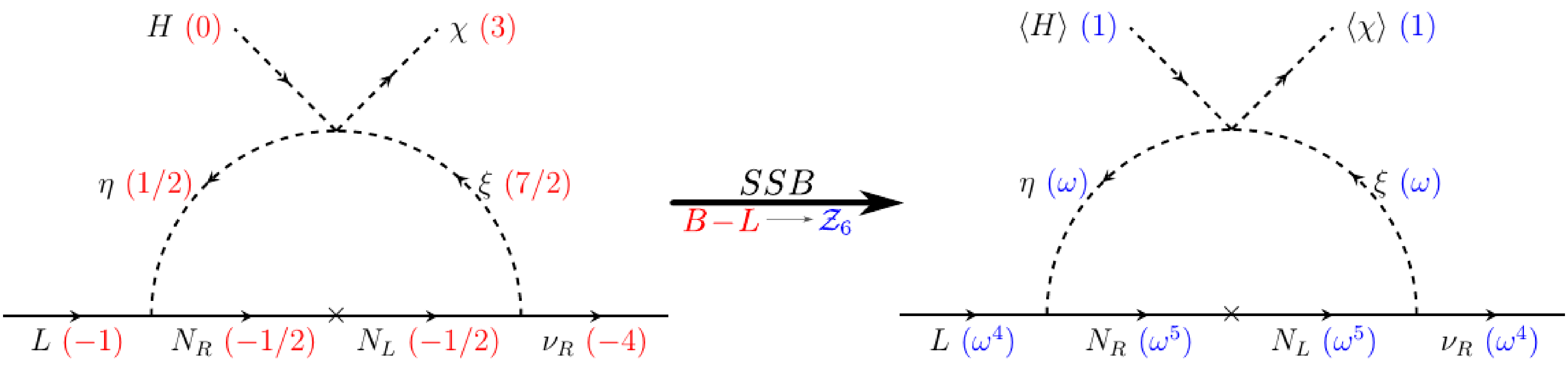

*[S.C.Chulia,R.Cepedello,E.Peinado,R.S '19]*

# Dark Matter Stability




*[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,R.S '18,'19]*

# B-L Symmetry as Scotogenic Symmetry: Dirac Neurinos

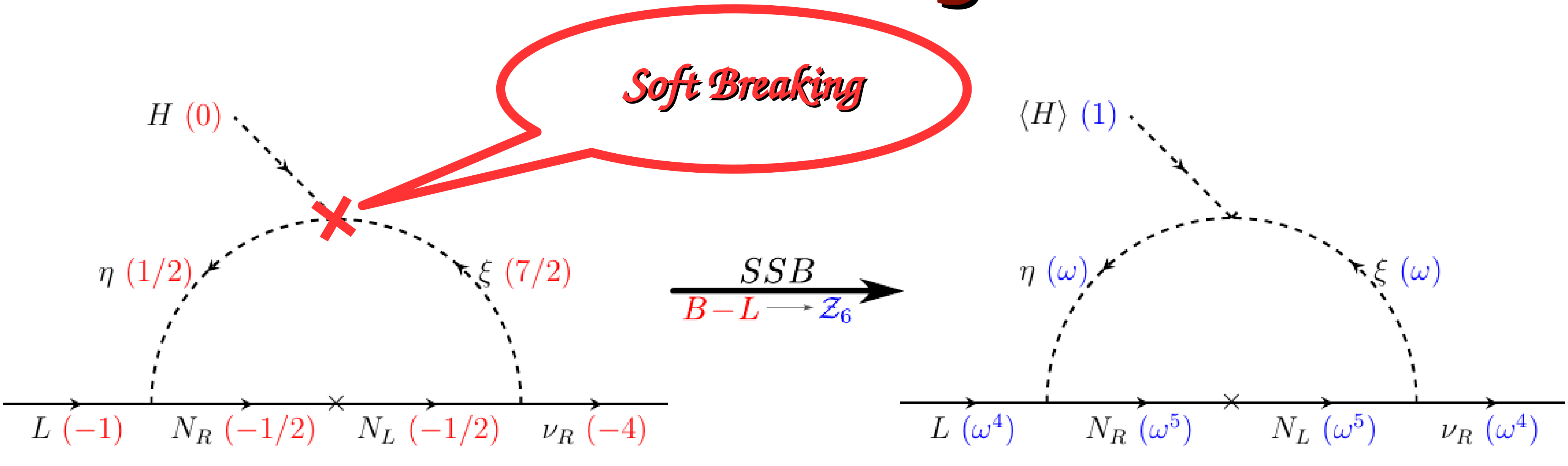



(a) $U(1)_{B-L}$ charge assignment. (b) Residual $\mathcal{Z}_6$ charge assignment.

