

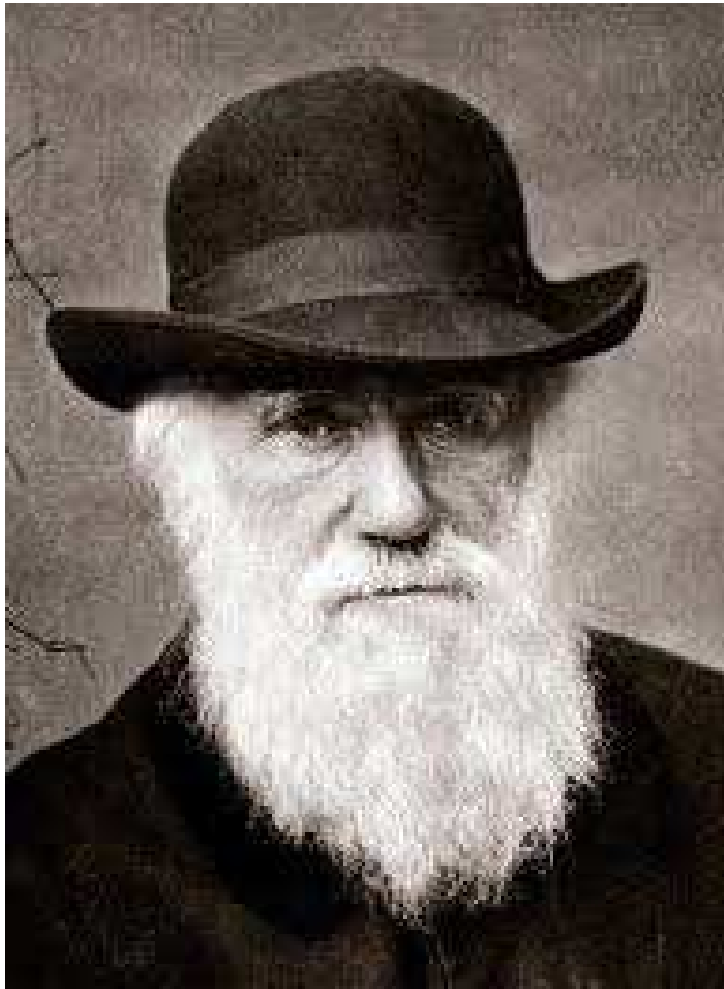
Adaptation in changing environments

Kavita Jain

J. Nehru Centre

Bangalore

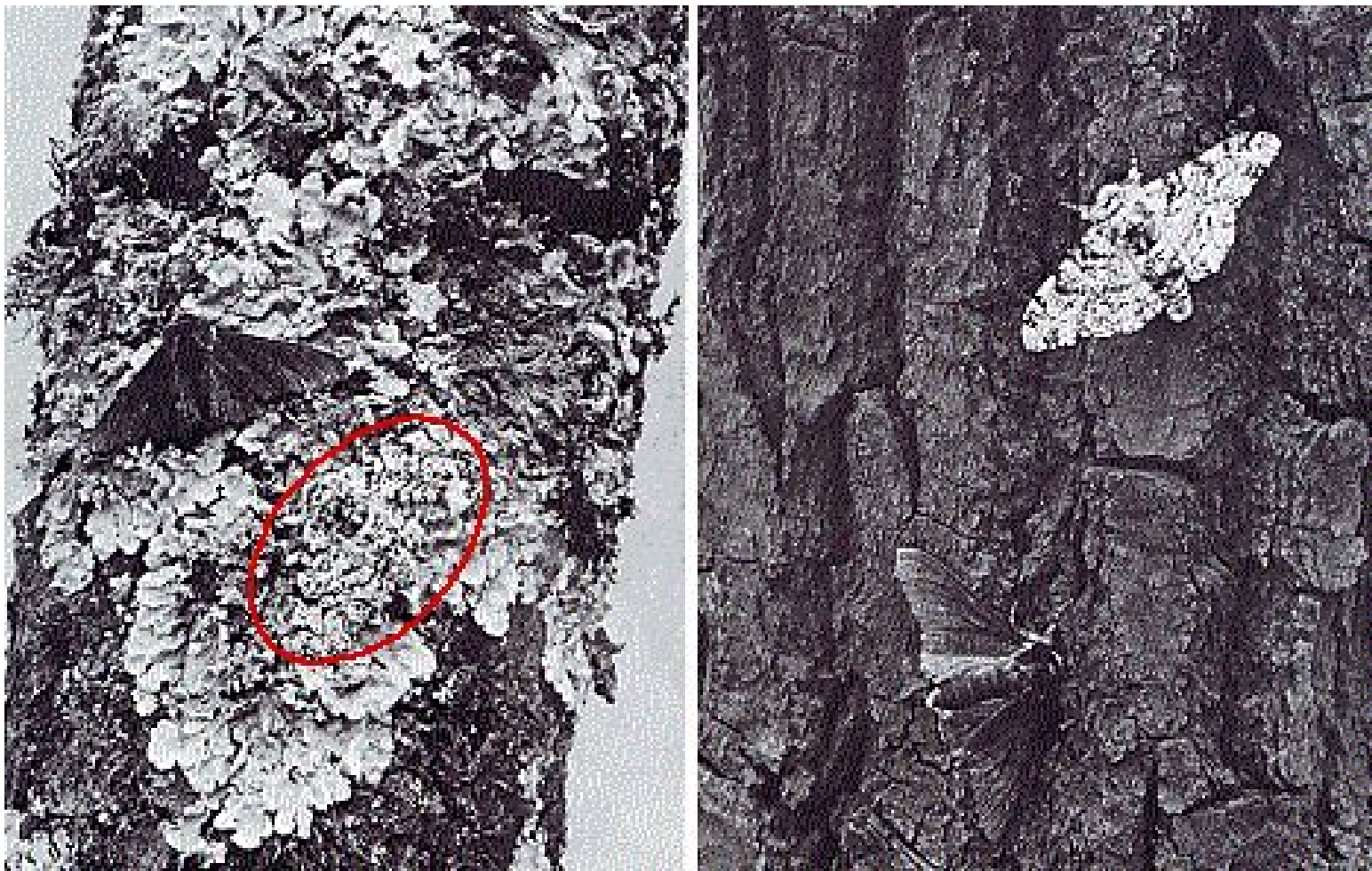
Glacial pace of evolution (Darwin 1859)



