Gravitational waves from cosmological phase transitions

Thomas Konstandin

ICTS Bengalore, Jan 8, 2025

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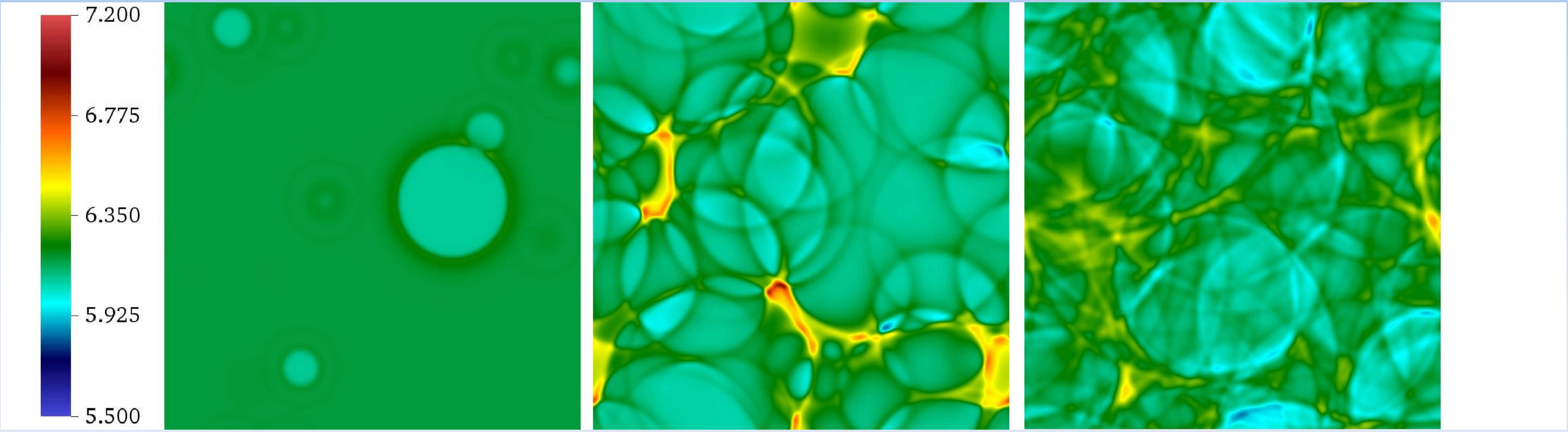
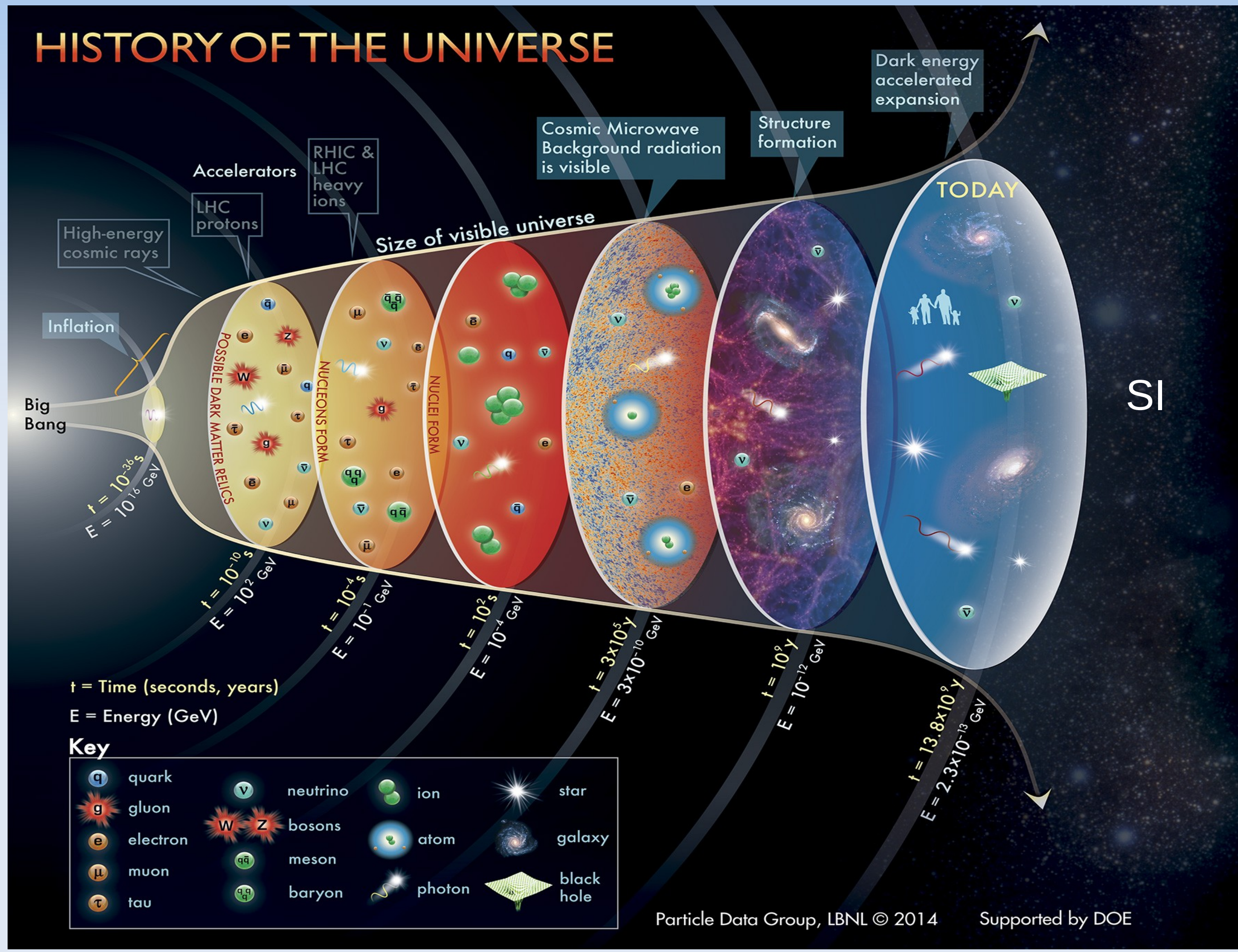


I. Introduction
II. Recent results
III. NanoGrav (tomorrow)

Standard Cosmology

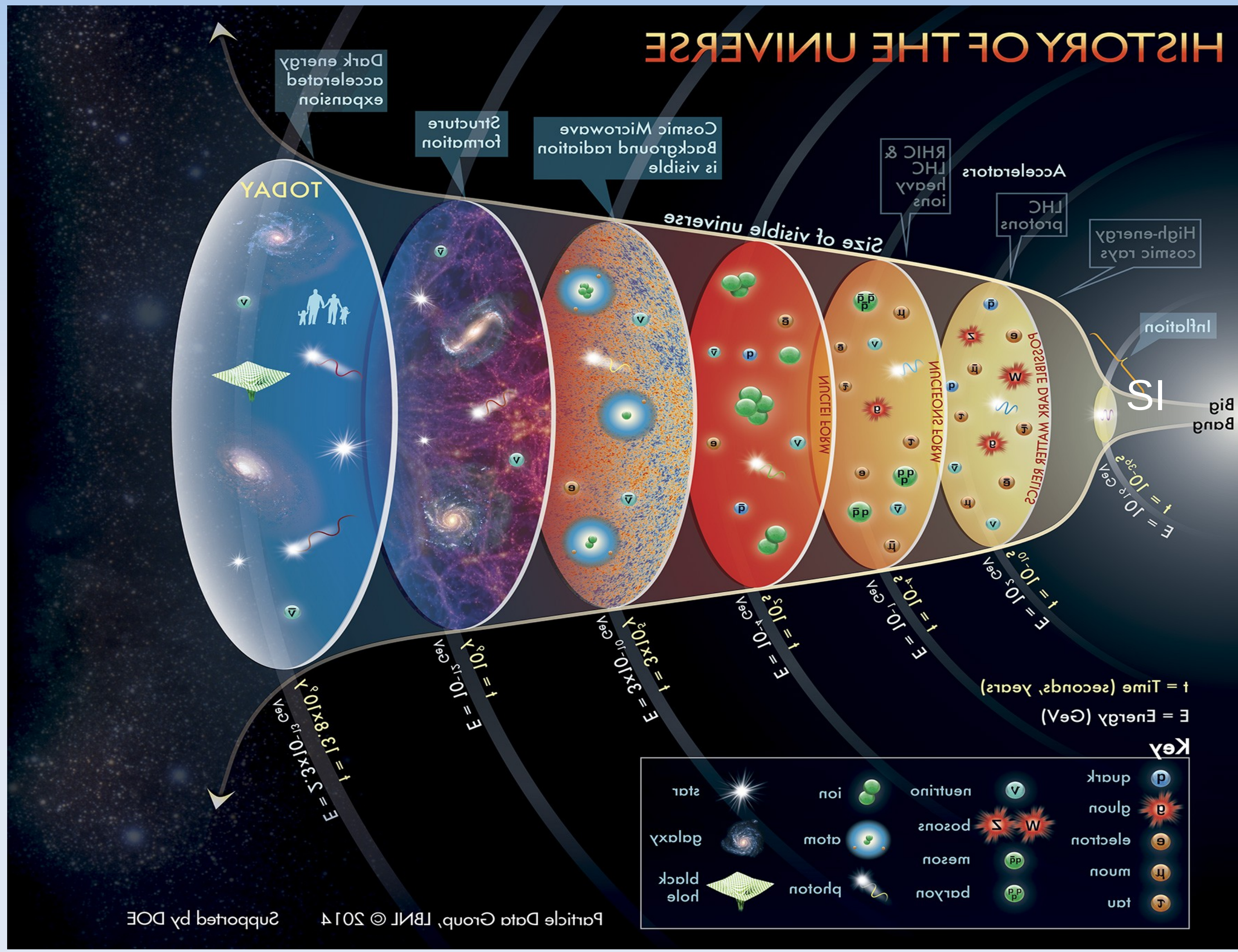


SI

time →

← temperature

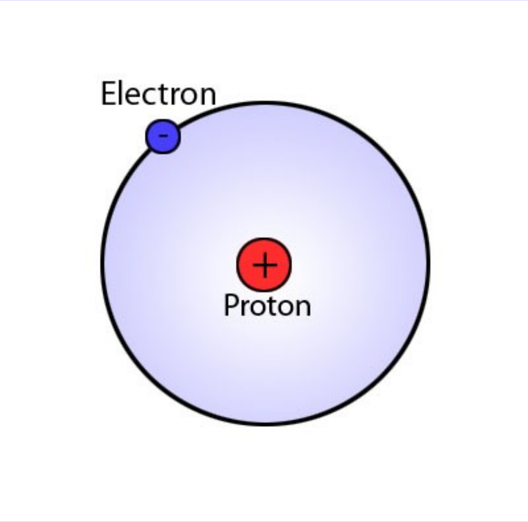
Standard Cosmology

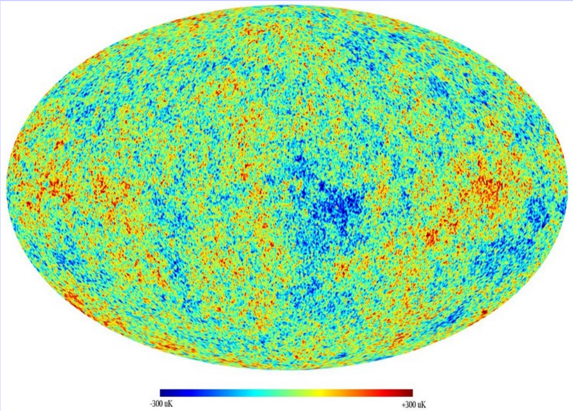


temperature →

← time

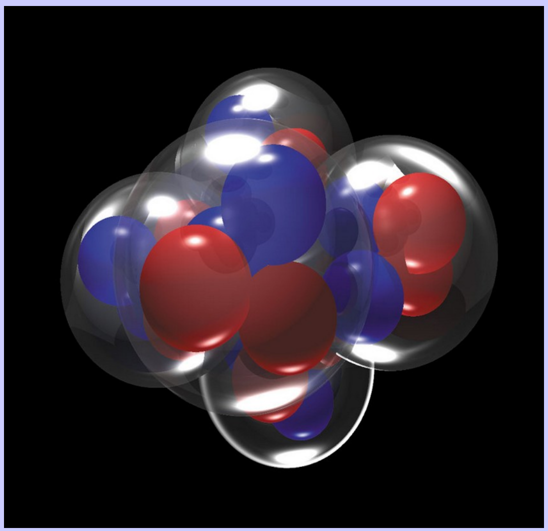
Atomic physics at T~eV

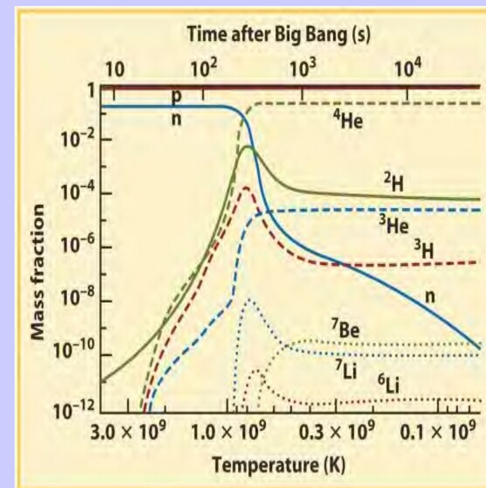




The Cosmic Microwave Background **links** atomic physics to cosmology at temperature T~eV