*[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,R.S '18,'19]*

- Here $\omega = e^{2\pi I/6}; \omega^6 = 1$ is the 6$^{th}$ root of unity.
- All particles carrying fractional B-L charges belong to Dark Sector
  - **Lightest Dark Sector particle will be Stable Dark Matter Candidate**

# Dirac Scotogenic




(a) $U(1)_{B-L}$ charge assignment.

(b) Residual $\mathcal{Z}_6$ charge assignment.

*[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,R.S '18,'19]*

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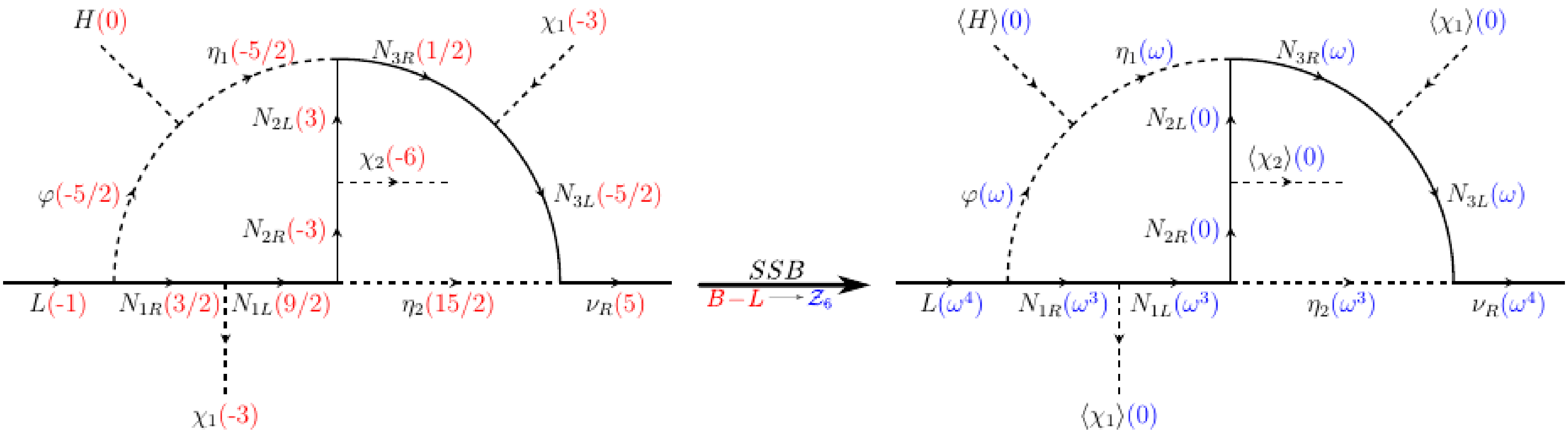
# Two Loop Model

