GWs from PTs and PTAs

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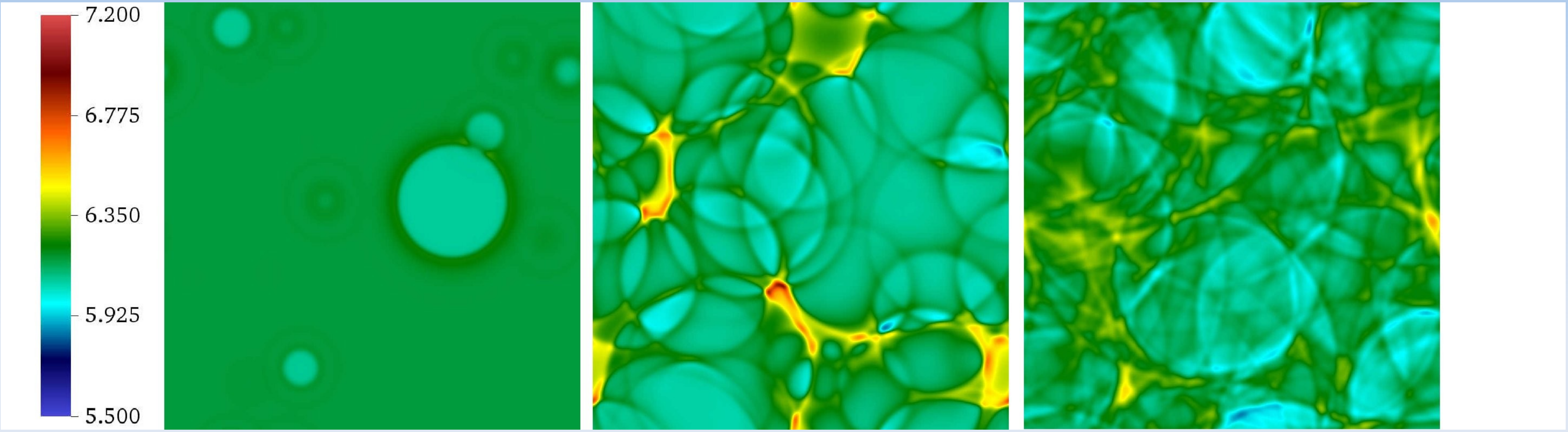
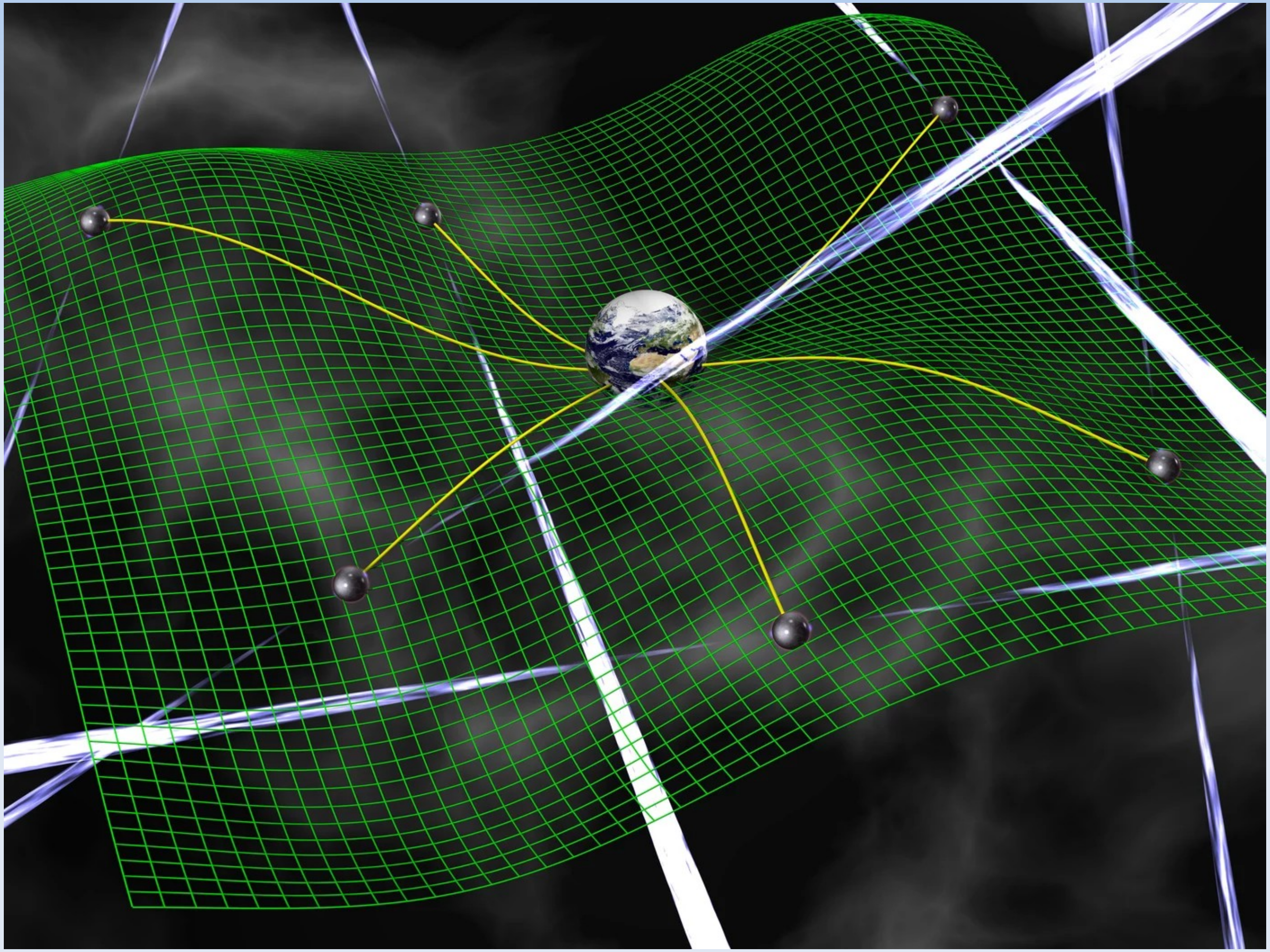