*natural **selection** is ... scrutinizing
... the slightest **[heritable] variations**;
rejecting that which is bad, preserv-
ing and adding up all that is good...*

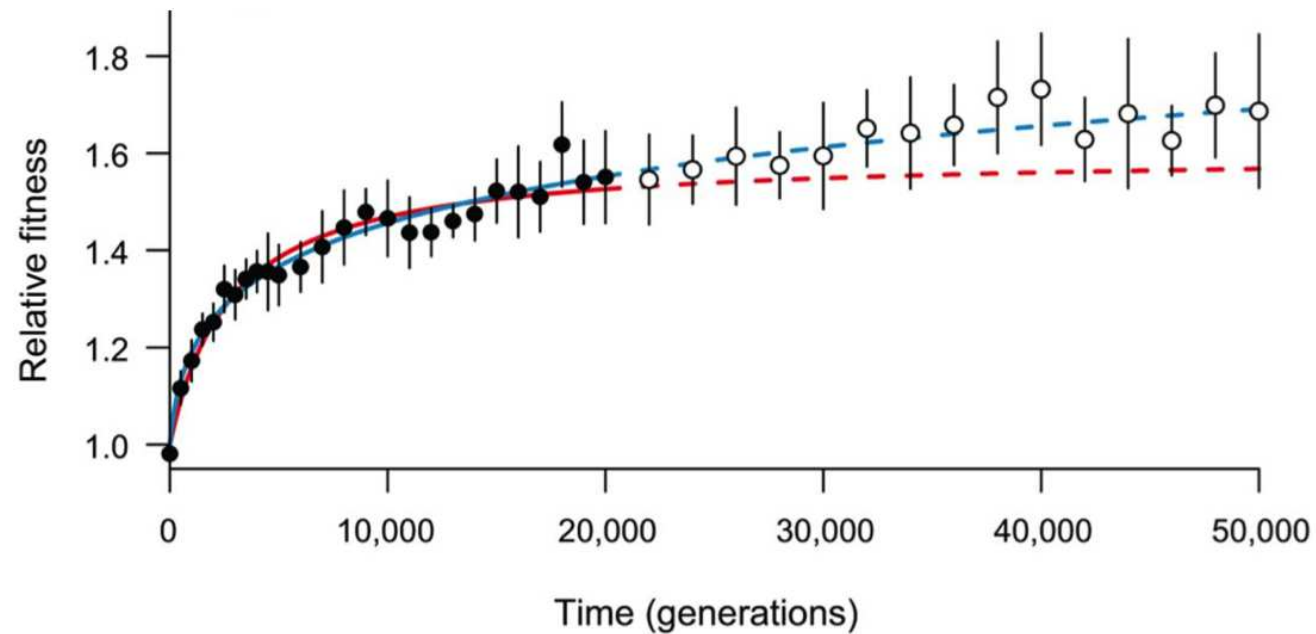
*We see nothing of these slow
changes in progress ... we only see
that the forms of life are now different
from what they formerly were.*

The story of peppered moth (Kettlewell 1955; Cook & Saccheri 2013)



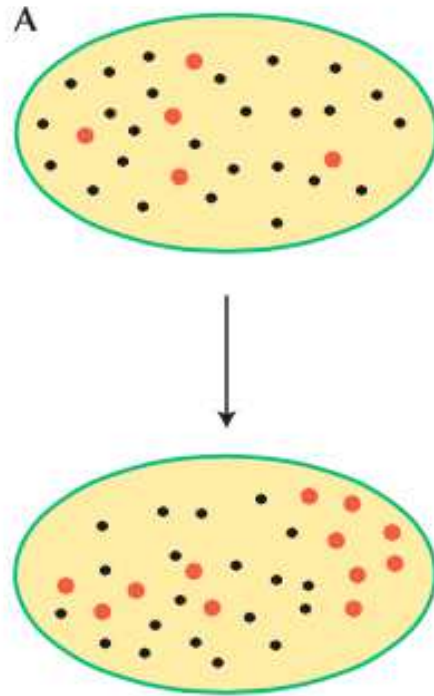
Rise and fall of the black-peppered moth in a span of 100 years

Long term microbial experiment (Lenski lab, 1988-)