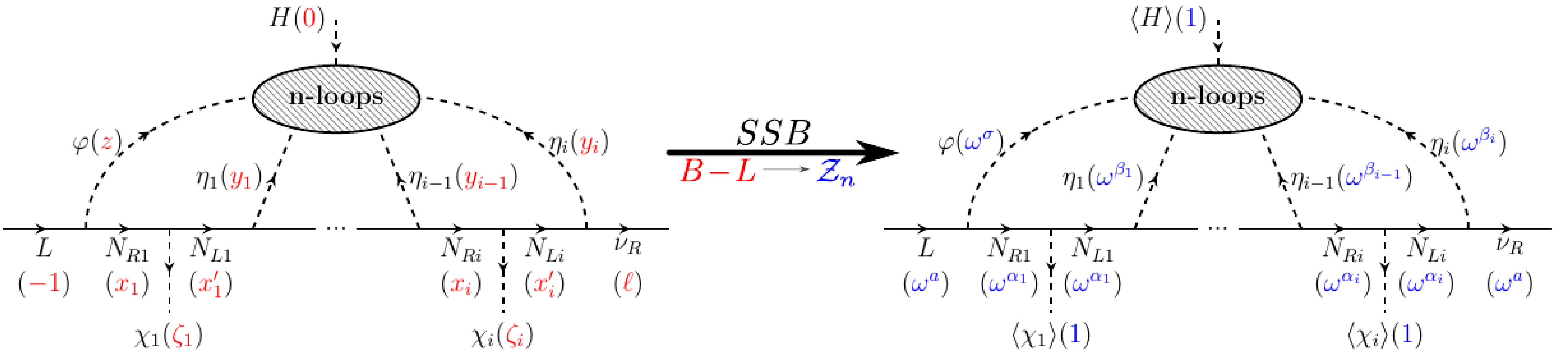



(a) $U(1)_{B-L}$ charge assignment.

(b) Remnant $\mathcal{Z}_6$ charge assignment.

*[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,R.S, '19, '19]*

# Completely General N Loop Formalism




(a) General $U(1)_{B-L}$ charge assignment. (b) General residual $\mathcal{Z}_n$ charge assignment.

Figure 1: General charge assignment for any topology and its spontaneous symmetry breaking pattern.

*[C.Bonilla,S.C.Chulia,R.Cepedello,E.Peinado,R.S '19, '19]*

# Exotic Proton Decays

- Proton decay is a key prediction of Grand Unified Theories
- Several past experiments and currently Super-Kamiokande experiment is looking for it
- No signal has been observed so far *[SK, '17]*
- GUTs typically predict 2-body decays of proton such as