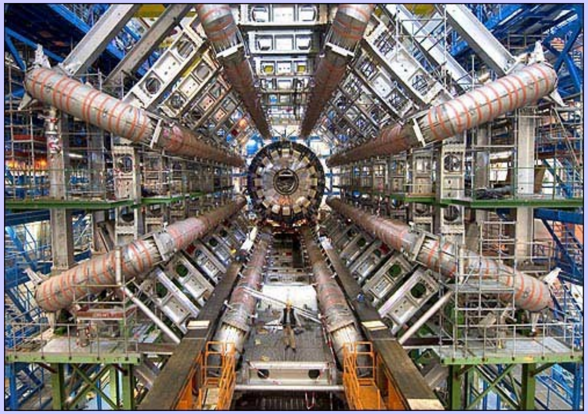
Nuclear physics at T~MeV

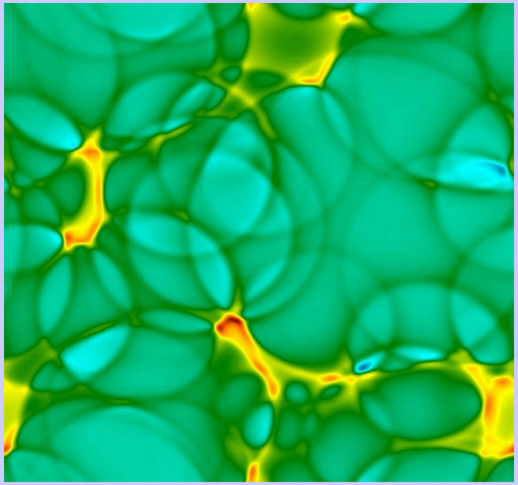




Big bang nucleosynthesis **links** nuclear physics to cosmology at temperature T~MeV

Phase transition at T~100 GeV?





Possibly, the electroweak phase transition drove the Universe **out-of-equilibrium.** This would provide a link to current particle physics experiments.

Electroweak phase transition

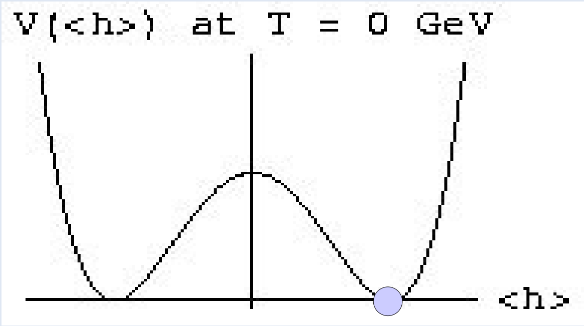
gravitational waves

baryogenesis

Electroweak symmetry breaking

The **Mexican hat** potential is designed to lead to a finite Higgs vacuum expectation value (VEV) and break the electroweak symmetry

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

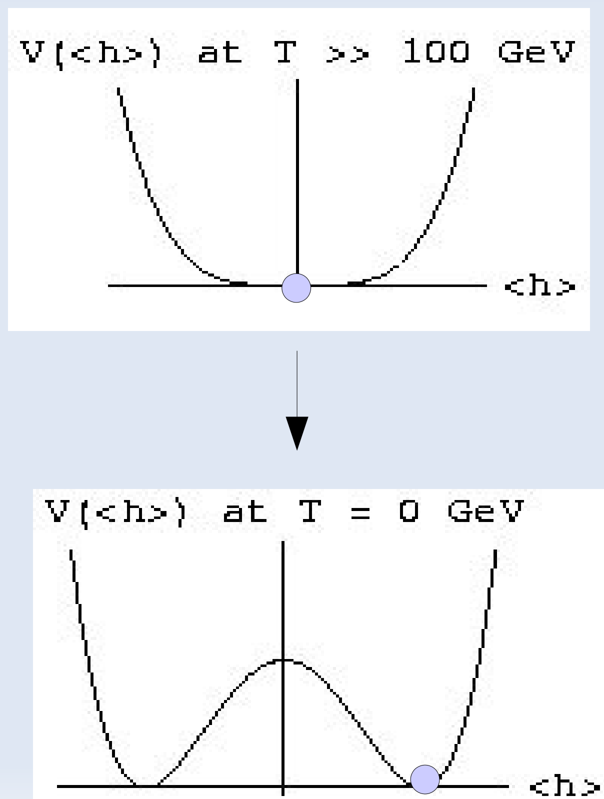


Electroweak symmetry breaking

[Weinberg '74]

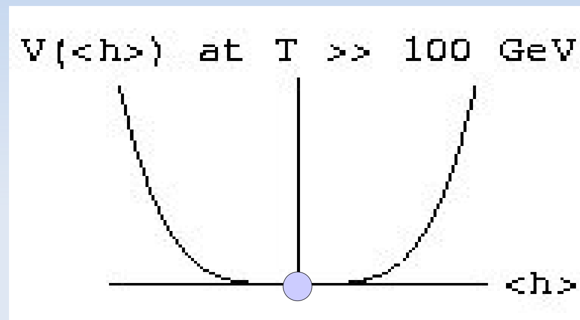
At large temperatures the symmetry is restored

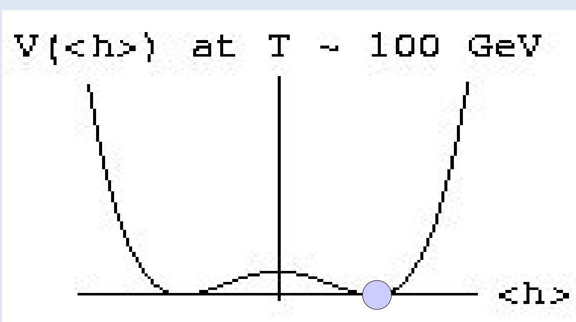
$$V(h,T) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + \text{const} \times h^2 T^2 + \text{details}$$

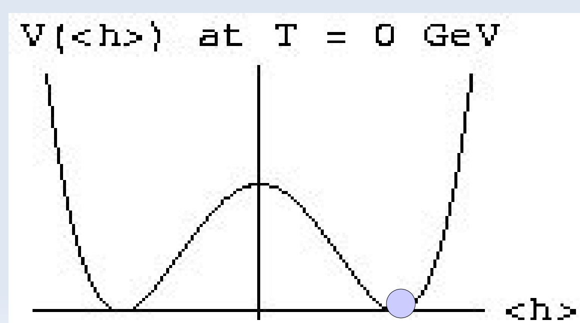


Electroweak symmetry breaking

Depending on the details, the phase transition can be very weak or even a cross over