I. Introduction
II. New results for strong PTs
III. Anisotropies in PTAs

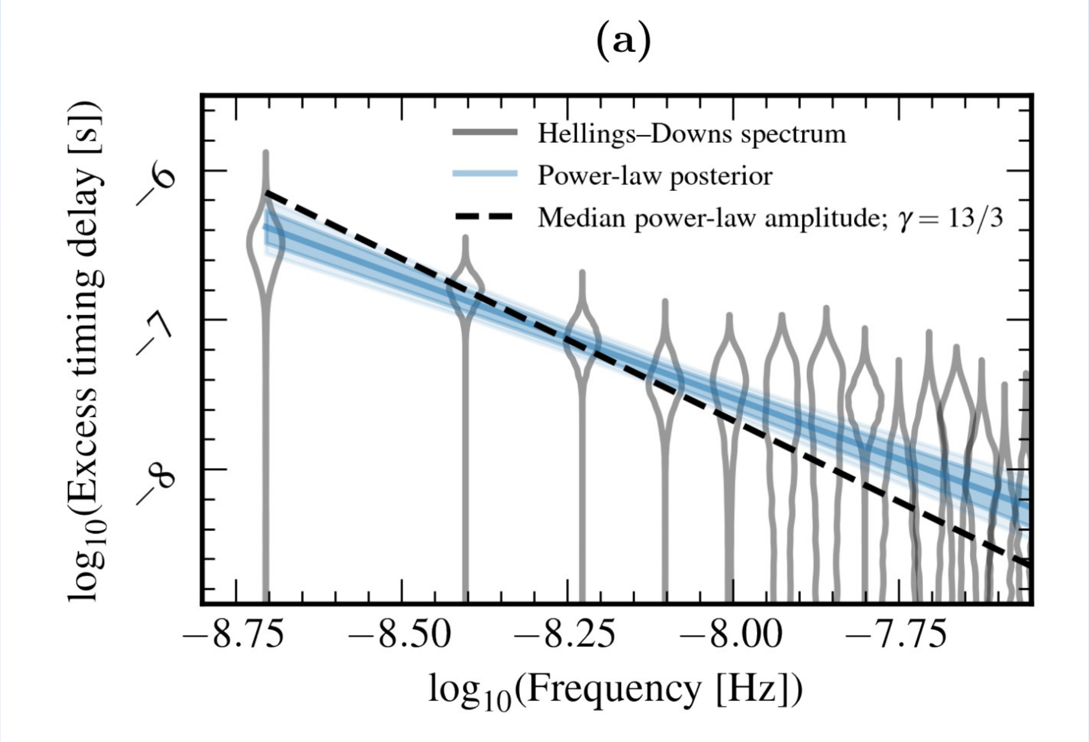
Pulsar timing array



Credit: David Champion/Max Planck Institute for Radio Astronomy

New data release

NANOGrav measured a common red noise spectrum in the nHz regime (1/10 year)

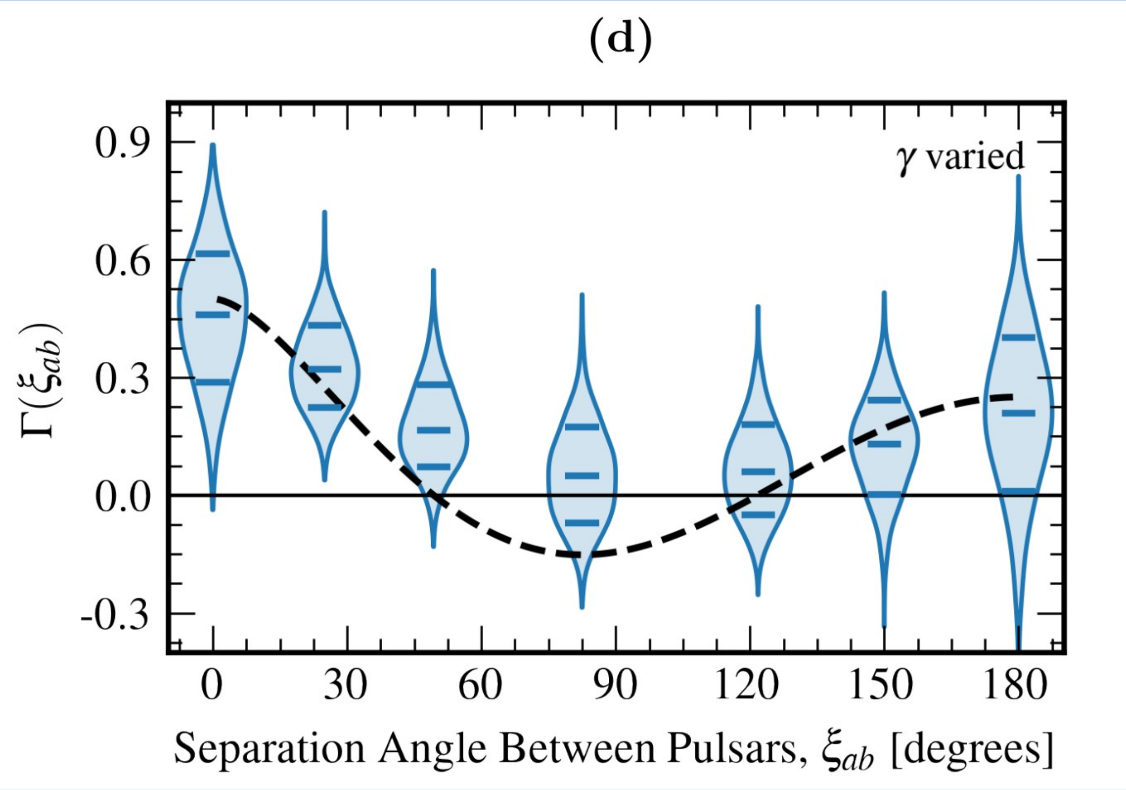


Other PTA experiements had similar results with somewhat less statistics (EPTA, PPTA, CPTA)

[NANOGrav 2023]

Are these really GWs?

The smoking gun for a stochastic GW source is a correlation that follows the Hellings-Downs curve

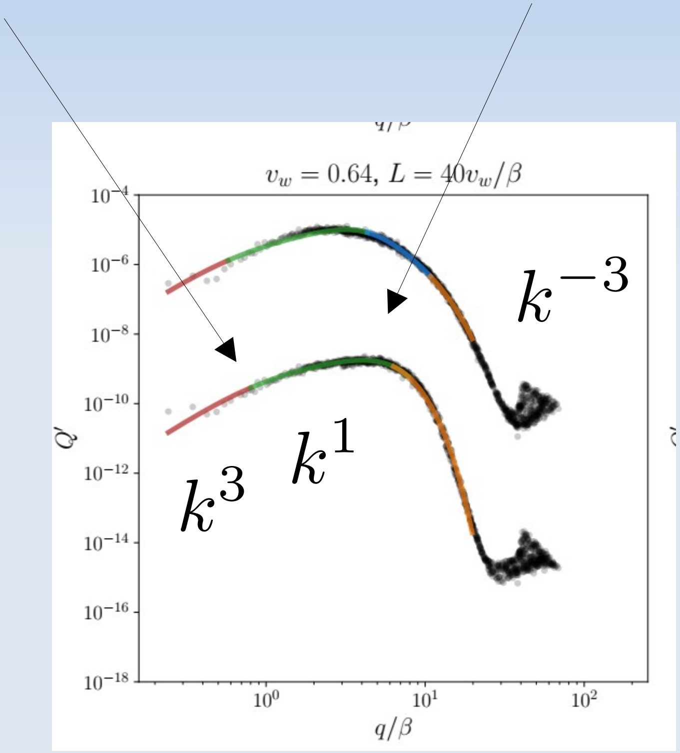


The data seems to support a Hellings-Downs curve, even though there is also a quite large monopole.