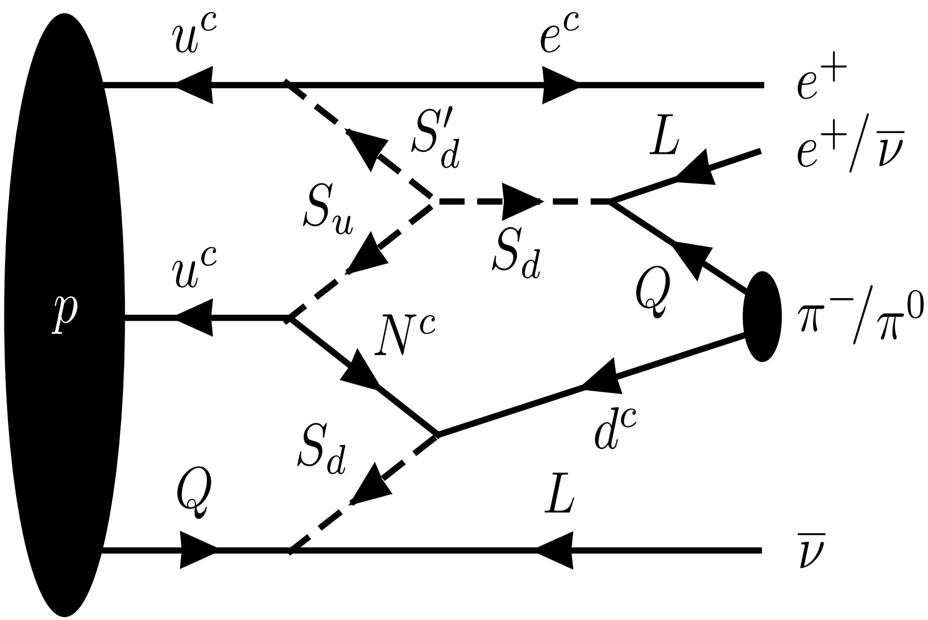
$$p \to \pi^0 e^+$$

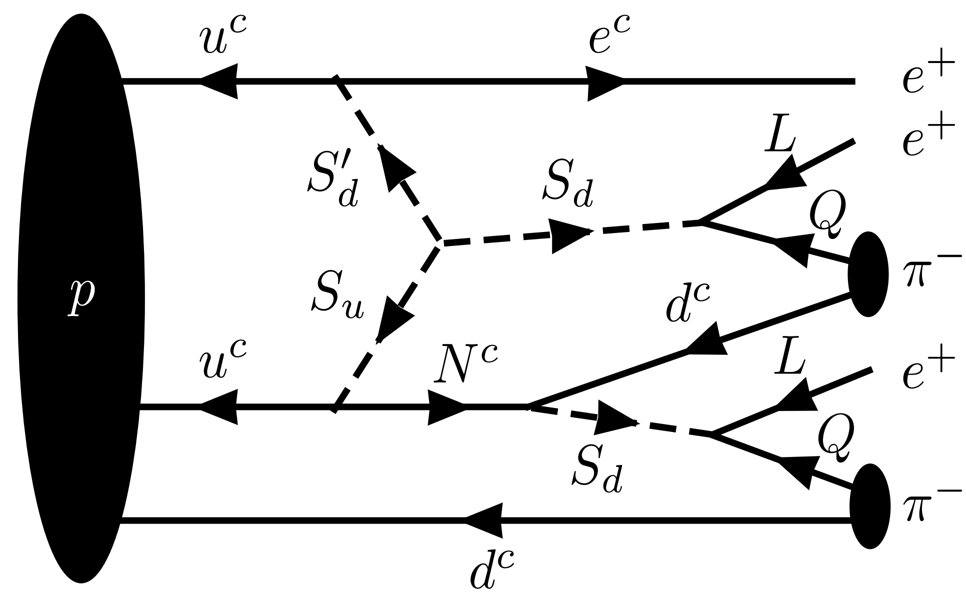
- However, we recently pointed out that if neutrinos are Dirac then proton might decay in modes like *[R. Fonseca, M. Hirsch, RS '18]*

$$p \to \pi^0 \pi^0 \bar{\nu} \bar{\nu} e^+ \quad p \to \pi^- \pi^0 \bar{\nu} e^+ e^+ \quad p \to \pi^- \pi^- e^+ e^+ e^+$$

# Exotic Proton Decays

- Possible when residual $Z_3$ symmetry is preserved : Dirac neutrinos [R. Fonseca, M. Hirsch, RS '18]
- $Z_3$ also forbids the canonical two-body decays