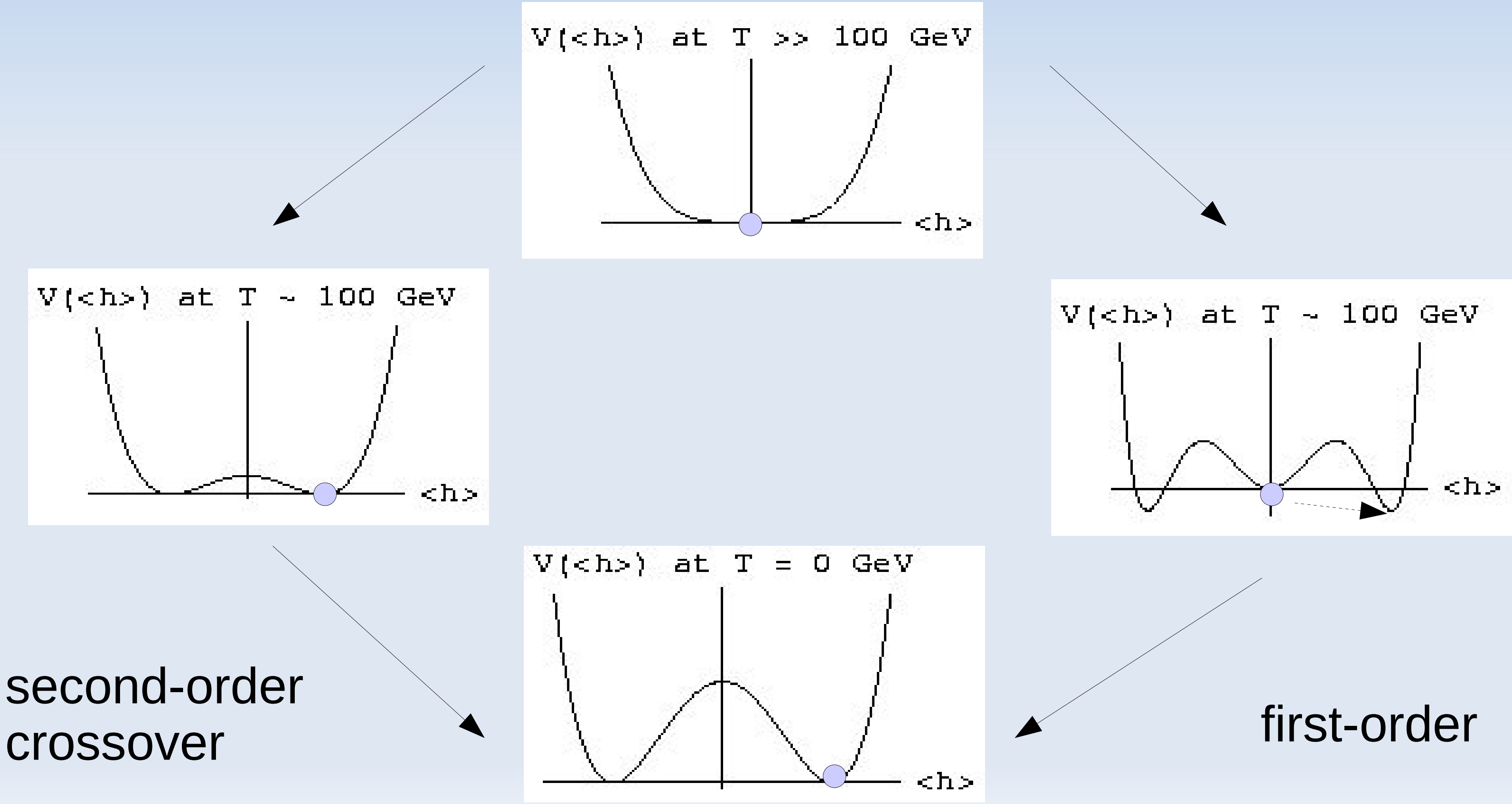




second-order
crossover

Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



Electroweak symmetry breaking

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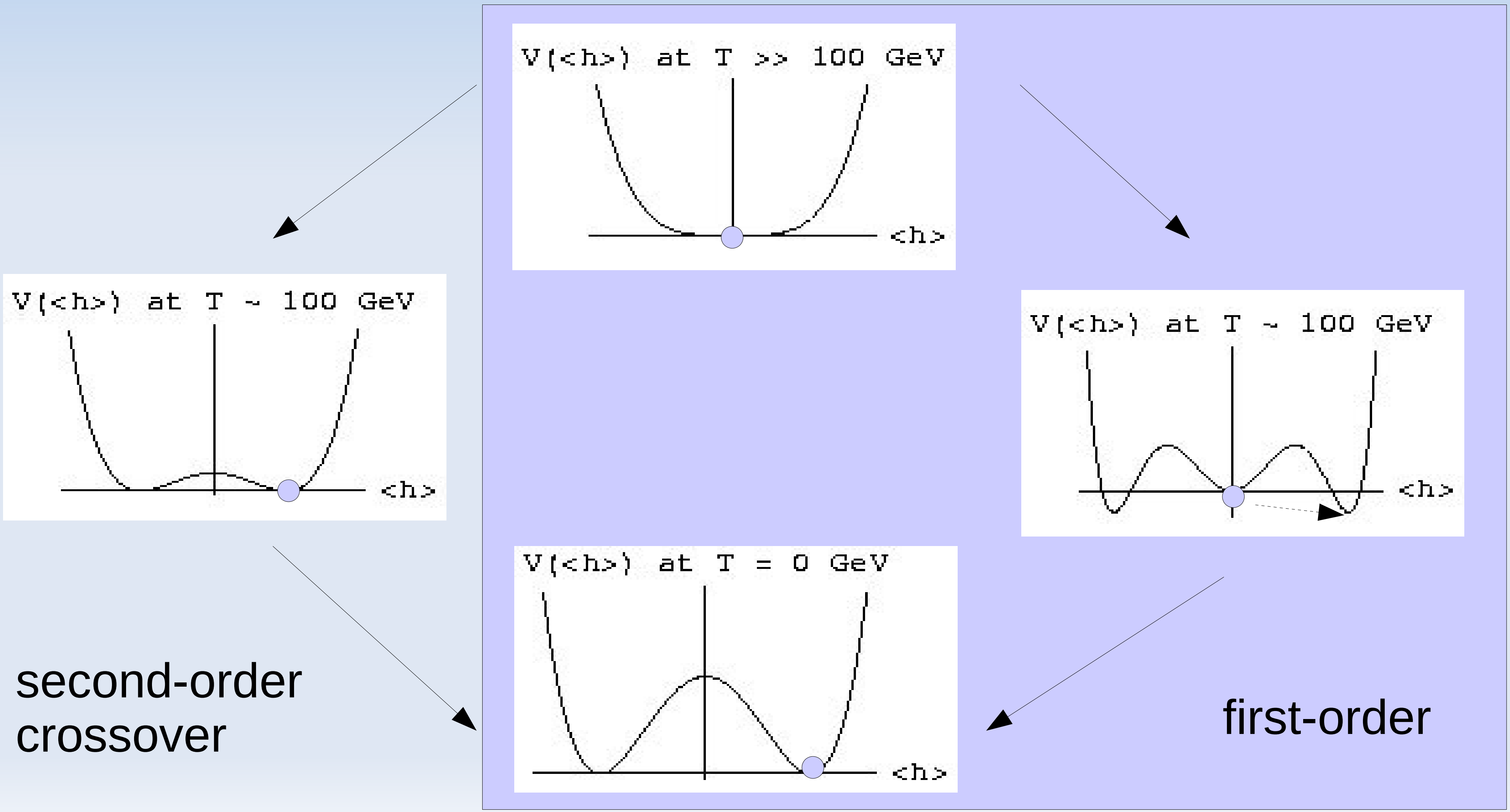
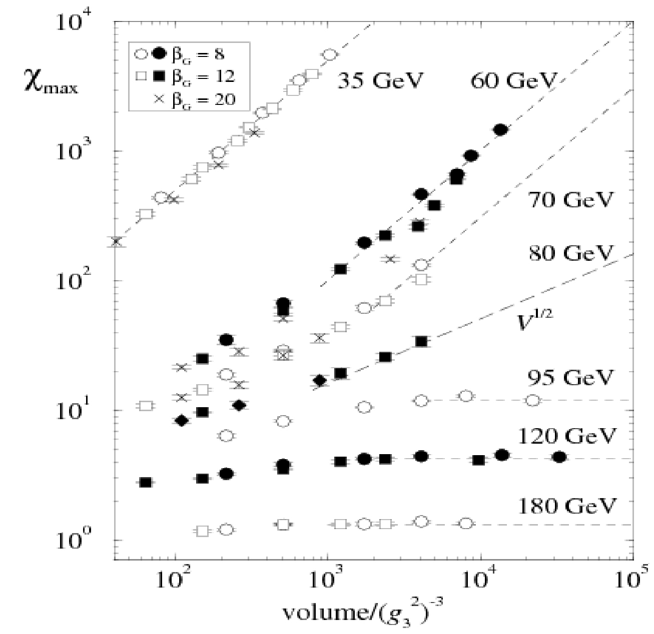


Electroweak phase transition in the SM

The effective potential is the standard tool to study phase transition at finite temperature.

Lattice studies show that there is a crossover in the SM.

A light Higgs would lead to a 1st-order PT.



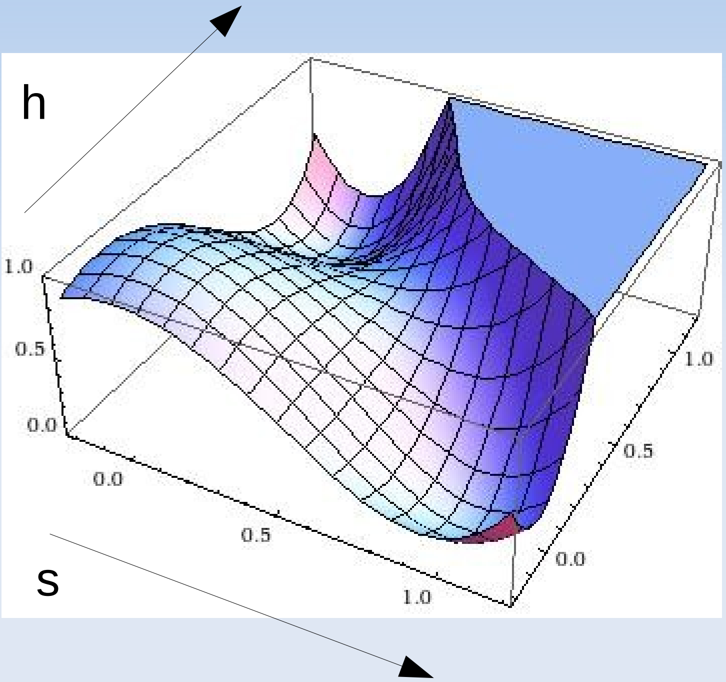
[Kajantie, Laine, Rummukainen, Shaposhnikov '96]

Singlet extension

The Standard Model only features a electroweak crossover.

A potential barrier and hence first-order phase transitions are quite common in extended scalar sectors:

$$V(h,s) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + m_s^2 s^2 + \lambda_s s^4 + \lambda_m s^2 h^2$$



The singlet field has an additional $\mathbb{Z}_2$ symmetry and is a viable DM candidate.

The phase transition proceeds via

$$(h,s) = (0,w) \to (h,s) = (v,0)$$

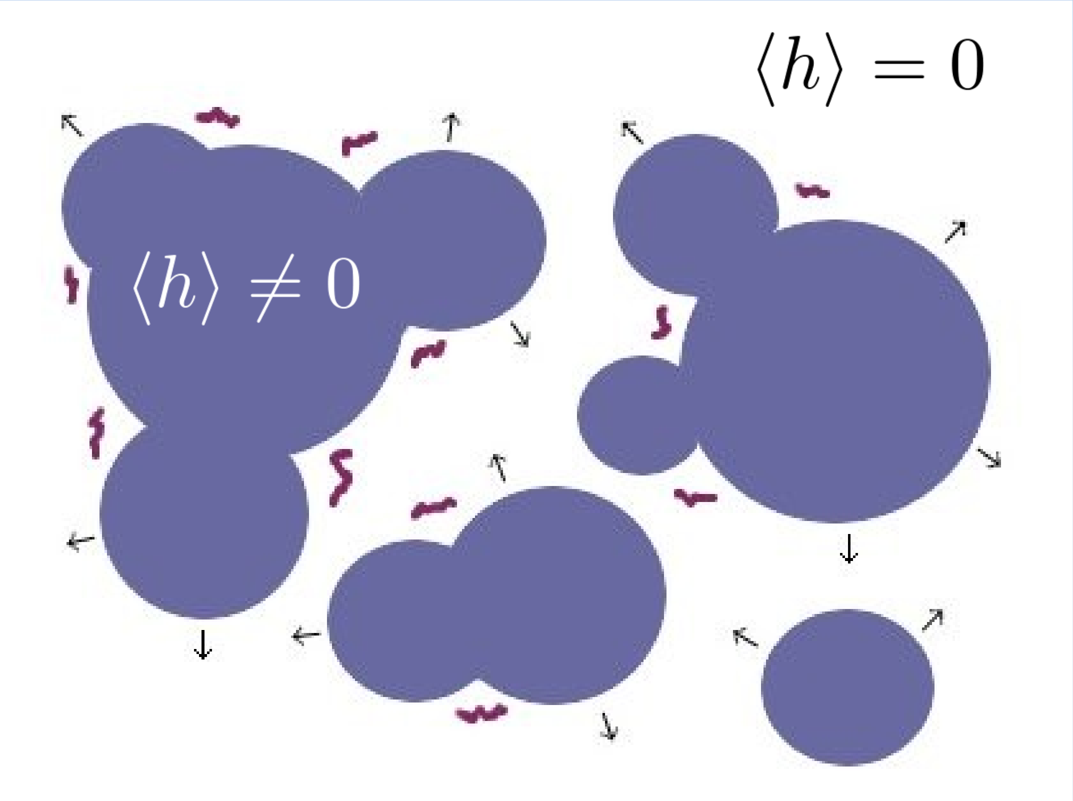
First-order phase transitions

• first-order phase transitions proceed by bubble nucleations

• in case of the electroweak phase transition, the ”Higgs bubble wall” separates the symmetric from the broken phase

• this is a violent process $(v_{wall} \simeq O(c))$ that drives the plasma out-of-equilibrium and sets the fluid into motion

Gravitational waves



During the first-order phase transitions, the nucleated bubbles expand. Finally, the colliding bubbles break spherical symmetry and generate stochastic gravitational waves.