[NANOGrav 2023]

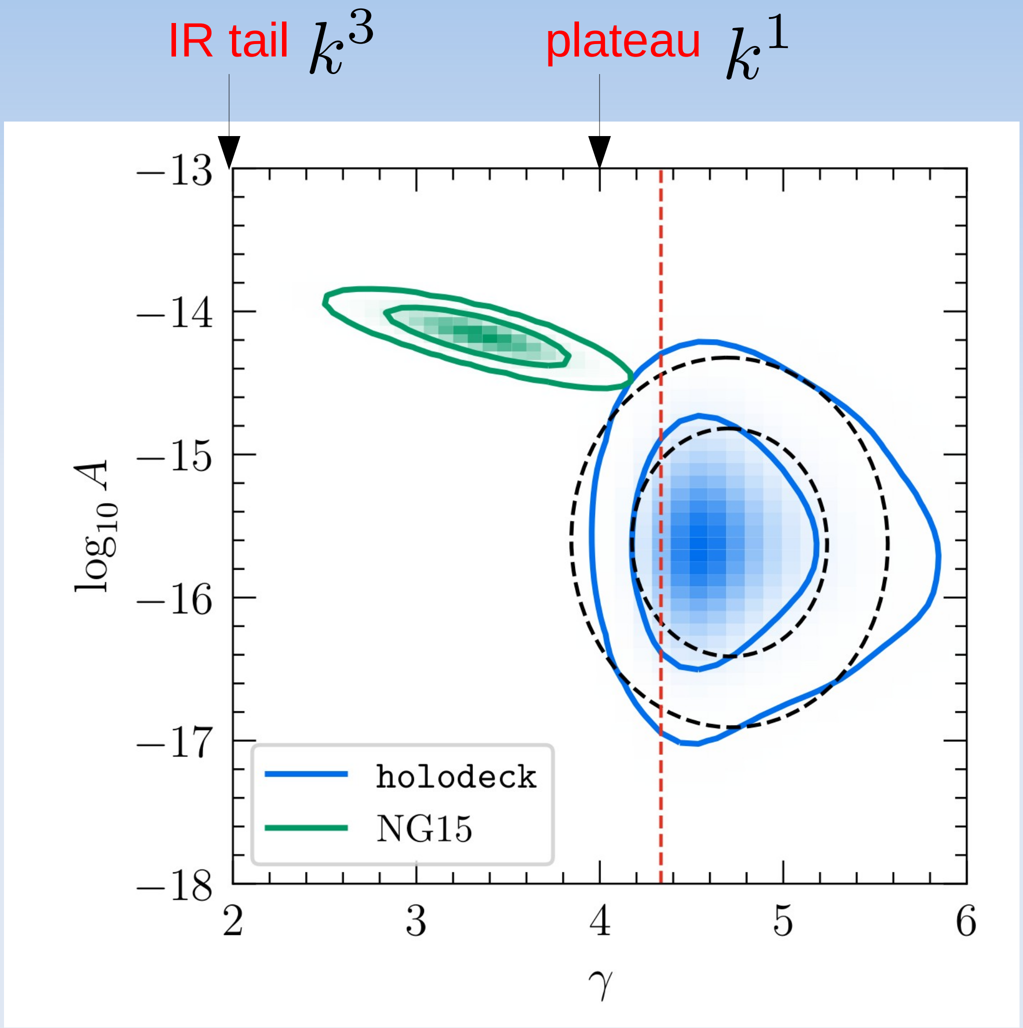
Simulation of cosmological phase transitions

The spectra have two features due to the bubble size and the shell thickness.



[Jinno, TK, Rubira, Stomberg 2022]
[Hindmarsh 2016]

Where do they come from?



The power law fit is somewhere between the IR tail and plateau. So the fit will probably further improve with the new spectra. *[NANOGrav 2023]*

How to distiguish SMBHs from cosmological backgrounds?

There are in principle different ways to distinguish a background from supermassive black holes from a stochastic cosmological background

1) In principle the shape of the power spectrum can provide information.

2) For a SMBH background, isolated point sources should be at some point identifyable

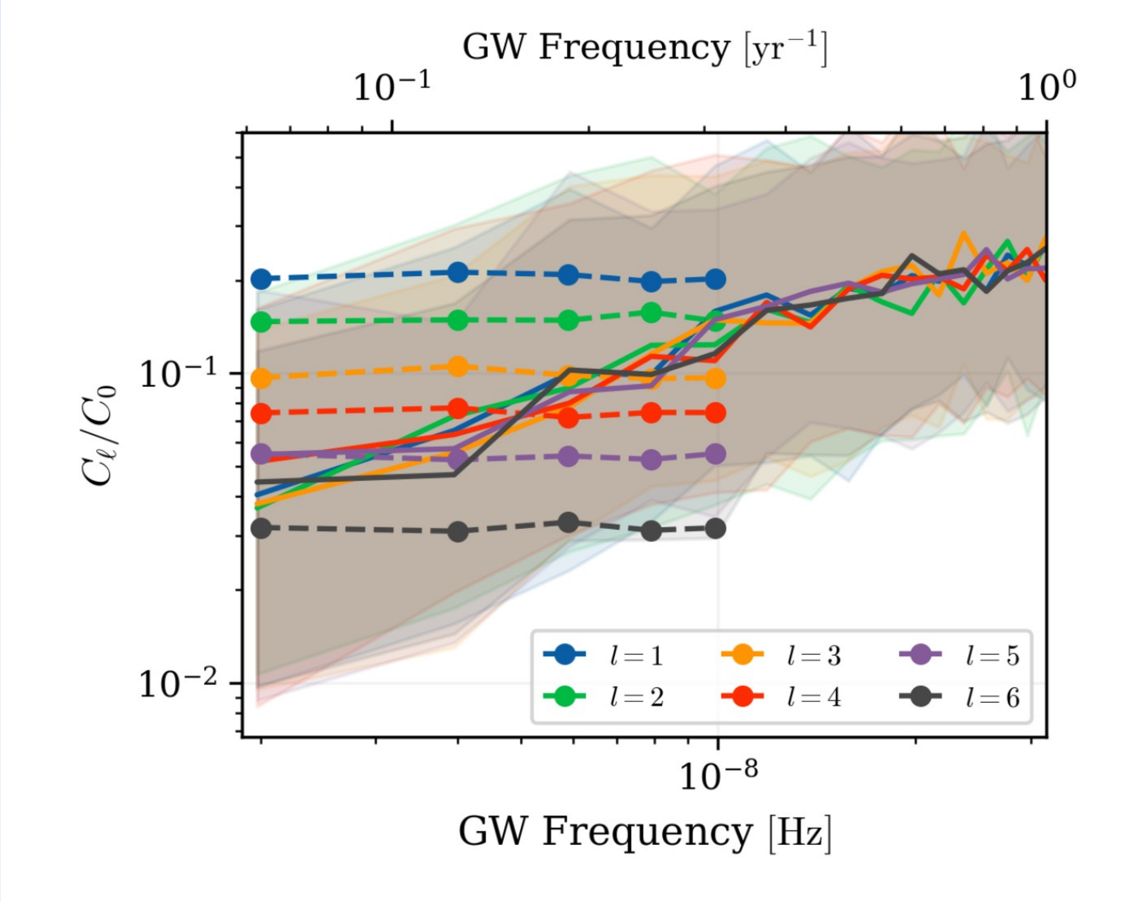
3) more general, one would expect some anisotropies for SMBHs