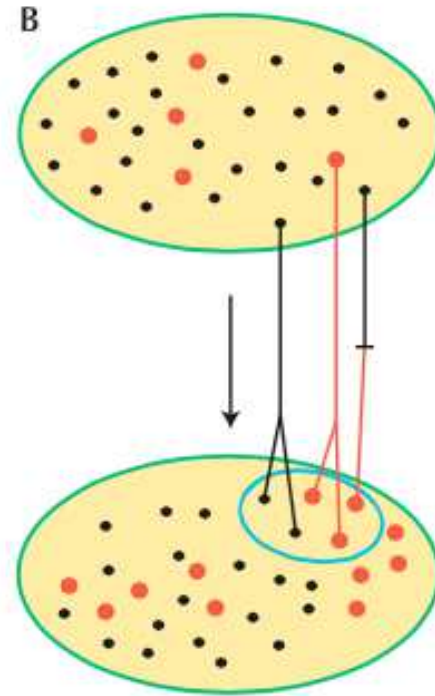
Qualitative, verbal explanations → Quantitative modeling in evolution

Modeling in evolution



Predict the future
(classical pop gen)

Time
↓



Infer the past
(coalescent theory)

(*Evolution*, Barton et al.)

Basic evolutionary processes

- Natural selection
- Mutation
- Stochasticity (random genetic drift)
- Population structure (age-structured, spatially-structured, ...)

Due to interplay of these processes, how does allele frequency change?

Selection

- Suppose two variants (allele) of a gene: ○ and ●
- Simple model (Punnett/Norton 1915; Haldane 1924)

Allele	Fitness	Frequency
●	$1 + s$	x
○	1	$1 - x$

- Mutant can be **beneficial** ($s > 0$), neutral ($s = 0$), **deleterious** ($s < 0$)
- Fitness can be measured in experiments, e.g., growth rate

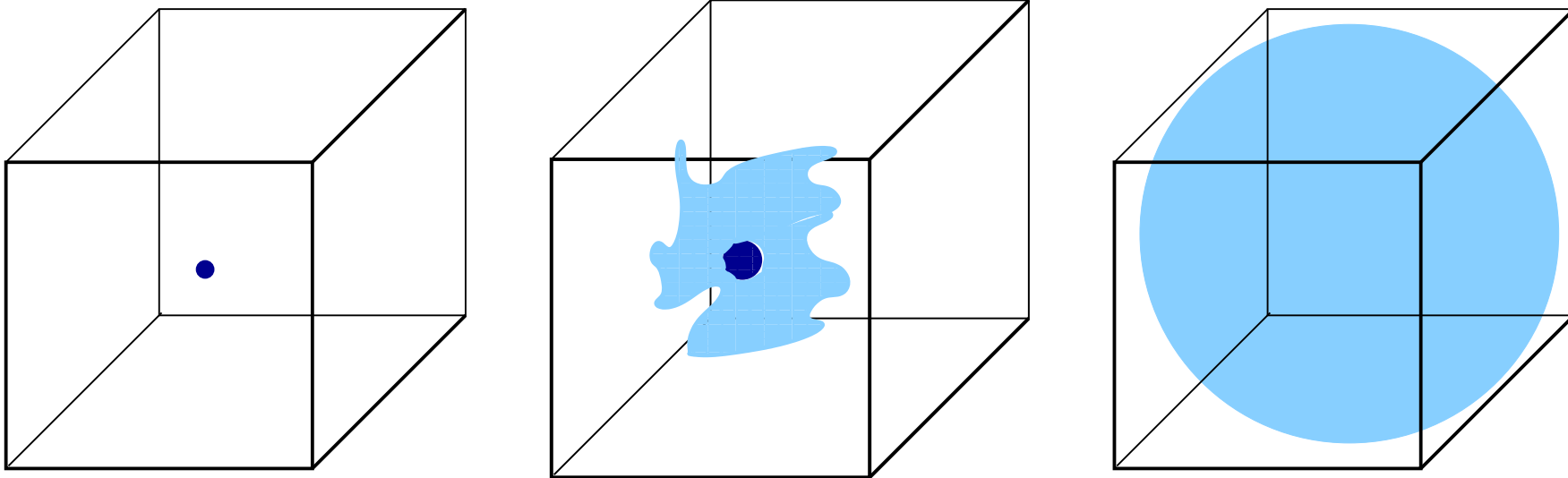
(de Visser & Krug 2014; Fragata et al. 2019)

Mutation-selection balance

- Alleles can change: $\bullet \xrightarrow{\nu} \circ, \circ \xrightarrow{\mu} \bullet$
- Selection and mutation act in opposite manner