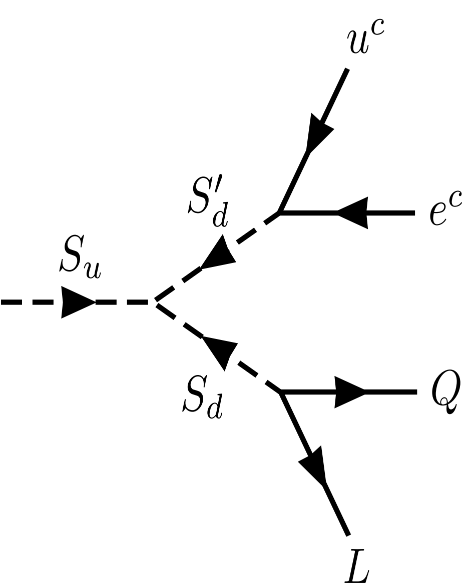


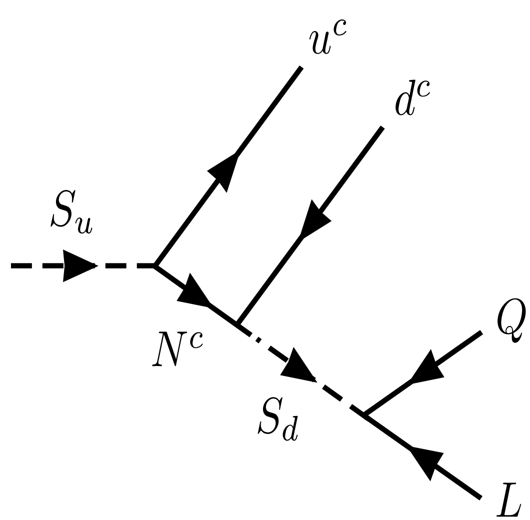



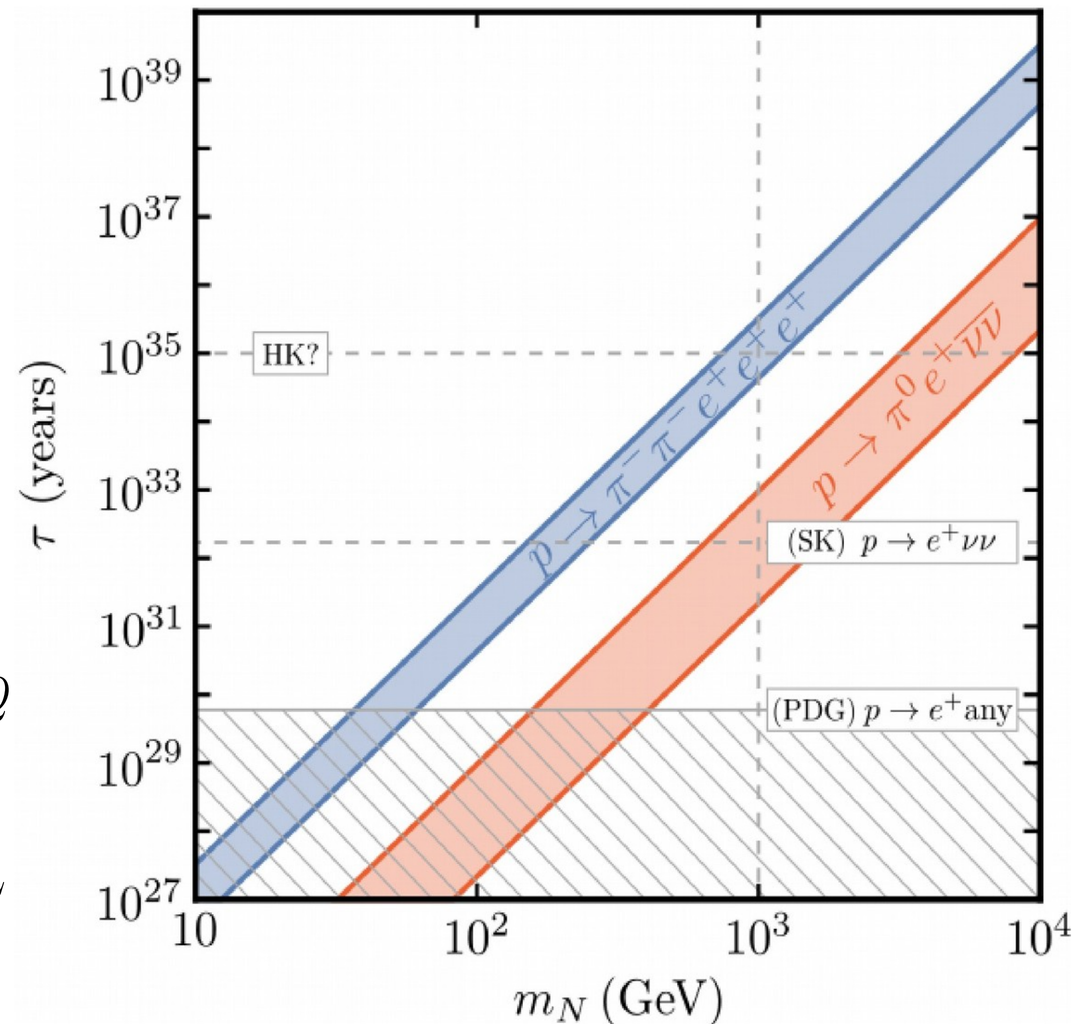
# Exotic Proton Decays

- Induced by very high dimensional operators:
  - Dim-10 or above
- Scale of new physics can be well within LHC range
- Unique signature at LHC

*[R. Fonseca, M. Hirsch, RS '18]*



# Gauged Lepton Number to Z3

- One can also obtain exotic proton decay in a gauged model
  - Break $U(1)_L \to Z_3$ **[Duttatreya, O. Popov, RS '22]**
- Scale of new physics can be well within LHC range
- Unique signature at LHC