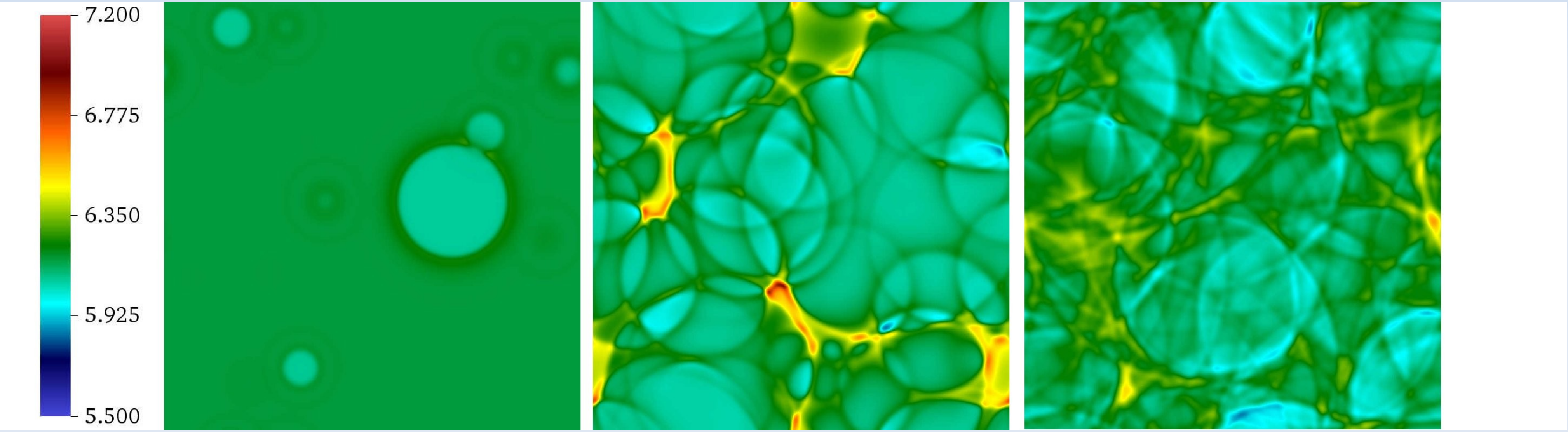
Sources of GWs from PTs

During and after the phase transition, several sources of GWs are active

- Collisions of the scalar field configurations / initial fluid shells
- Sound waves after the phase transition
  (long-lasting → dominant source)
- Turbulence
- Magnetic fields

In the last 10 years, simulations became the main tool to incorporate all these effects.

GWs from cosmological phase transitions



[Hindmarsh, Huber, Rummukainen, Weir '15]

Back of the envelope

There are several quantities that can enter in the determination of the GW spectrum:

The temperature of the phase transition T.

The (inverse) duration of the phase transition

$P \propto \exp(\beta t)$ and typically $\beta / H \sim O(100)$

The bubble size is $R_* \sim 3/\beta$ and the wall velocity v_w.

The amount of latent heat Λ that is transformed into kinetic energy *K* in the plasma:

$$\Lambda \to K \,, \quad \alpha = \frac{\Lambda}{\rho_{\rm tot}}$$

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Back of the envelope

The peak frequency at production is linked to the bubble size or the duration of the phase transition

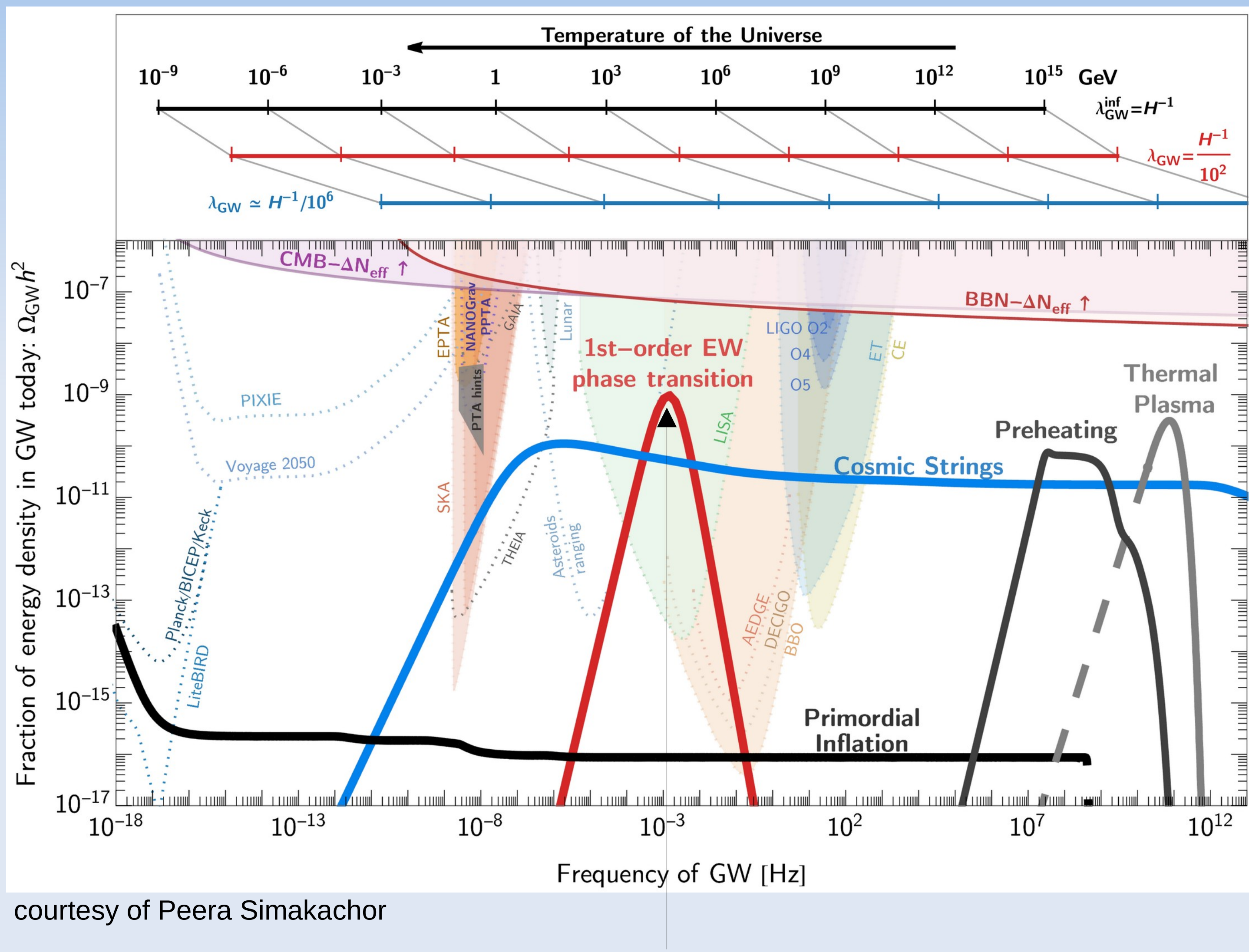
$$\omega^*_{\rm peak} \simeq 1/R_* \simeq \beta \simeq O(100)\, H$$

After the redshift, this amounts to

$$\omega_{\rm peak} \simeq \frac{\beta}{100\, H} \frac{T}{100 {\rm GeV}} \ {\rm mHz}$$

Since GWs behave as radiation, Ω_{GW} is only redshifted after the transition to matter domination by a factor ~10^{-5}.