4) specific cosmological models might have additional signatures (e.g. beam dump or N_{eff})

5) Signal in LISA/LIGO

Anisotropies

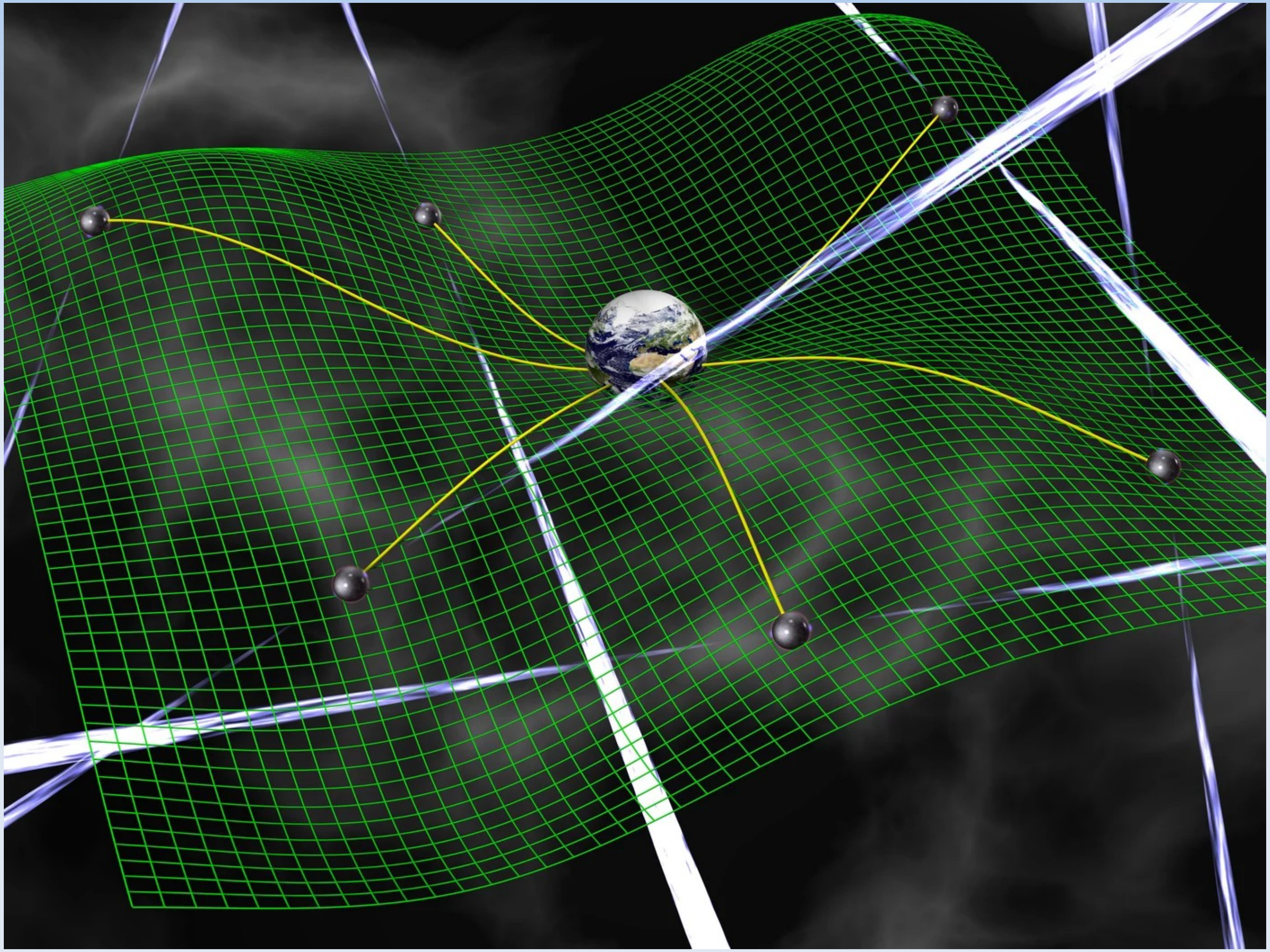
No anisotropies have been found so far.



The bands denote expectations from SMBH.
The measurements are upper limits.

[NANOGrav 2023]

Cosmic Variance in anisotropy searches at PTAs



Credit: David Champion/Max Planck Institute for Radio Astronomy

Let's dig into the details a bit ...

We observe time-delays in pulsar *a* that is in direction p_a in a metric background. The time delay is the metric projected onto the line of sight, integrated along the path of the pulsar

$$\delta t_a(t) = \frac{p_a^i p_a^j}{2} \int_{t-L_a}^{t} dt' h_{ij}\left[t', (t-t')\boldsymbol{p}_a\right]$$

Assuming the background is a collection of GWs this can be integrated

$$h_{ij}(t, \boldsymbol{x}) = \sum_A \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega}\, \tilde{h}_A(f, \hat{\Omega}) e^{i2\pi f(t-\hat{\Omega}\cdot\boldsymbol{x})} e_{ij}^A(\hat{\Omega}) \,,$$

Here e_{ij}^A is the polarization tensor that is orthogonal to Ω.

Let’s dig into the details a bit ...

$$\delta t_a(t) = \int df \int d\hat{\Omega} \sum_A \tilde{h}_A(f,\hat{\Omega}) R_a^A(f,\hat{\Omega}) \frac{e^{2\pi i f t}}{2\pi i f} ,$$

R is called **response function**.

$$R_a^A(f,\hat{\Omega}) \equiv F_a^A(\hat{\Omega}) \left[1 - e^{-2\pi i f L_a (1+\hat{p}_a\cdot\hat{\Omega})}\right] , \qquad F_a^A(\hat{\Omega}) \equiv \frac{p_a^i p_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)} e_{ij}^A(\hat{\Omega}) .$$

The two terms are called **earth** and **pulsar** terms and neglecting the puslar term is in principle O(1).