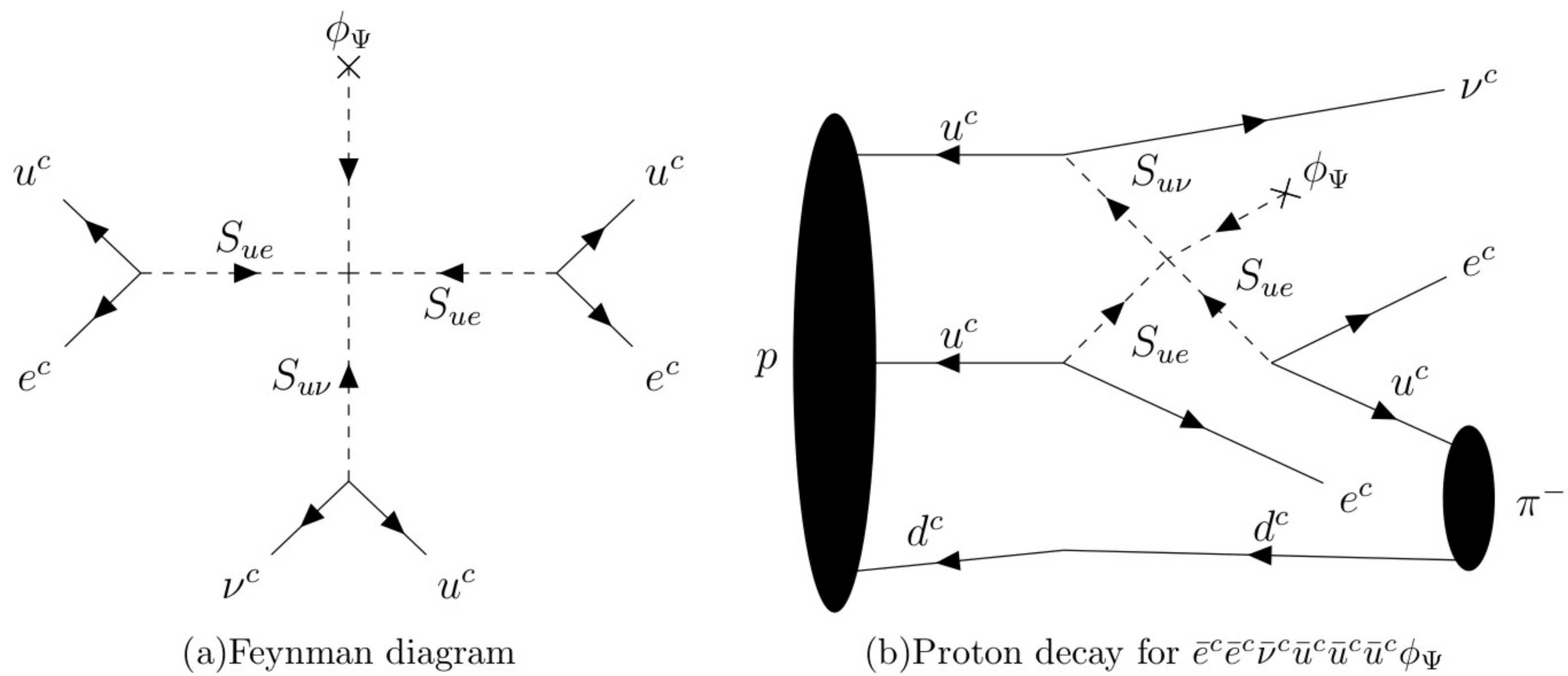



FIG. 2: Feynman diagram of a proton decay via only LQs induced by the effective operator $\bar{e}^c\bar{e}^c\bar{\nu}^c\bar{u}^c\bar{u}^c\bar{u}^c\phi_\Psi$.