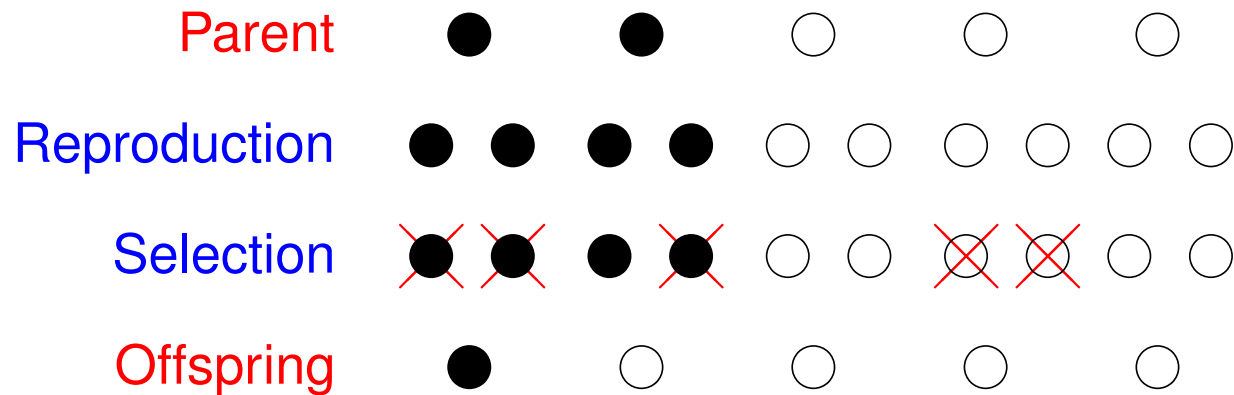
→ Phase transitions can occur

(Eigen 1971; Peliti, Franz,... \gtrsim 1995; review - Jain & Krug 2007)



Random genetic drift

Stochastic evolution because of finite resources (food...)

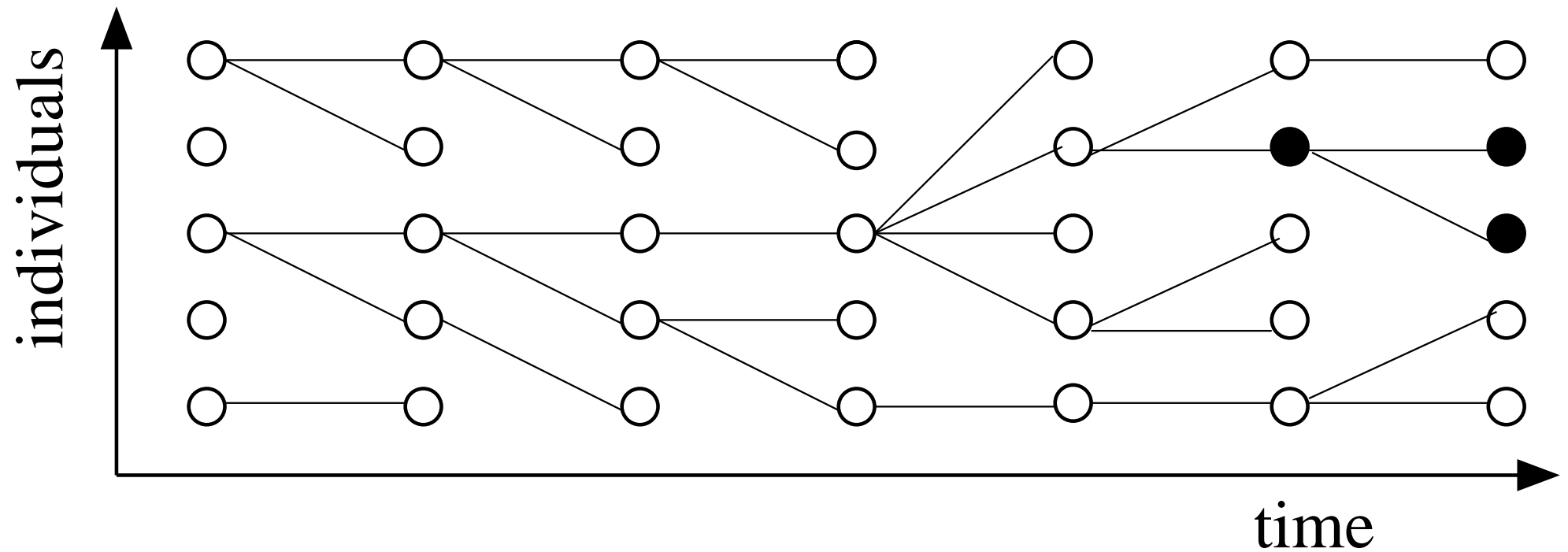


To maintain population size N , sample offspring with

$$\text{Prob} \propto \text{Fitness of parent}$$

“Ising model” of pop genetics: Wright-Fisher process

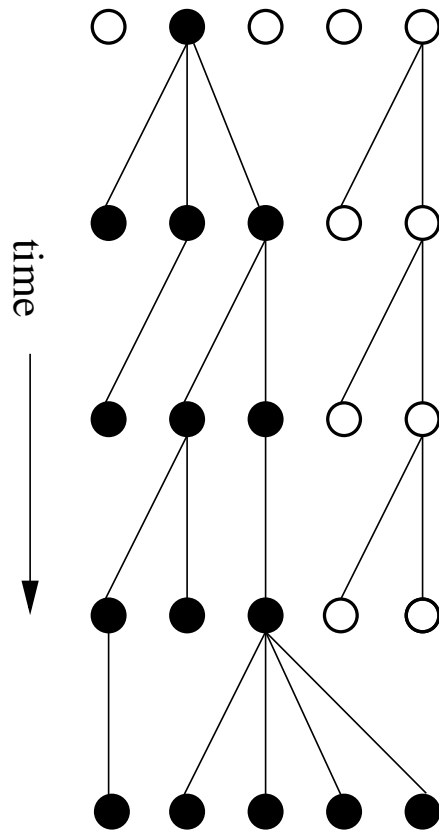
(Fisher 1922, Wright 1924)