Collinear limits

The prefactor in F can develop a divergence which however cancels due to the pulsar term.

$$R_a^A(f,\hat{\Omega}) \equiv F_a^A(\hat{\Omega})\left[1 - e^{-2\pi i f L_a(1+\hat{p}_a\cdot\hat{\Omega})}\right], \qquad F_a^A(\hat{\Omega}) \equiv \frac{p_a^i p_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)} e_{ij}^A(\hat{\Omega})\,.$$

Consider no two cases: the pulses ride on the wave or ride against the wave

$$\hat{p} \to -\hat{\Omega}: \qquad R_a^A \to i\,\pi\,f\,L_a\,\hat{p}_a^i\hat{p}_a^j e_{ij}^A(\hat{\Omega})$$

$$\hat{p} \to \hat{\Omega}: \qquad \text{no simplification}$$

So PTAs are sensitive to the direction of the GWs. Notice also that $p^i p^j e_{ij}^A = 0$ for both cases.

Large wavelength / small frequency

$$R_a^A(f,\hat\Omega) \equiv F_a^A(\hat\Omega)\left[1 - e^{-2\pi i f L_a(1+\hat p_a\cdot\hat\Omega)}\right], \qquad F_a^A(\hat\Omega) \equiv \frac{p_a^i p_a^j}{2(1+\hat\Omega\cdot\hat p_a)} e_{ij}^A(\hat\Omega)\,.$$

In the case of a large wavelength / small frequency

$$L_a\omega \to 0: \qquad R_a^A \to i\,\pi\,f\,L_a\,\hat p_a^i \hat p_a^j e_{ij}^A(\hat\Omega)$$

So in this limit one cannot distinguish between p and -p.

Notice that in groundbased experiments, one measures the phase difference from the round-trip and is hence anyway hardly sensitive to the direction flip anyway.

Correlation function

Many of the noise sources are specific to the pulsar, so in order to reduce the noise one considers the correlation of the time delays across several pulsars

$$\rho_{ab} = \frac{1}{T}\int_{-T/2}^{T/2} dt\ \delta t_a(t)\delta t_b(t)\,,$$

When the source is transformed into a discrete Fourier sum (in time), this yields

$$\rho_{ab} = \sum_j \sum_{kk'} \sum_{AA'} R^{A*}_{akj} R^{A'}_{bk'j}\, h^{A*}_{kj} h^{A'}_{k'j} \exp(i\Delta\phi^{AA'}_{kk',j})\, \frac{\Delta\hat{\Omega}^2 \Delta f}{4\pi^2 f_j^2} + \text{c.c.}\,,$$

k and k' is the sum over sources

So one has a double sum over sources and polarizations.

To what extent is interference between sources important?

Hellings-Downs Curve

The Hellings-Downs curve can be obtained in two cases

1) An **infinite sum** over a large number of random **uncorrelated** sources for a **single pair of pulsars**

$$h_{k,A} h^*_{k',A'} = \delta_{kk'} \delta_{AA'} P_0$$

the correlation function then reduces to the sum

$$\rho_{ab} \propto \sum_{k,A} R^A_{ak} R^{*A}_{bk} \simeq 2 \sum_{k,A} F^A_{ak} F^{*A}_{bk}$$

This exactly gives the Hellings-Downs correlation.

Hellings-Downs Curve

The Hellings-Downs curve can be obtained in two cases

2) If the sources is a **single plane wave** and one averages over a **large number of pulsars** and then **bins by angular separation**.

Both cases have in common that **interference is irrelevant**. However, including interference $\boldsymbol{\rho = HD}$ is not exactly recovered even for an **infinite number of sources and pulsars**! (~cosmic variance)

(see *”Variance of the Hellings-Downs correlation”*, B. Allen)

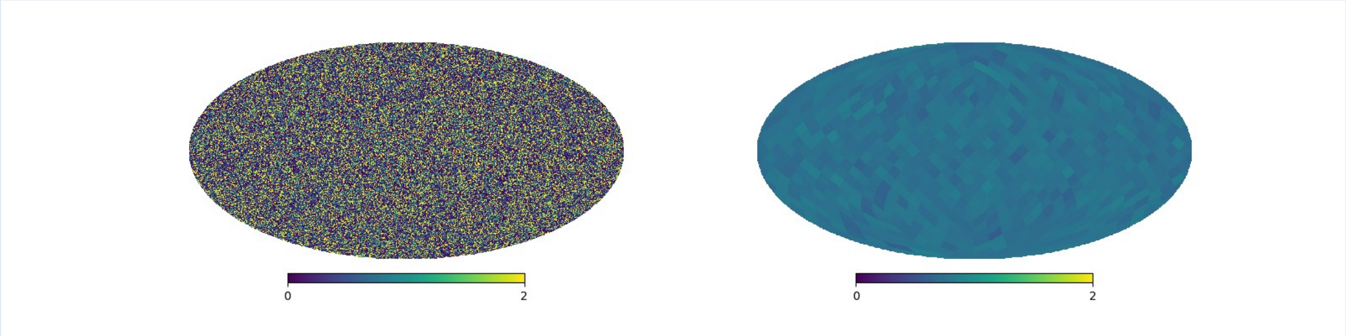
Neglecting interference

The analysis of anisotropies often neglect interference from the start

$$h_{k,A}h^*_{k',A'} = \delta_{kk'}\delta_{AA'}P(k)$$

which is motivated by the fact that this is often true in the average of ensemble statistical models. All the information is in the power spectrum.

For many sources (or small patches) the power seen by the experiment is limited by the resolution → coarse graining

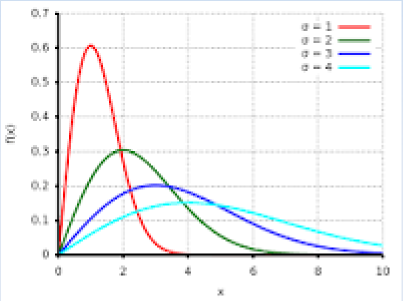


Any stochastic source looks isotropic **when interference is neglected**.