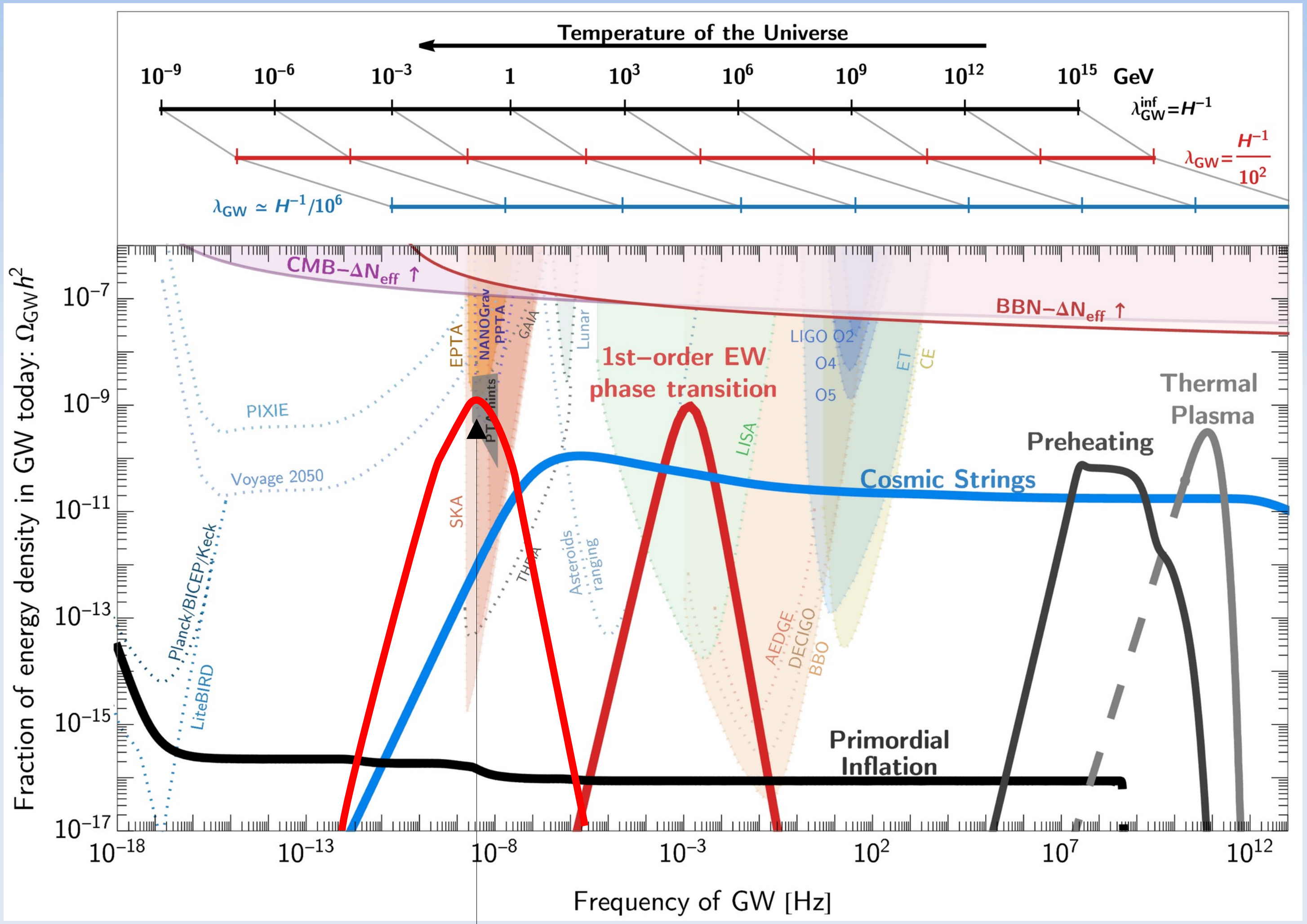
Stochastic GW landscape



courtesy of Peera Simakachor

PT at around T ~ 100 GeV

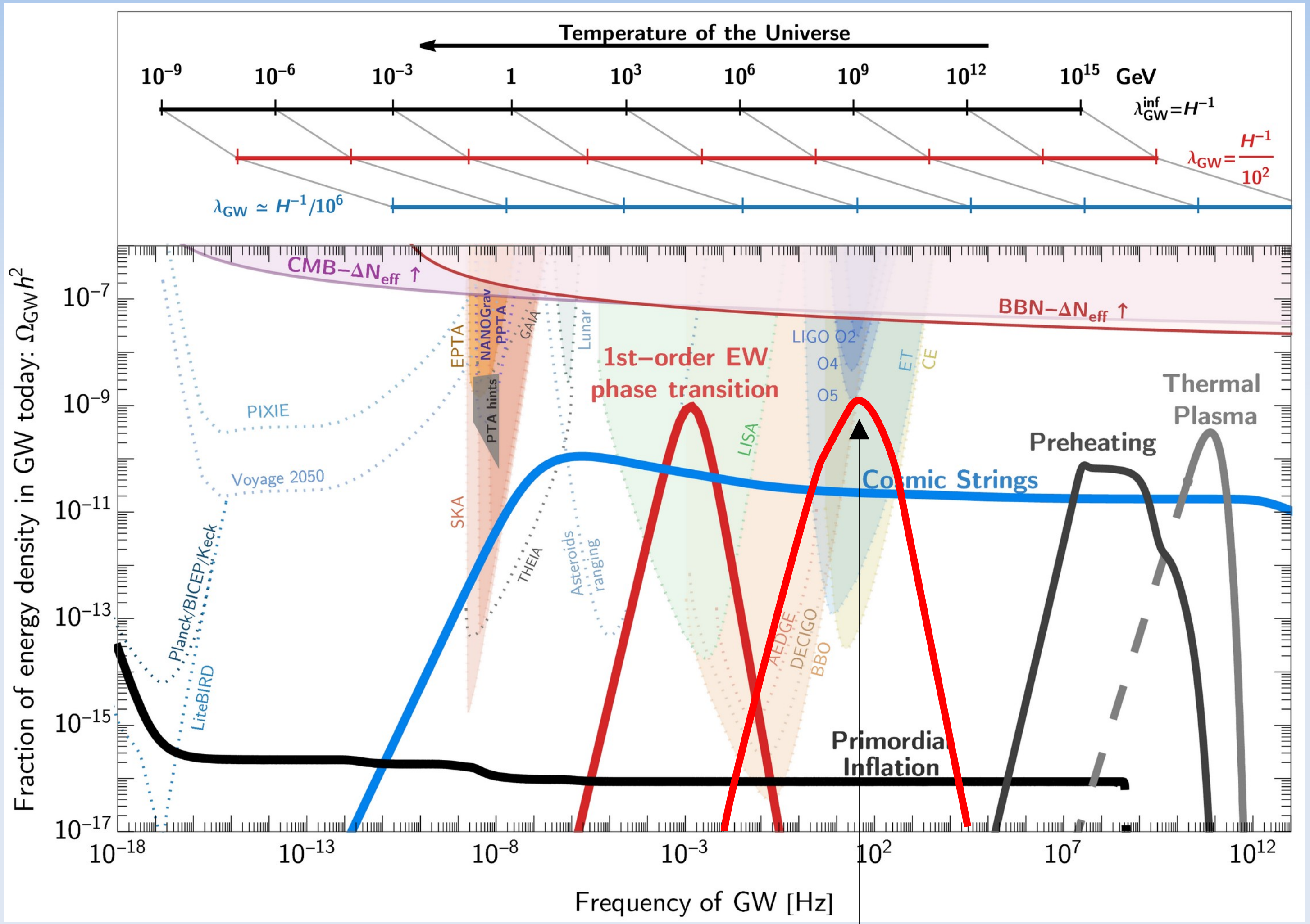
Stochastic GW landscape



courtesy of Peera Simakachor

PT at around T ~ 10 MeV

Stochastic GW landscape

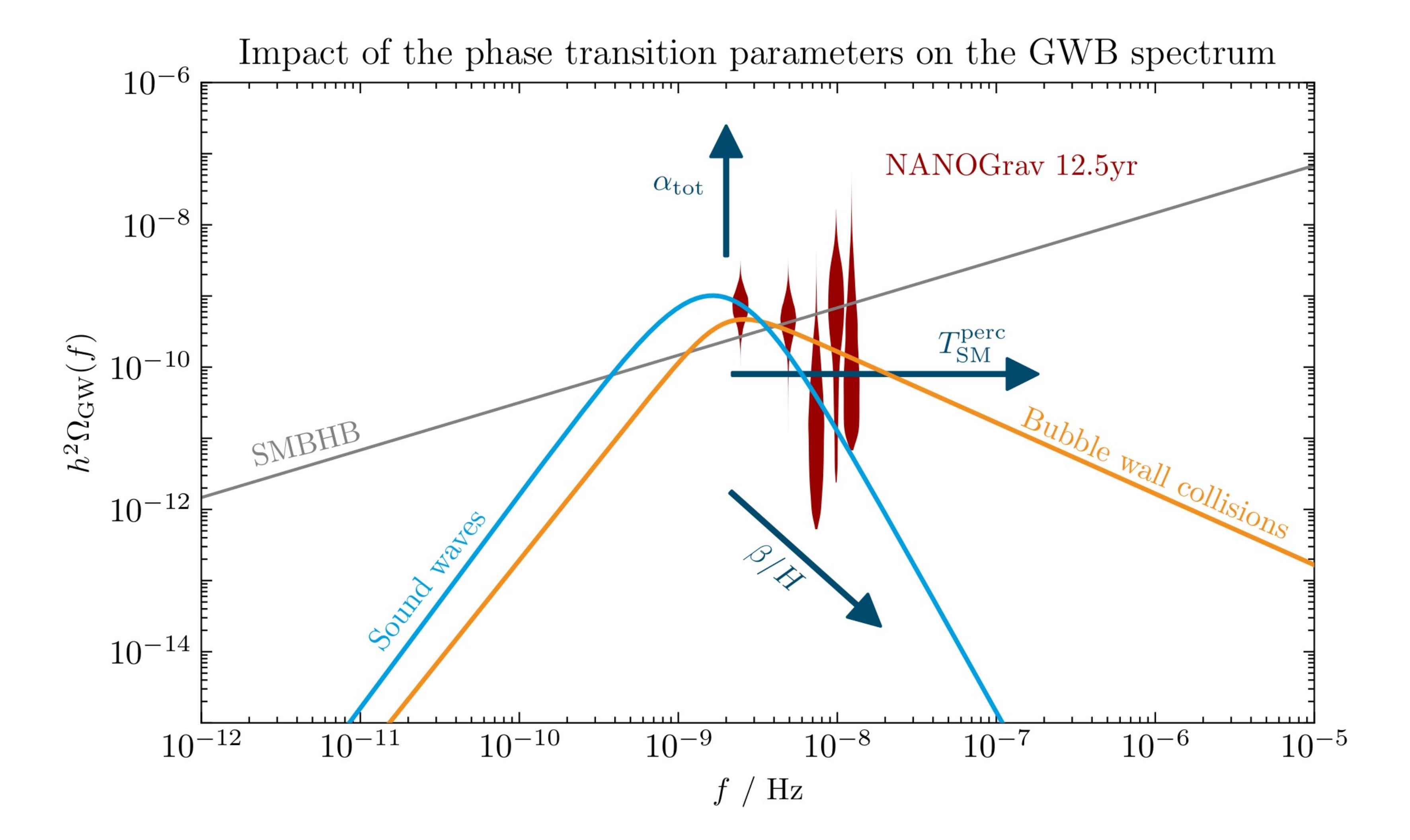


courtesy of Peera Simakachor

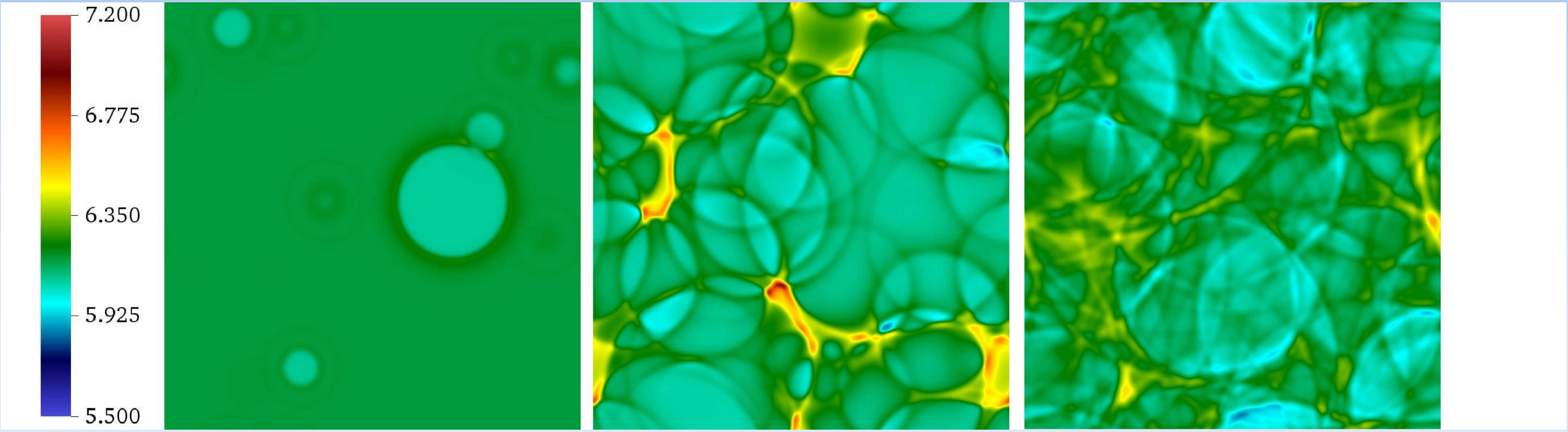
PT at around T ~ 10.000 TeV

Observation

dark sector phase transition, T ~ 10 MeV, v_w = 1



Gravitational waves from cosmological phase transitions



I. Introduction
II. Recent results
III. NanoGrav

The conventional approach

In the conventional approach, one solves the equation for the conservation of the total energy momentum tensor

$$\partial_\mu (T^{\mu\nu}_{fl} + T^{\mu\nu}_{\phi}) = 0$$

with

$$T^{\mu\nu}_{fl} = u^\mu u^\nu w - g^{\mu\nu} p, \qquad p = \frac{1}{3} a T^4$$

$$T^{\mu\nu}_{\phi} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left[\frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi + V_T(\phi, T) \right]$$

In combination with the Klein-Gordon equation including a friction term

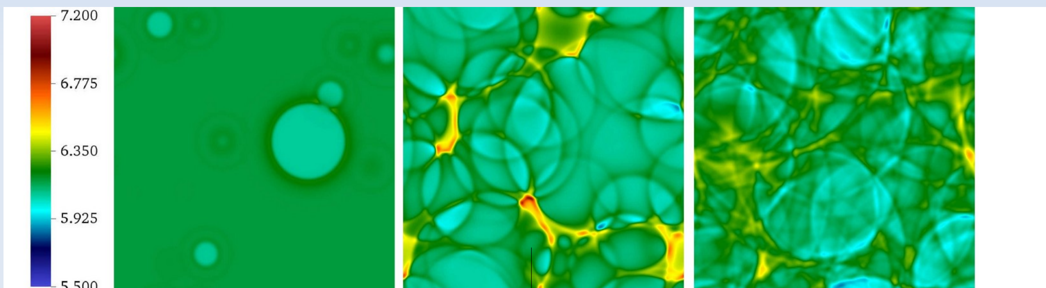
$$\Box \phi + dV/d\phi = \eta \, u^\mu \partial_\mu \phi \, .$$

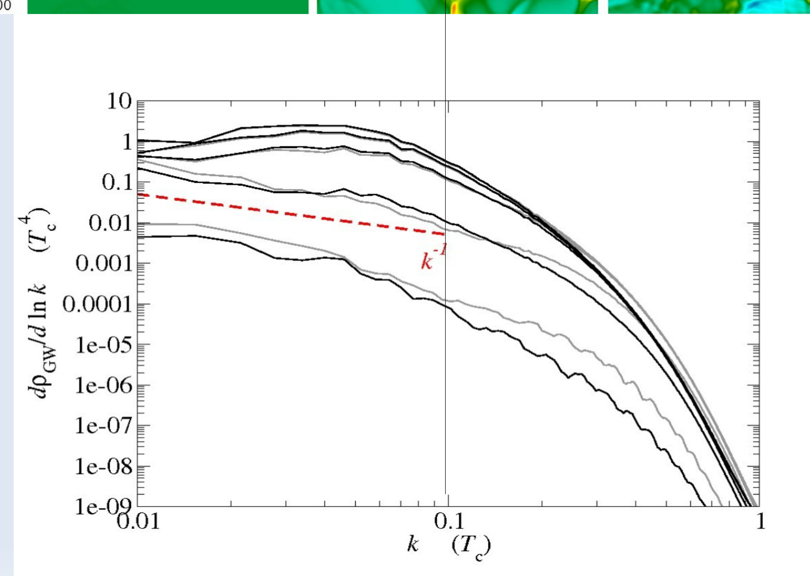
The friction coefficient η is tuned to obtained the required wall velocity. The simulation has to resolve the Higgs wall that transitions between the two phases.