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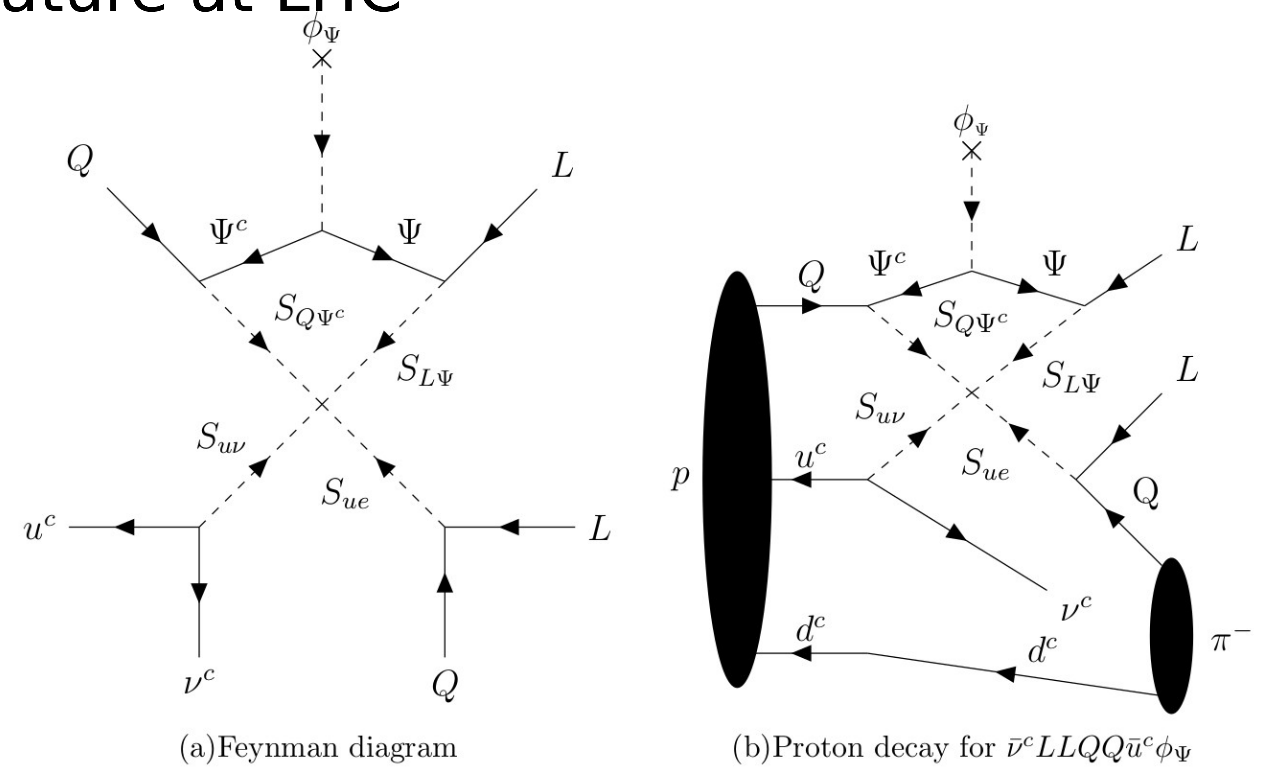


FIG. 4: 1-loop Feynman diagram of a proton decay via new fermions induced by the effective operator $\bar{\nu}^c LLQQ\bar{u}^c\phi_\Psi$.

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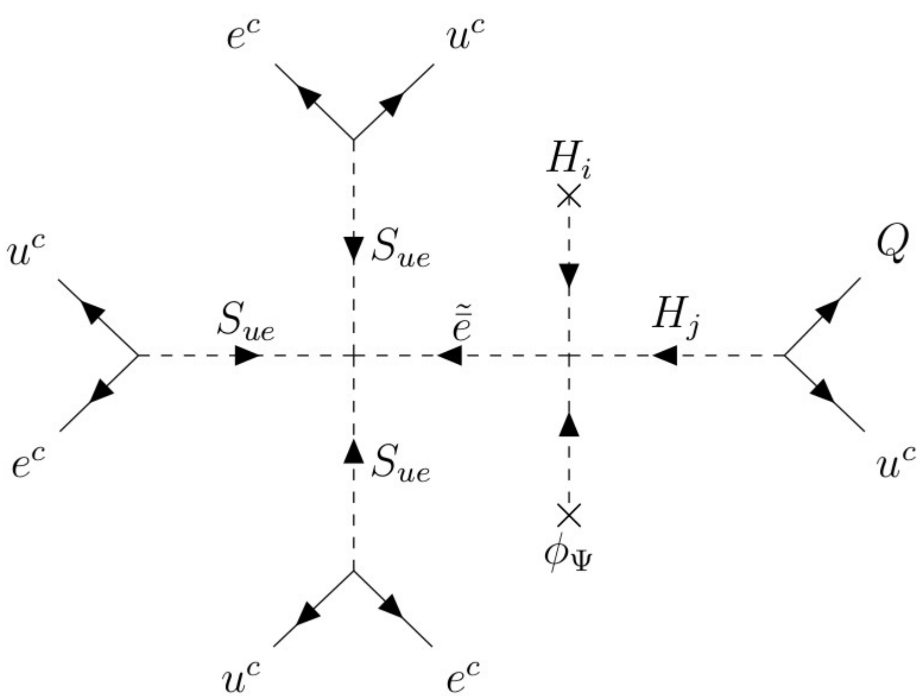