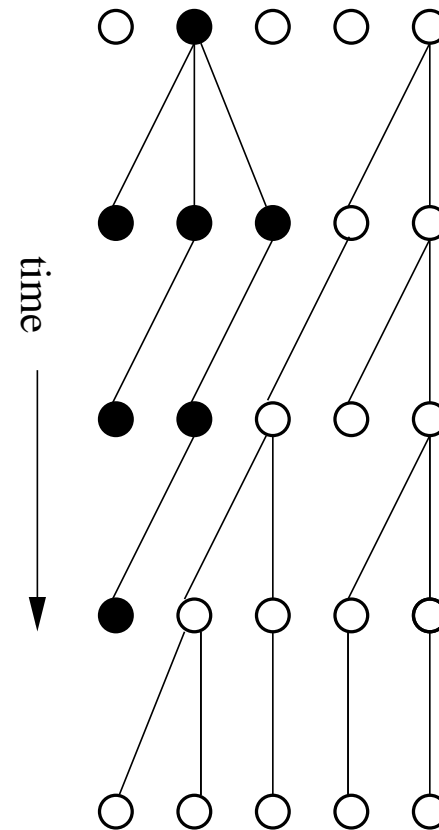
Rule: Choose a parent with prob \propto parent's fitness

First passage problem

Because mutation rates are low, ignore $\bullet \leftrightarrow \circ$ on time scales of interest

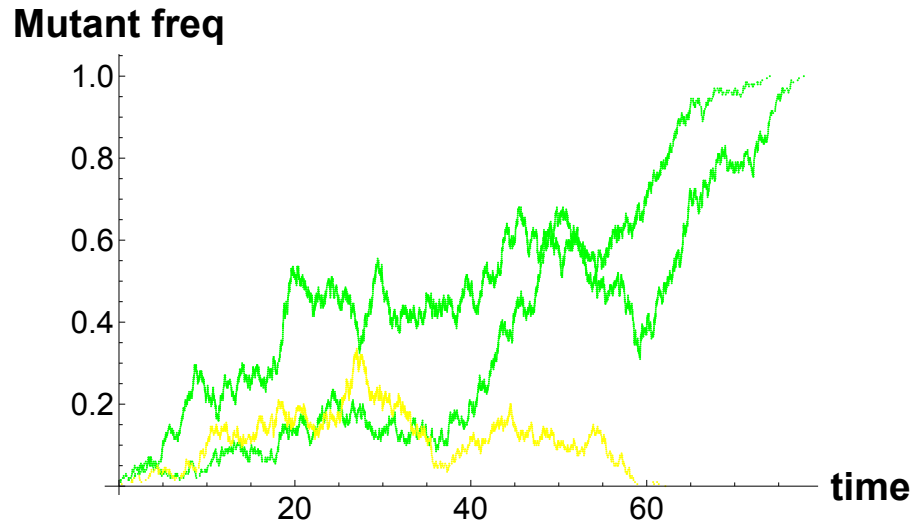


Mutant "fixes"



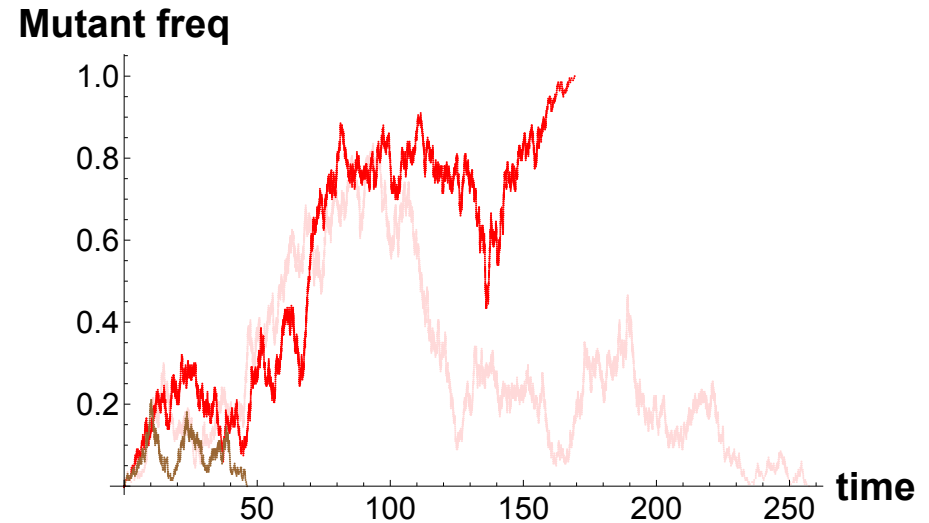
Mutant extinct

Stochastic trajectories in Wright-Fisher process



Beneficial mutant

- Beneficial mutant can get lost
- Once reaches finite frequency, marches ahead



Deleterious mutant

- Deleterious mutant can spread
- No such luck!

Basic questions

Today: Chance that an initially rare mutant spreads in the population?

(fixation probability)

How long does it take for the successful mutant to spread? (fixation time)

Put another way,

- Model with two absorbing boundaries
- First passage probability? First passage time?