State-of-the-art: simulations

[Hindmarsh, Huber , Rummukainen, Weir '13, ’15, ’17]
[Weir ’16] [Gould, Sukuvaara,Weir ’21] [Cutting, Hindmarsh, Weir ’18&’19]
[Cutting, Escartin, Hindmarsh, Weir ’20]

Depending on the context, the system can be descibed using hydrodynamics (fluid + Higgs) or just a scalar field





The produced GW spectrum can be read off from the simulation.

Really robust results, not many a priori assumptions.
But very costly.
How to extrapolate to other models and parameters?

Bubble wall thickness

The main challenge in the hydrodynamic simulation is to cover very different length scales.

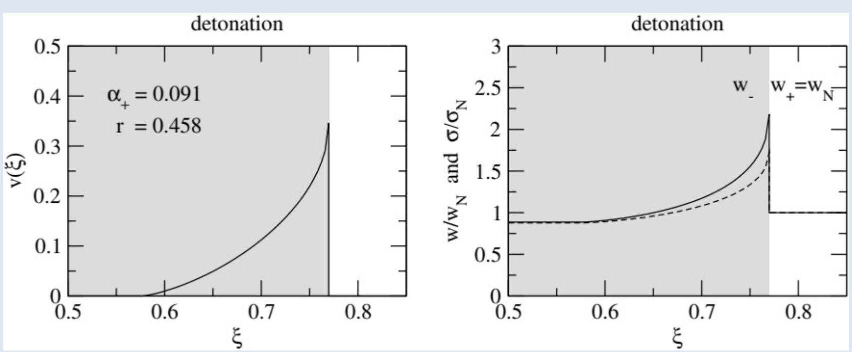
In the physical phase transition

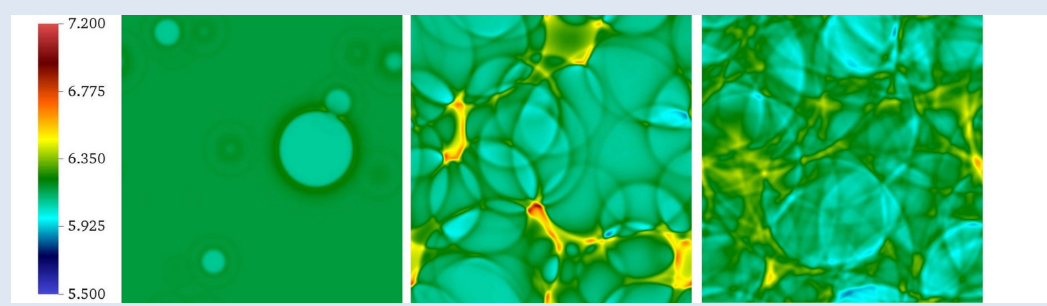
wall thickness <<<<<<< fluid shell thickness < bubble size

1/100GeV % of Hubble radius

In simulations:

grid spacing < (wall thickness < fluid shell thickness < bubble size) < box size





Higgsless approach

In the ”Higgsless approach” the scalar field is not dynamical. One solves the coservation equation of the fluid

$$\partial_\mu T^{\mu\nu}_{fl} = 0 \qquad\qquad T^{\mu\nu}_{fl} = u^\mu u^\nu w - g^{\mu\nu} p,$$

and the information of the Higgs field is encoded in the equation of state of the plasma using the bag equation of state

$$p = \frac{1}{3} a T^4 + \epsilon(t, \vec{x})$$

The bag constant is

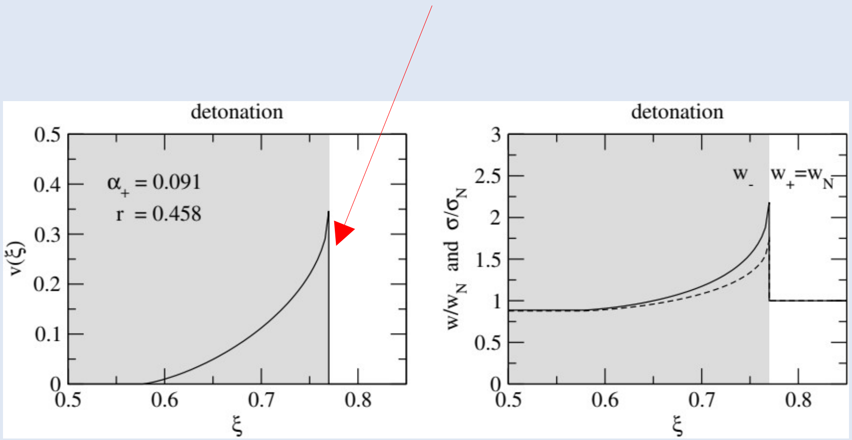
$$\epsilon(t, \vec{x}) = \begin{cases} \epsilon_0 & \text{outside bubbles} \\ 0 & \text{inside bubbles} \end{cases}$$

The nucleation time and place of the bubbles is (randomly) pre-calculated and the expansion of the bubbles uses a fixed wall velocity.

Higgsless simulations

Hence, we perform simulations that are agnostic about the wall thickness. This would resemble an *EFT* where the Higgs field was integrated out.

However, this requires a hydrodynamic numerical framework that can deal with *shocks* and other discontinuities:



Higgsless simulations

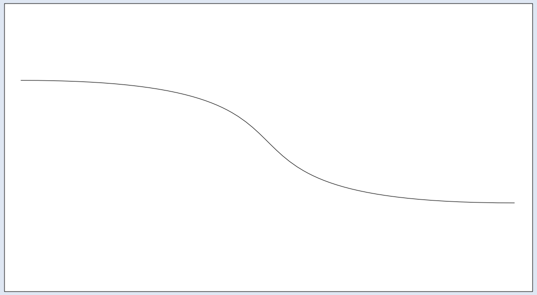
Consider the differential equation of a right-mover

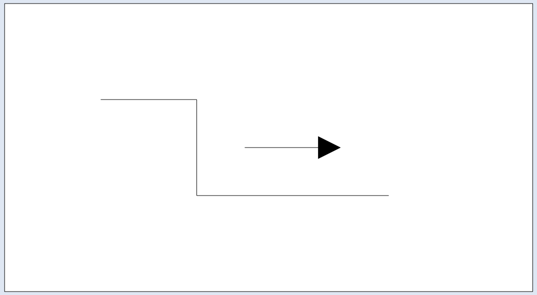
$$(\partial_t + \partial_x)\, f(t,x) = 0$$

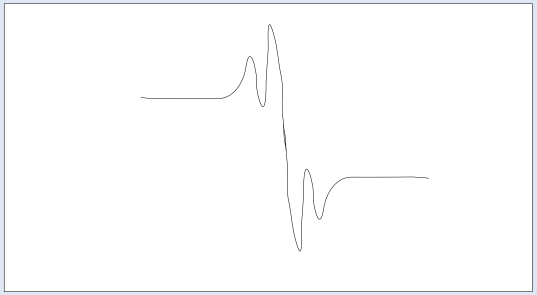
With the solution

$$f(t,x) = g(x-t)$$

When this equation is numerically solved, typically one of two issues occurs







too much viscosity
damping

not enough viscosity
Gibbs oscillations

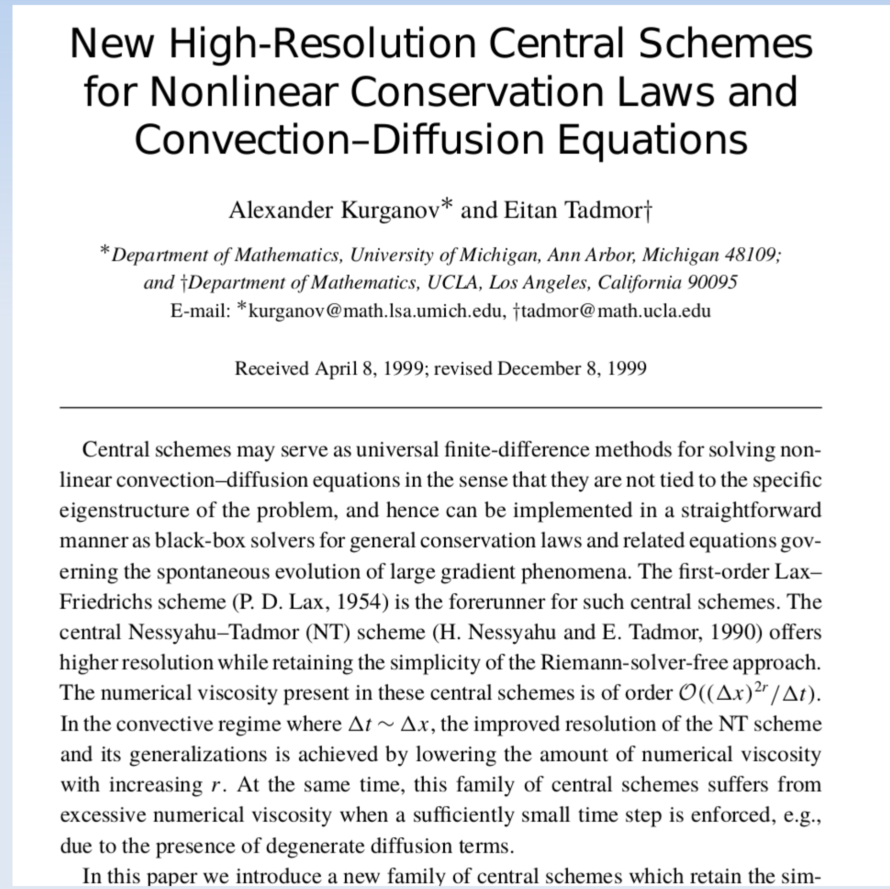
Higgsless simulations

Ideally one wants to have a scheme that abides to total variation diminishing to avoid oscillations.

Viscosity should be minimal to reduce damping.

This can be achieved via hybridization (adding non-linear terms) in a semi-discrete scheme.

For conservation laws, this is for example possible via the Kurganov-Tadmor method.



New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection–Diffusion Equations

Alexander Kurganov* and Eitan Tadmor†

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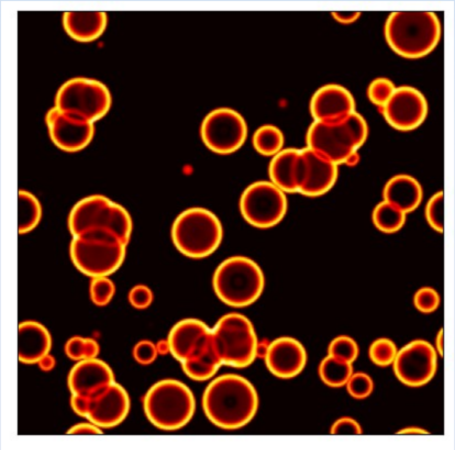
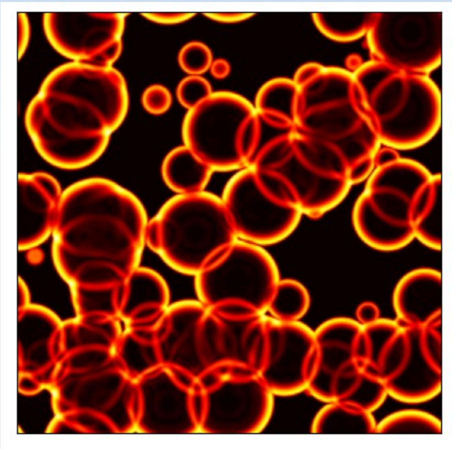
Central schemes may serve as universal finite-difference methods for solving nonlinear convection–diffusion equations in the sense that they are not tied to the specific eigenstructure of the problem, and hence can be implemented in a straightforward manner as black-box solvers for general conservation laws and related equations governing the spontaneous evolution of large gradient phenomena. The first-order Lax–Friedrichs scheme (P. D. Lax, 1954) is the forerunner for such central schemes. The central Nessyahu–Tadmor (NT) scheme (H. Nessyahu and E. Tadmor, 1990) offers higher resolution while retaining the simplicity of the Riemann-solver-free approach. The numerical viscosity present in these central schemes is of order $\mathcal{O}((\Delta x)^{2r}/\Delta t)$. In the convective regime where $\Delta t \sim \Delta x$, the improved resolution of the NT scheme and its generalizations is achieved by lowering the amount of numerical viscosity with increasing r. At the same time, this family of central schemes suffers from excessive numerical viscosity when a sufficiently small time step is enforced, e.g., due to the presence of degenerate diffusion terms.