Adding interference

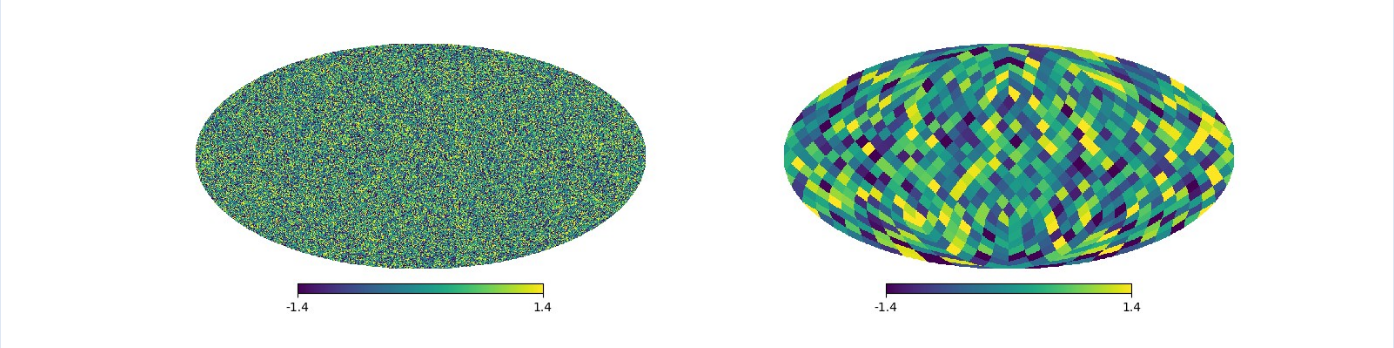
Stochastic sources are often modeled by the Gaussian ensemble. Every patch contains two complex amplitudes for the polarisation (per freqency).

The amplitude per patch is a Rayleigh distribution and the power is a χ^2 distribution



$$P(k) = \sum_A h_k^A h_k^{*A}$$

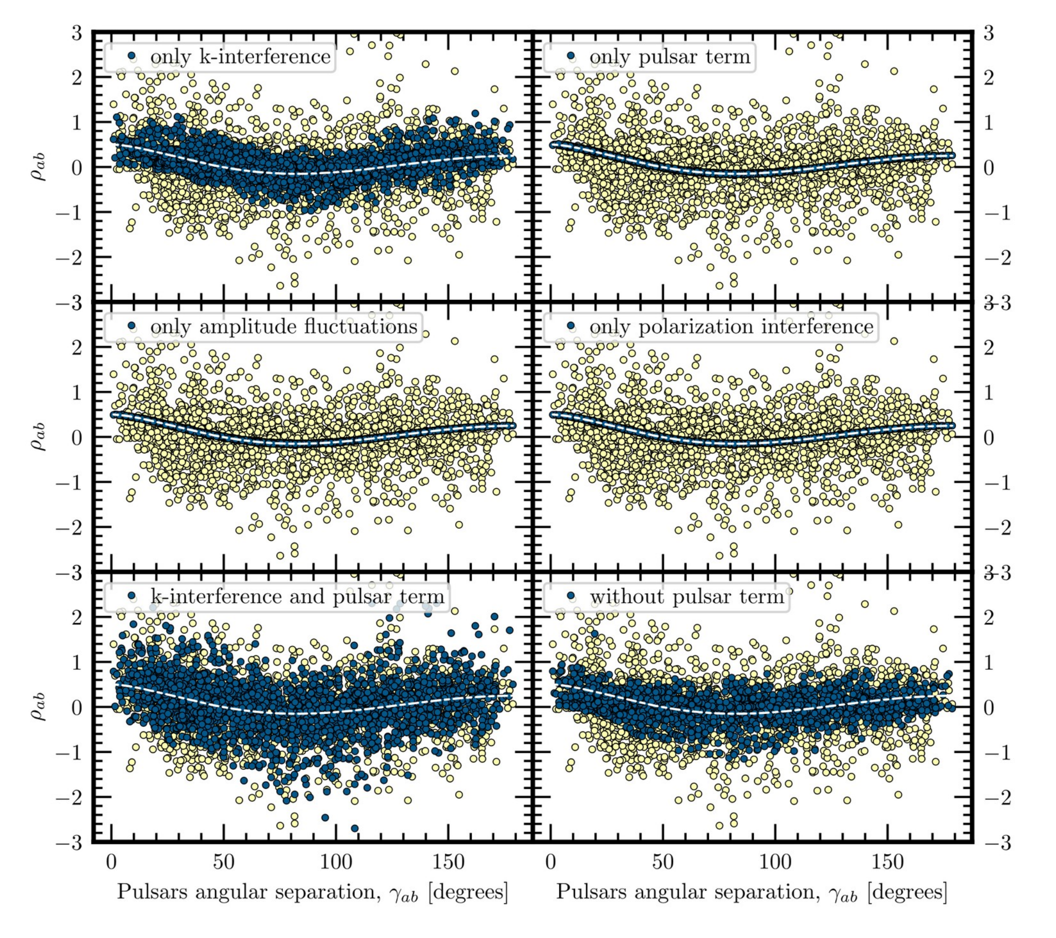
How does coarse graining work **including interference**?
The sum of Gaussians are Gaussian, e.g. for *Re* h_k^+



And the coarse-grained amplitude is still Rayleigh distributed.
Hence, **stochastic sources are not isotropic.**

Cosmic variance

There is another way to state the same fact: When interference is included, **cosmic variance is large** even in the limit of **many sources and many pulsars**.

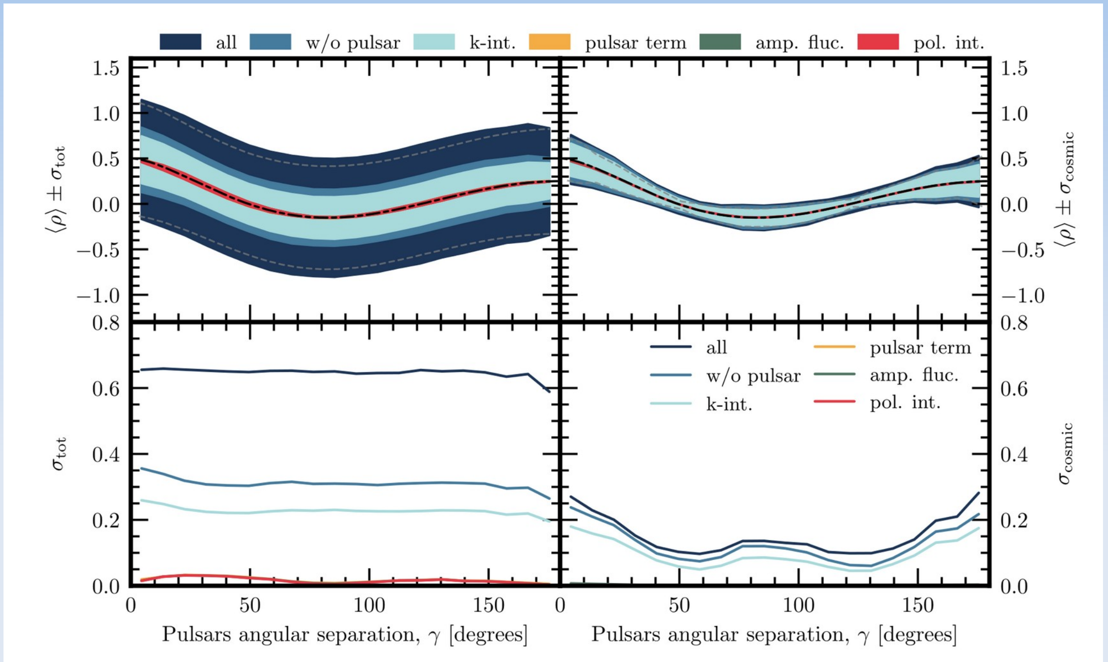


$$\rho_{ab} = \sum_j \sum_{kk'} \sum_{AA'} R^{A*}_{akj} R^{A'}_{bk'j} \, h^{A*}_{kj} h^{A'}_{k'j} \exp(i\Delta\phi^{AA'}_{kk',j}) \, \frac{\Delta\hat{\Omega}^2 \Delta f}{4\pi^2 f_j^2} + \text{c.c.} \, ,$$

Cosmic variance vs pulsar variance

The total variance of the HD-correlations has two components: Pulsar variance and cosmic variance.