Backward Fokker-Planck equation

- $P(x_0, 0|x, t) = \text{Prob}(\text{trajectories starting at } x_0 \text{ terminate at a given } x, t)$

$$-\frac{\partial P}{\partial t_0} = D_1(x_0, t_0) \frac{\partial P}{\partial x_0} + D_2(x_0, t_0) \frac{\partial^2 P}{\partial x_0^2}$$

- For Wright-Fisher process with large populations and small selection,

$$-\frac{\partial P}{\partial t_0} = \underbrace{s(t_0)x_0(1-x_0)}_{\text{SELECTION}} \frac{\partial P}{\partial x_0} + \underbrace{\frac{x_0(1-x_0)}{N}}_{\text{GENETIC DRIFT}} \frac{\partial^2 P}{\partial x_0^2}$$

- Eventual probability of fixation, $P_{\text{fix}} \equiv P(x_0, t_0 | x = 1, t \rightarrow \infty)$

- In constant environment, time-homogeneous Markov process

$$0 = s(t_0)x_0(1 - x_0)\frac{\partial P_{\text{fix}}}{\partial x_0} + \frac{x_0(1 - x_0)}{N}\frac{\partial^2 P_{\text{fix}}}{\partial x_0^2}$$

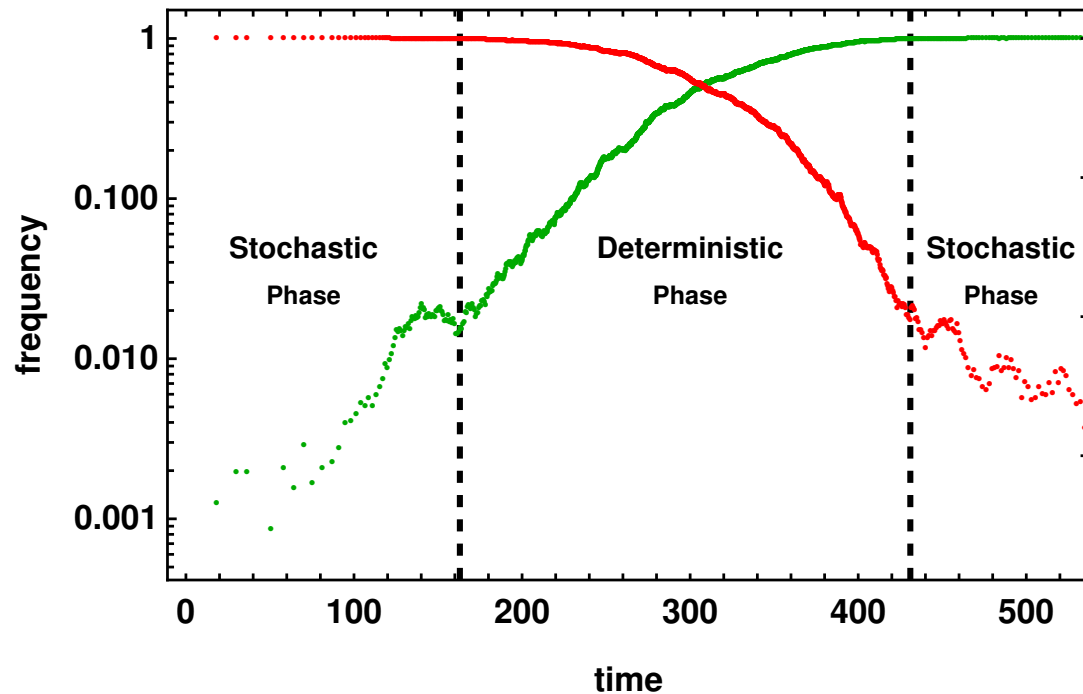
Reduces to an ODE! (Kimura 1962)

- In changing environment, time-inhomogeneous Markov process

$$-\frac{\partial P_{\text{fix}}}{\partial t_0} = s(t_0)x_0(1 - x_0)\frac{\partial P_{\text{fix}}}{\partial x_0} + \frac{x_0(1 - x_0)}{N}\frac{\partial^2 P_{\text{fix}}}{\partial x_0^2}$$

Have to deal with a PDE

Semi-deterministic approx for beneficial mutant (Desai & Fisher 2007)



Stochastic fluctn imp when
mutant/wildtype number is low

Once the frequency is finite,
mutant unlikely to get lost

- Deal with stochastic phase using branching-type process where lineages evolve independently
- Splice the stochastic and deterministic solutions

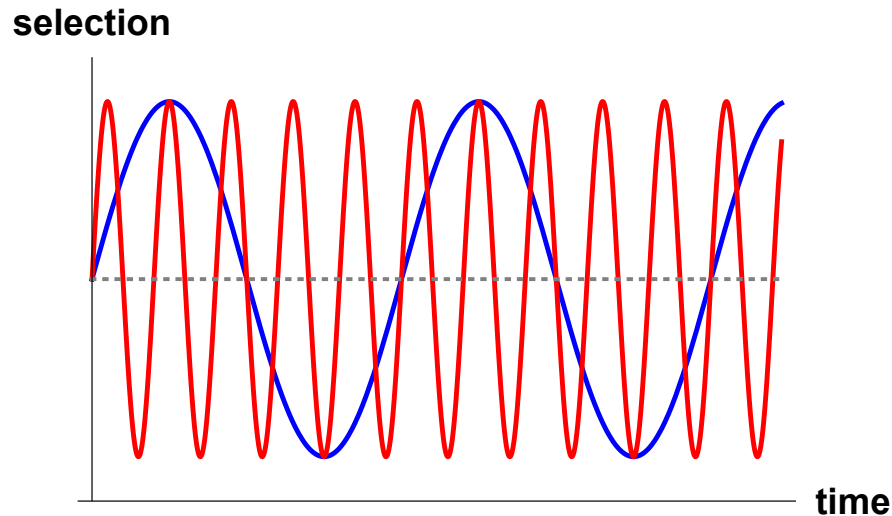
(Uecker & Hermisson 2011, Martin & Lambert 2015)