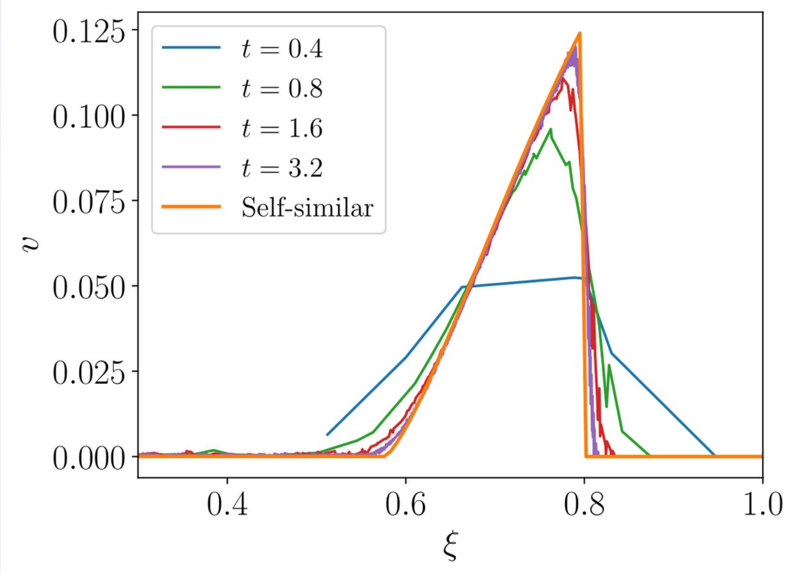
In this paper we introduce a new family of central schemes which retain the sim-

Simulation of cosmological phase transitions

Based on this, we developed a highly efficient scheme to simulate relativistic hydrodynamics during cosmological first-order phase transitions.



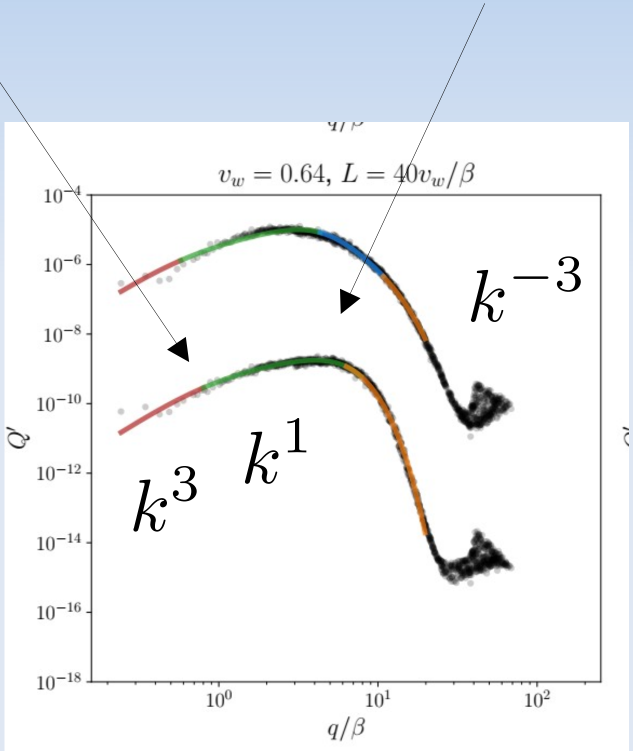




These simulations allow to extract GW spectra from the phase transition in a few hours instead of weeks (factor **2000 speed improvement** compared to former approaches)

Simulation of cosmological phase transitions

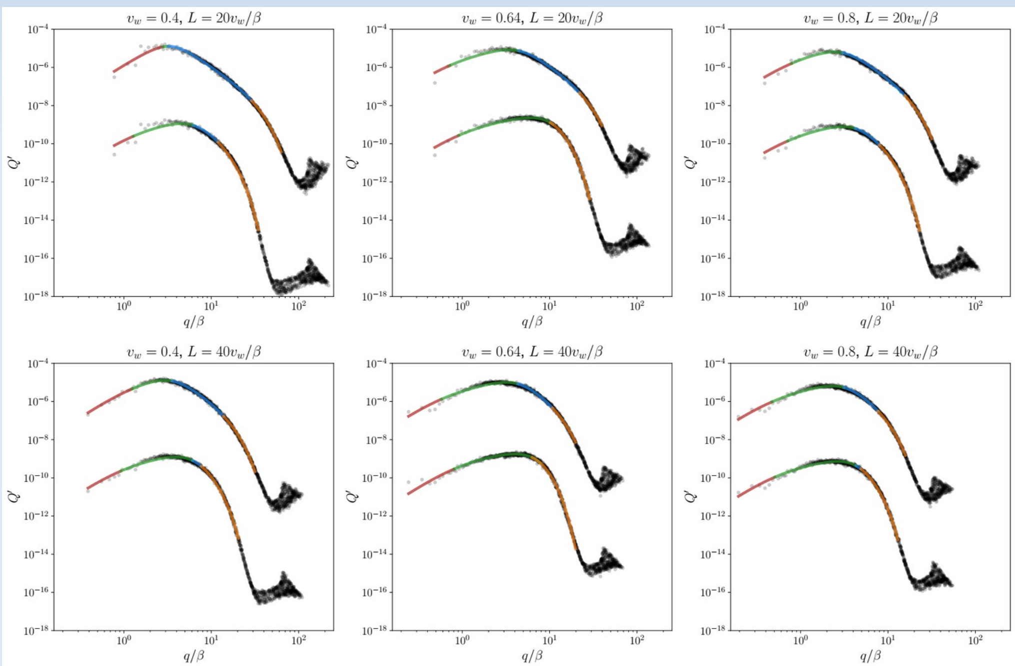
The spectra have two features due to the bubble size and the shell thickness.

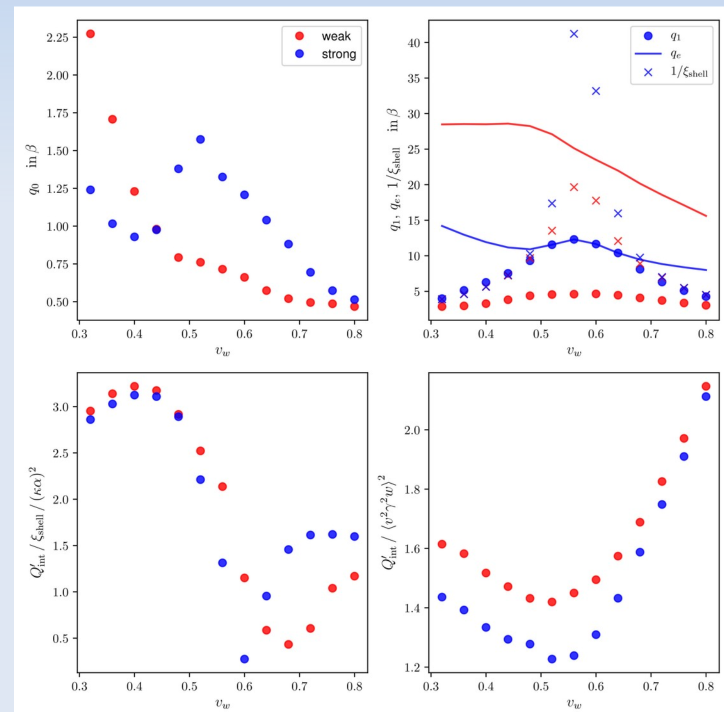


[Jinno, TK, Rubira, Stomberg 2022]
[Hindmarsh 2016]

Simulation of cosmological phase transitions

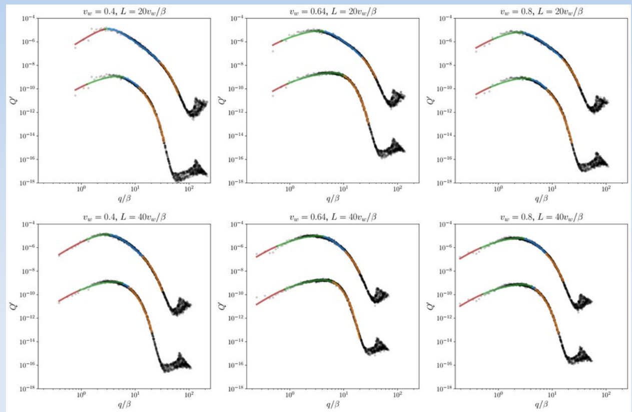
The setup allows to run many simulations a day and to extract the GW spectra as functions of the PT properties: wall velocity v_w, PT strength α

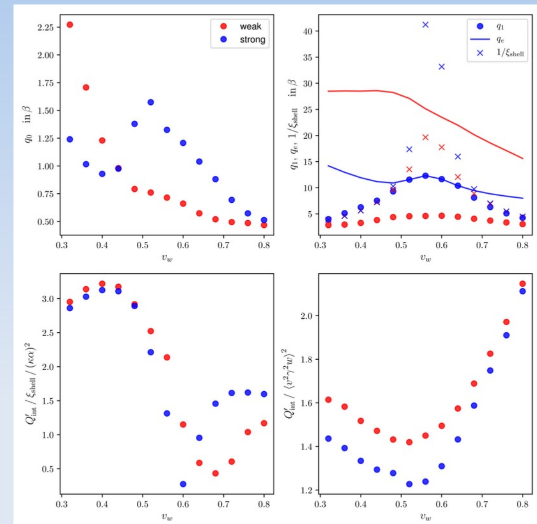




[Jinno, TK, Rubira, Stomberg 2022]

Some conclusions





Using these simultions, by now, we have very accurate and reliable predictions of GW spectra from cosmological phase transitions.

There are still some loose end though: deep IR, role of turbulence, non-linear regime

[Jinno, TK, Rubira, Stomberg 2022]

Entering the strong regime

We simulate for the first time the hydrodynamics of very strong PTs.

We observe systematically the decay of kinetic energy.

We also observe vorticity, possibly connected to turbulence.

Gravitational waves from decaying sources in strong phase transitions

Chiara Caprini,[a,b] **Ryusuke Jinno,**[c] **Thomas Konstandin,**[d] **Alberto Roper Pol,**[a,1] **Henrique Rubira,**[e] **Isak Stomberg**[d,2]

Abstract. We study the generation of gravitational waves (GWs) during a first-order cosmological phase transition (PT) using the recently introduced Higgsless approach to numerically evaluate the fluid motion induced by the PT. We present for the first time spectra from strong first-order PTs ($\alpha = 0.5$), alongside weak ($\alpha = 0.0046$) and intermediate ($\alpha = 0.05$) transitions previously considered in the literature. We test the regime of applicability of the stationary source assumption, characteristic of the sound-shell model, and show that it agrees with our numerical results when the kinetic energy, sourcing GWs, does not decay with time. However, we find in general that for intermediate and strong PTs, the kinetic energy in our simulations decays following a power law in time, and provide a theoretical framework that extends the stationary assumption to one that allows to include the time evolution of the source. This decay of the kinetic energy, potentially determined by non-linear dynamics and hence, related to the production of vorticity, modifies the usually assumed linear growth with the source duration to an integral over time of the kinetic energy fraction, effectively reducing the growth rate. We validate the novel theoretical model with the results of our simulations covering a broad range of wall velocities. We provide templates for the GW amplitude and spectral shape for a broad range of PT parameters.