Pulsar variance vanishes in the limit of many pulsars.



Cosmic variance vanishes in the limit of many realisations (but we only measure one! → irreducible)

Cosmic variance is mostly due to interference (not amplitude variations).

See also the very informative work
"Variance of the Hellings-Downs correlation", B. Allen

Anisotropies in NANOGrav

The anisotropy search in NANOGrav usues the following likelyhood function

$$p(\boldsymbol{\rho}|\boldsymbol{P}) = \frac{\exp[-\frac{1}{2}(\boldsymbol{\rho} - \boldsymbol{RP})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\rho} - \boldsymbol{RP})]}{\sqrt{\det(2\pi\boldsymbol{\Sigma})}} \,,$$

Where ***R*** is (another) response function and ***P*** parametrizes the power of the sky (e.g. pixel or spherical harmonics). The matrix **Σ** contains the cross-correlation uncertainties.

For a given cross-correlation ***ρ***, maximizing with respect to ***P*** reconstructs the sky map.

(notice the confusing notation $\boldsymbol{R}_{abk} \sim \boldsymbol{R}_{ak}\boldsymbol{R}_{bk}$ when interference neglected)

Anisotropies in NANOGrav

In order to decide if a sky is isotropic, one needs a **detection statistics**

$$\mathrm{SNR} = \sqrt{2 \ln \left[\frac{p(\boldsymbol{\rho}|\hat{\boldsymbol{P}})}{p(\boldsymbol{\rho}|\boldsymbol{P}_{\mathrm{iso}})} \right]} ,$$

In order to obtain a p-value, one determines the SNR distribution of a null-distribution

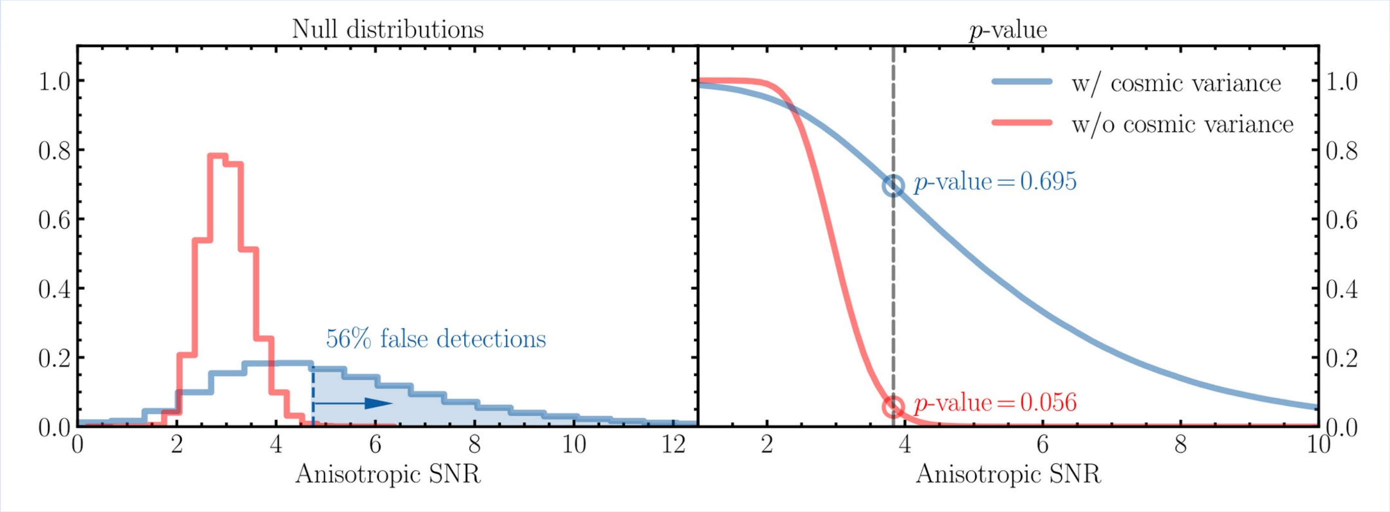
$$\rho_{ab} = HD_{ab} + Gauss(\Sigma_{ab})$$

But this neglects cosmic variance and also the correlations between the cross-correlations.

Instead, one can use the Gaussian ensemble to calculate null-distributions.