Specific model

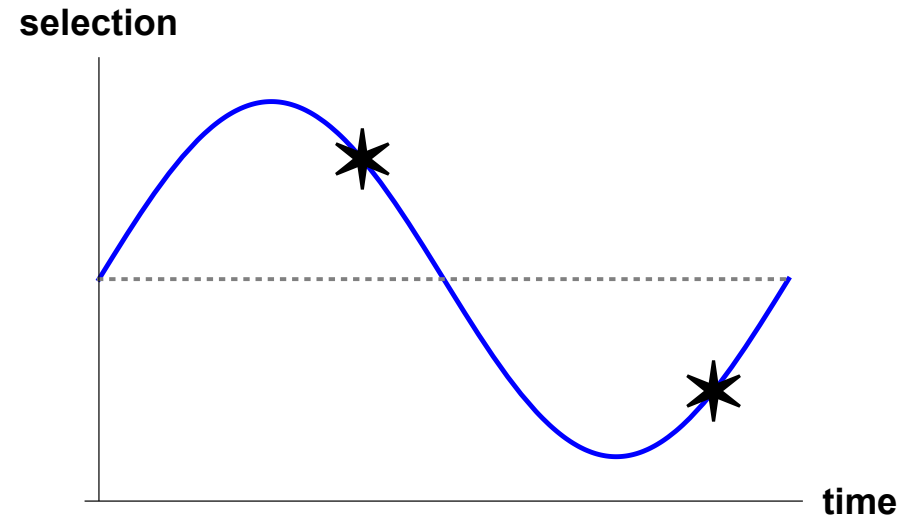
Adaptation in periodically changing environment (Devi & Jain 2020)

Time-dependent growth rate due to seasonal cycle, drug cycling...

$$s(t) = \bar{s} + \sigma \sin(\omega t)$$



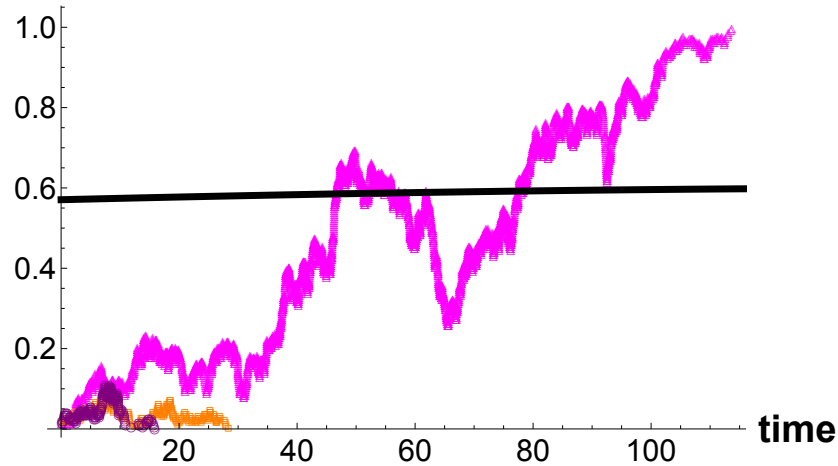
Effect of rate of environmental
change (ω)?



Does time of arrival matter?
(time-inhomogeneous equation)