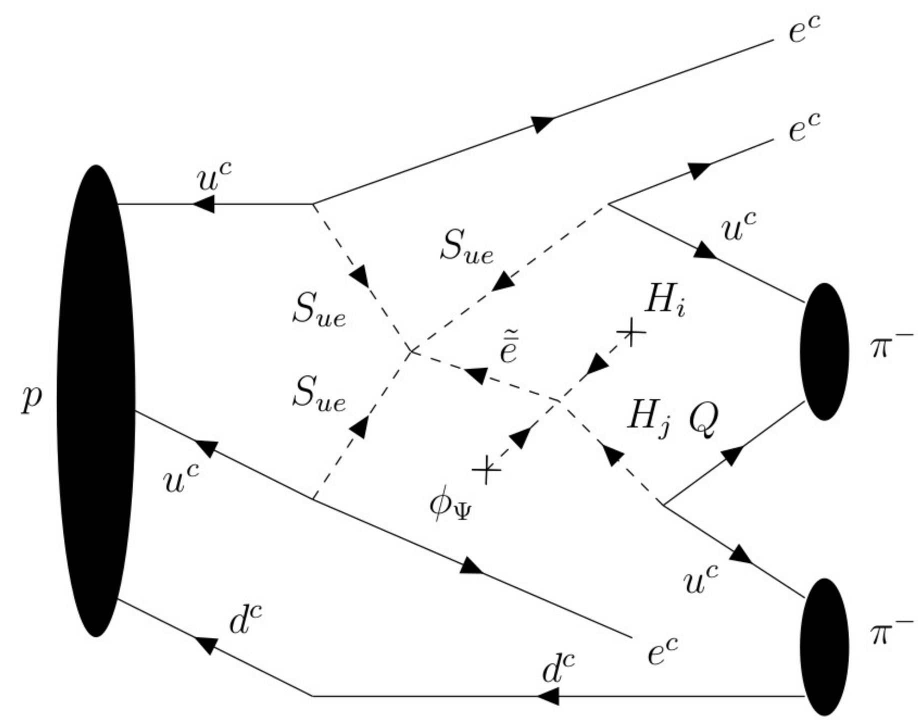


(b)Proton decay for $\bar{e}^c\bar{e}^c\bar{e}^c\bar{u}^c\bar{u}^c\bar{u}^c\bar{u}^c Q H \phi_\Psi$

# Important Omissions

- Lack of time: Certain important developments not covered. Some important ones are:
  - Loop Models not covered in details
  - Systematic ways to obtain all mass models for a given effective operator not discussed
- Fermion mixing matrices and family symmetries
  - Non-abelian flavor symmetries
  - Modular symmetries
  - Generalized CP symmetries etc
- Cosmological imprints of neutrinos
  - Baryogenesis via Leptogenesis
  - First Order Phase Transitions and Gravitational Waves
  - Extra relativistic degrees of freedom and $\Delta N_{\text{eff}}$

# Future Directions

- Several unanswered questions remain about neutrinos
- Here I touched only few of them
- One important development in last few years is that people have started looking seriously into Dirac neutrino mass models
  - This has opened up a plethora of unexplored possibilities
- Neutrinos can be the portal for elusive new physics
  - Neutrinos may be intimately connected with Dark Matter
  - Higgs vacuum stability depends crucially on UV completion of neutrino mass models
  - Neutrinos have various interesting cosmological imprints
  - They can have other consequences like exotic proton decays
- Exciting times in neutrino physics lie ahead

# *Thank You*