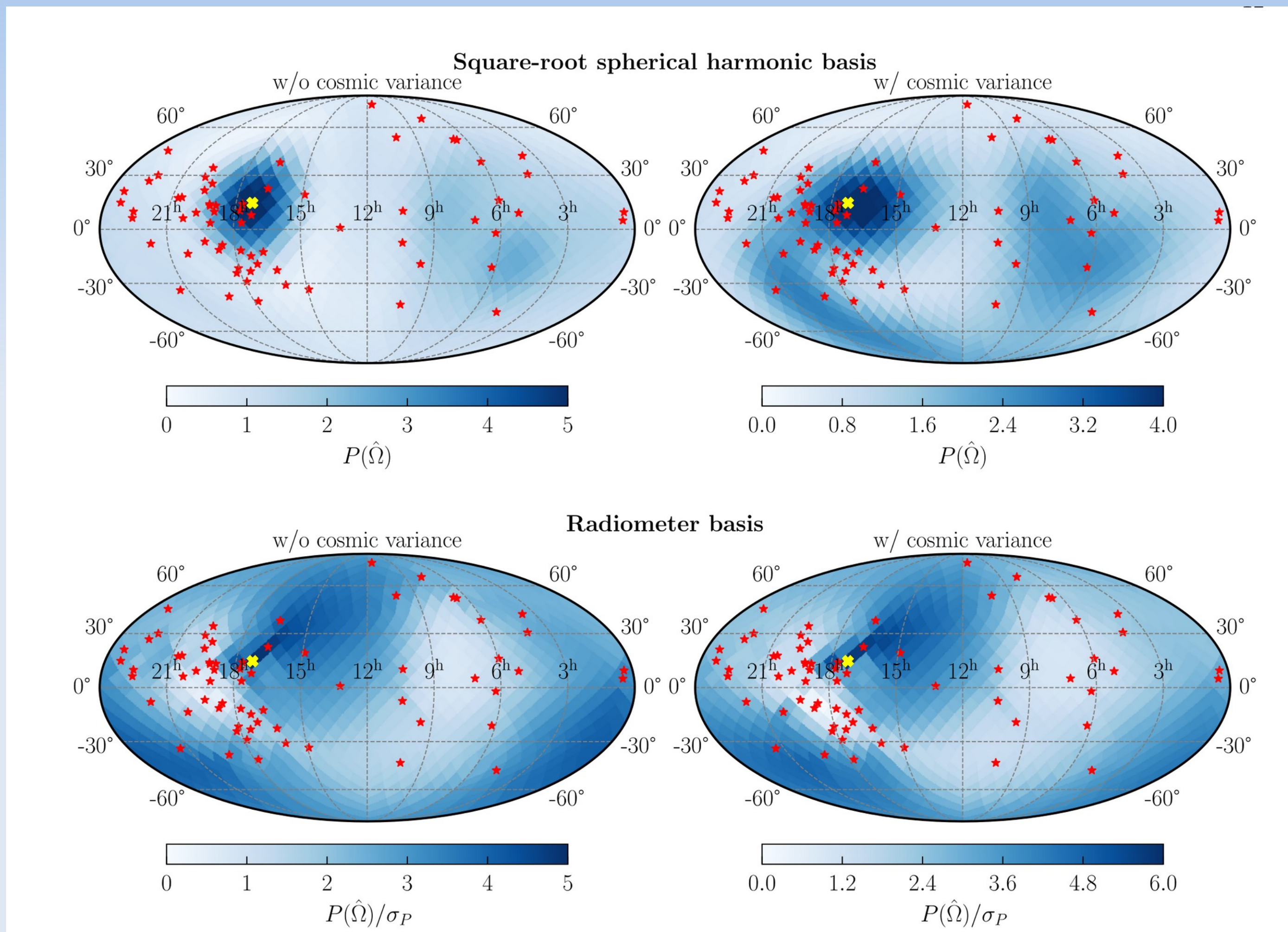
Anisotropies in NANOGrav

Including interference in this null-distribution makes a big difference:



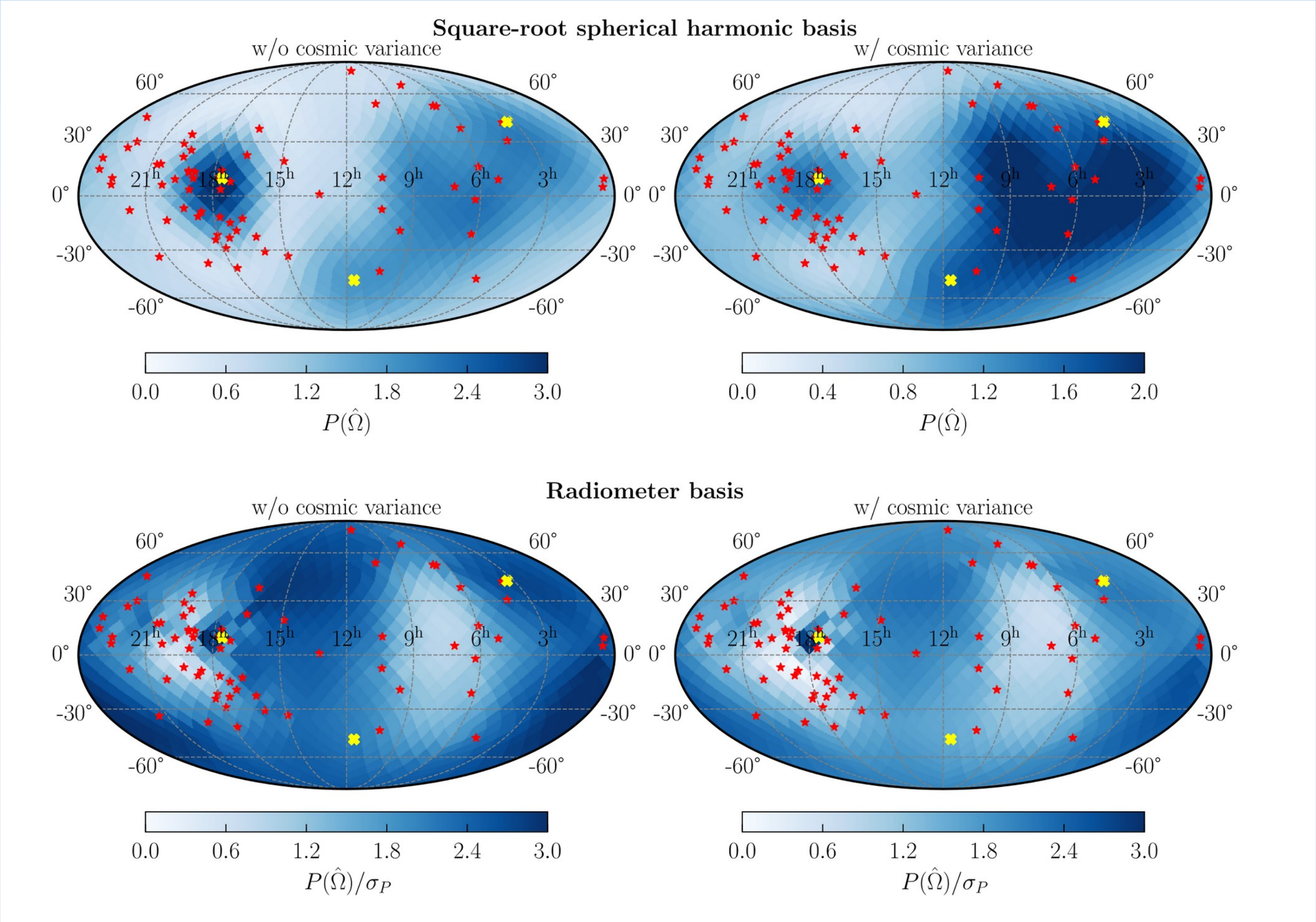
So including interference introduces a **confusion noise** that makes it harder to distinguish stochastic backgrounds from astrophysical backgrounds.

Reconstructing hot spots



Reconstructing a single hot spot in the sky is not inhibited by cosmic variance, since intereference is irrelevant.

Reconstructing hot spots



Reconstructing multiple hot spots in the sky can be **inhibited** by cosmic variance, since intereference is **relevant**.

Conclusion

Neglecting interference in the null-distribution of anisotropy searches leads to inflated p-values for the observation of astro-physical backgrounds.

Overall, interference makes it much harder to distinguish a stochastic (Gaussian ensemble) model from an astrophysical one (with hot spots).

Notice that ”cosmic variance” in the current context can be overcome by measuring anisotropies at different frequencies. (Instead of using the broadband approach, see Pitrou and Cusin, 2024).

This will however take much more data.