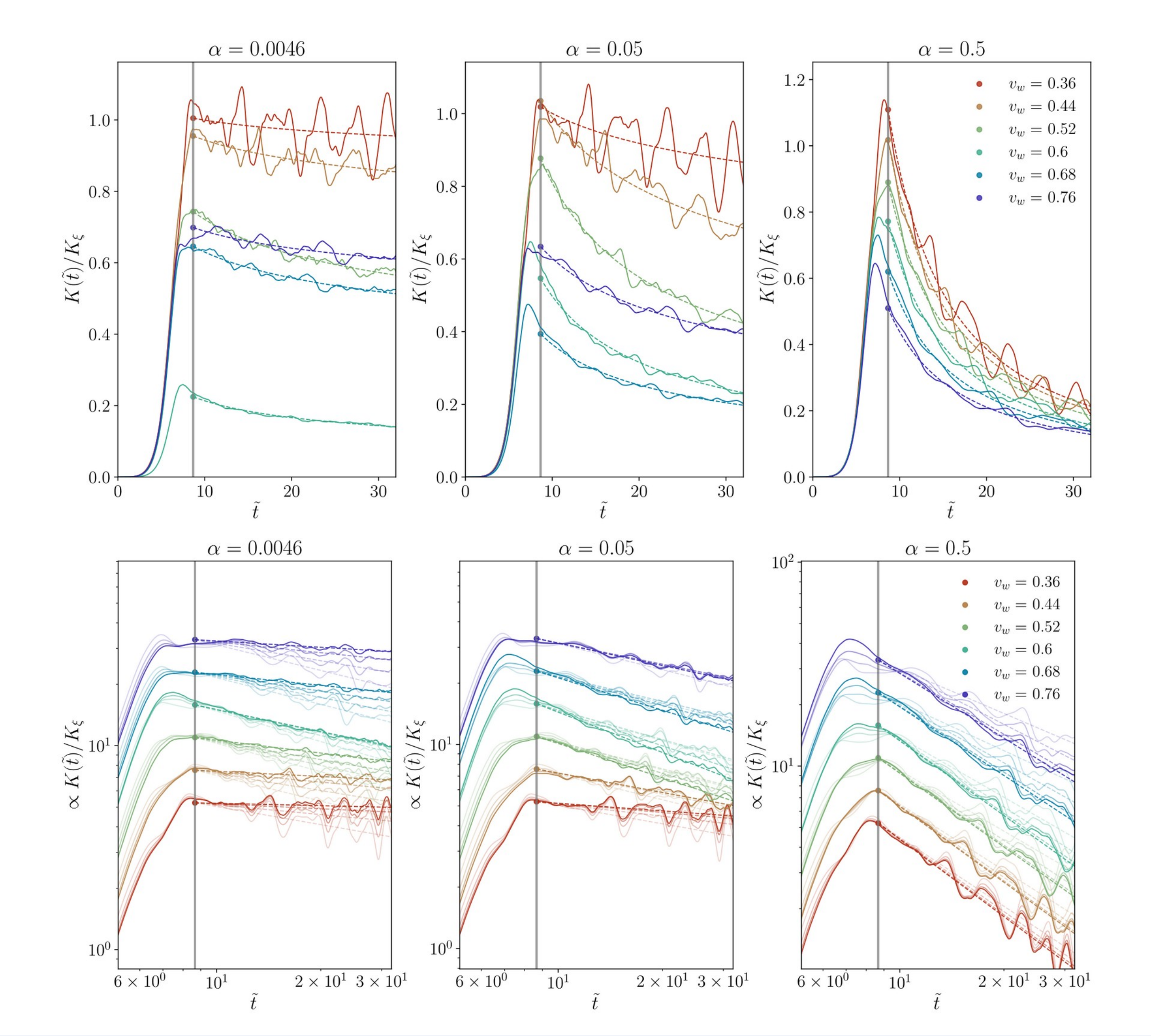
Decay of kinetic energy

The kinetic energy tends to decay over time for strong PTs.

We systematically study this by increasing the grid resolution.

Total energy is conserved, so this is a conversion: kinetic energy → heat

We also observe a lift of the spectral tilt in the UV and vorticity in the plasma → turbulence?



Incorporating decay

The parametric dependence of the GWs on the PT parameters is

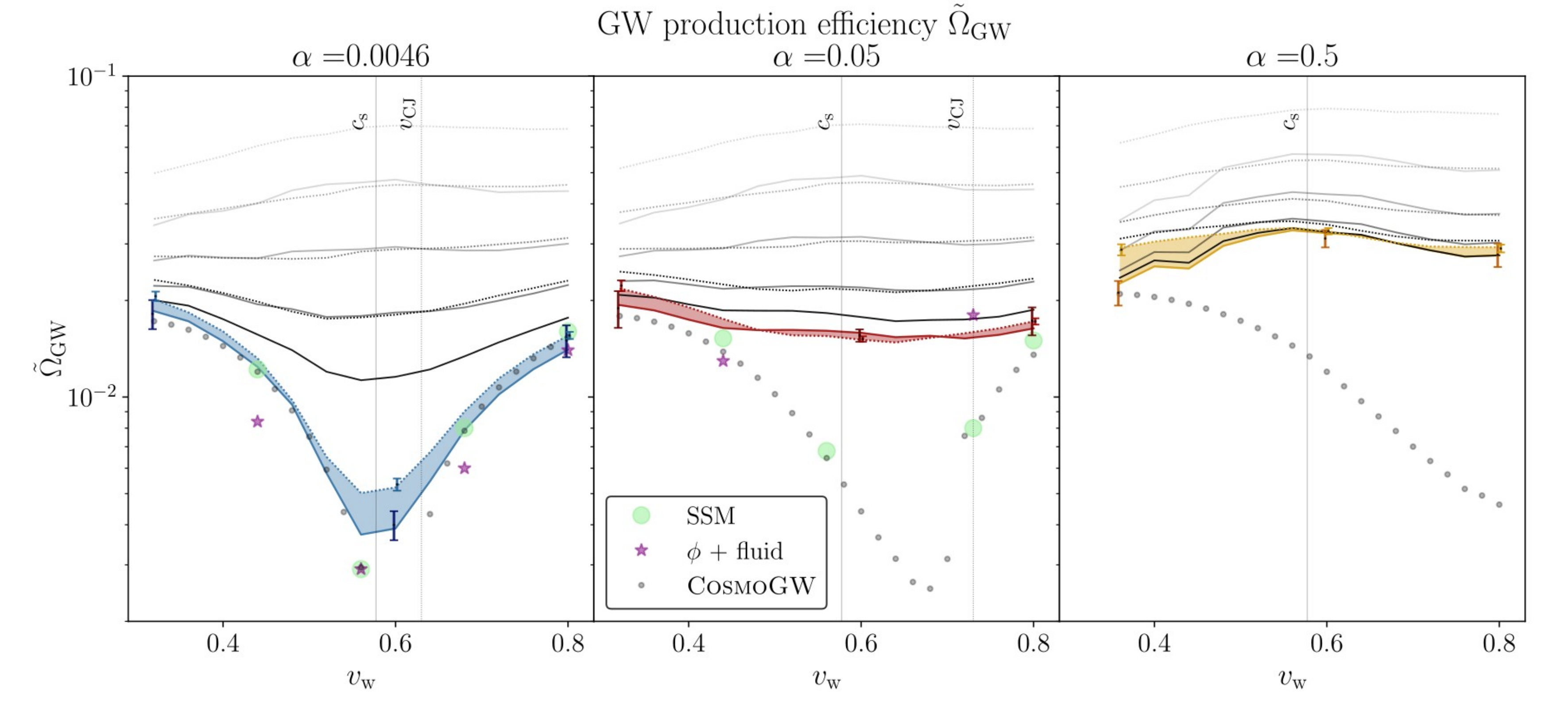
$$\Omega_{GW} \propto (R_* \beta)(\tau \beta) K^2 \, S_f(k/\beta)$$

where β is the duration of the PT, K is the fraction in kinetic energy, S_f is the spectral information, R_* is the mean bubble radius and τ is the lifetime of the source.

We found that due to the decay, this should be replaced by

$$\Omega_{GW} \propto (R_* \beta) \left[\beta \int dt \, K(t)^2 \right] S_f(k/\beta)$$

Main results

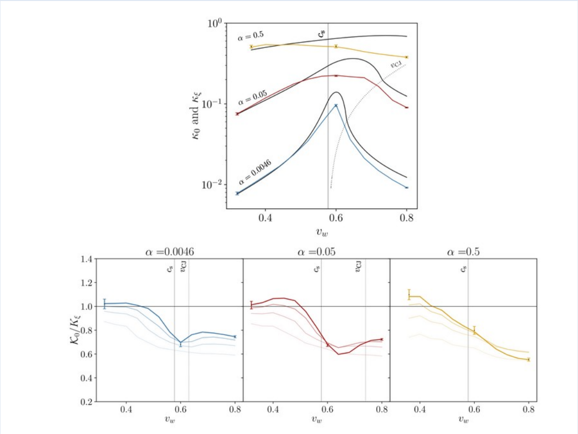


The proportionality factor only varies significantly for weak phase transitions and is quite universal for strong ones.

Other questions

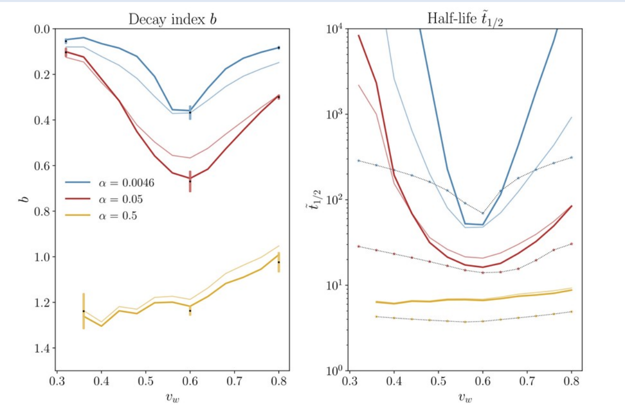
How to obtain the kinetic (peak) energy?

→ spherical bubbles



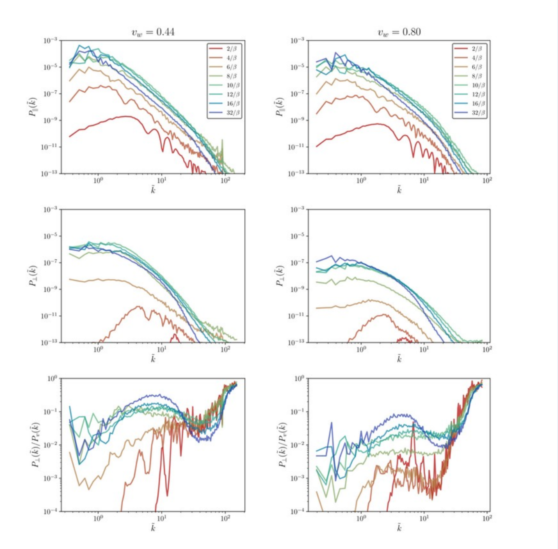
What is the lifetime?

→ some preliminary data



Is this really turbulence?

→ more work to do



We plan to provide a template that represents all our numerical results in the near future.

Thank you!