Qualitative picture

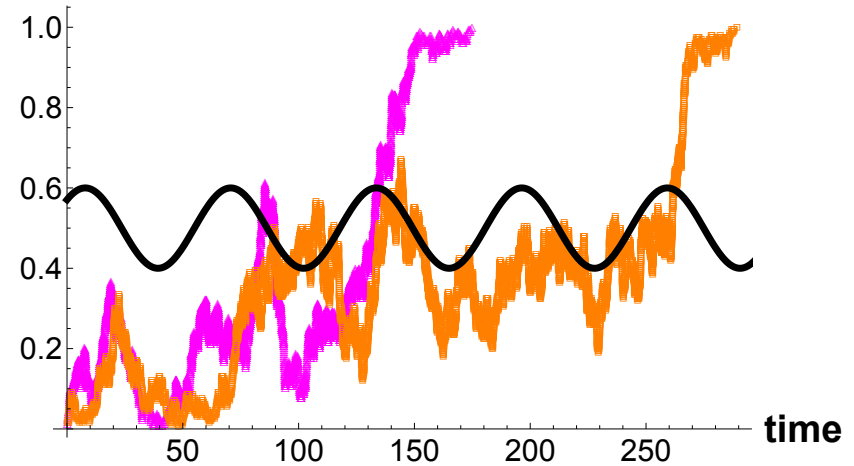
Mutant freq



$$T_{\text{env}} \gg T_{\text{fix}}$$

- Almost static environ
- Arrival time obviously imp

Mutant freq

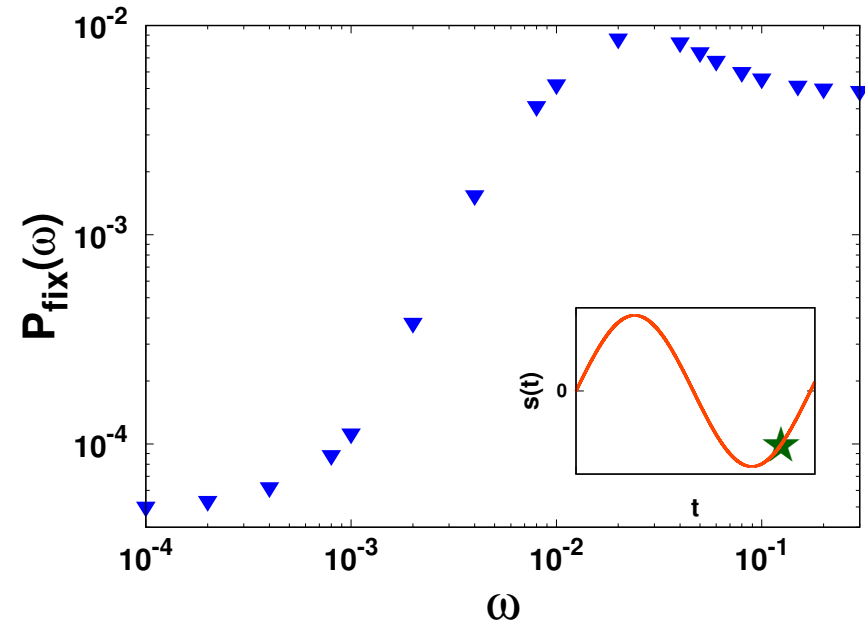
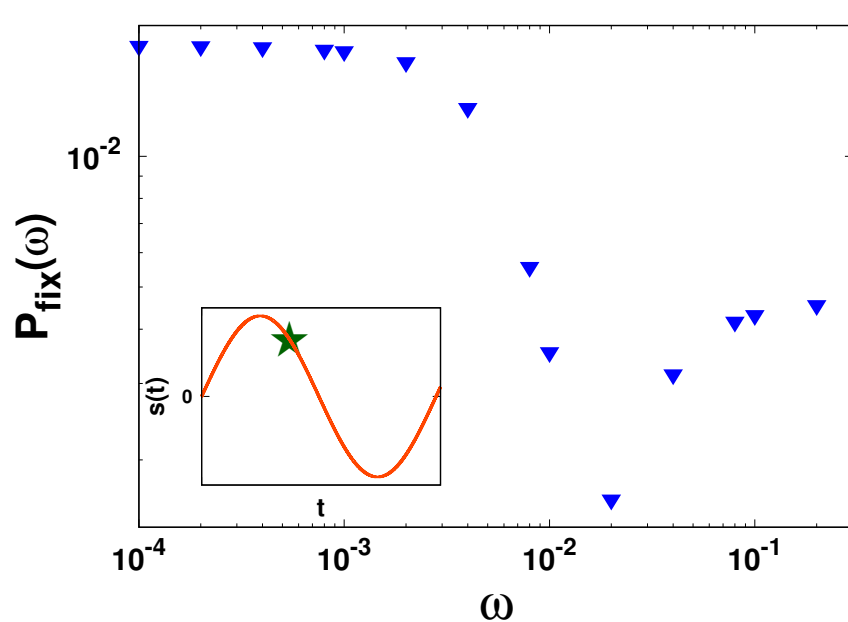


$$T_{\text{env}} \ll T_{\text{fix}}$$

- Sensitive to average environ
- Must survive at short times

→ Time-inhomogeneity is imp at all frequencies

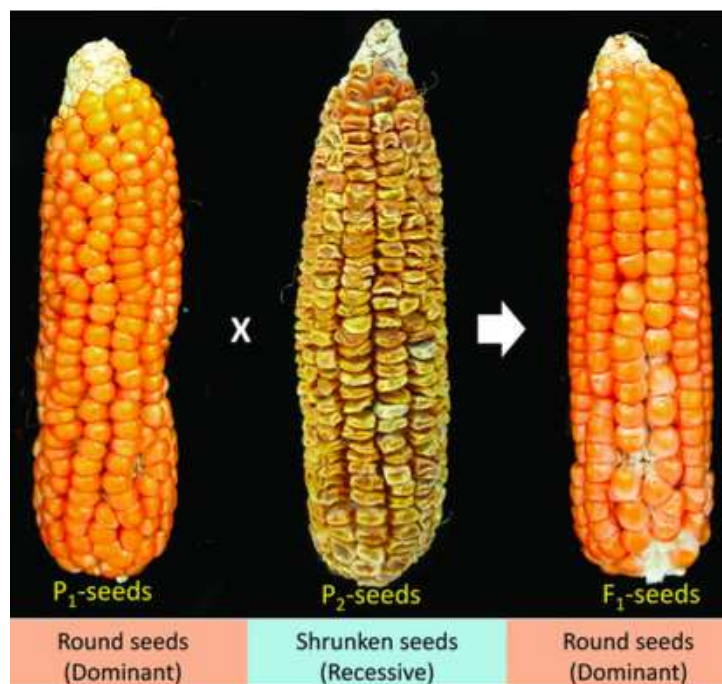
Fixation probability in changing environments



$$P_{\text{fix}} = \begin{cases} P_{\text{fix}}(\text{static}) + \omega \cos(\omega t_0) & , \omega \ll \omega_r \\ P_{\text{fix}}(\text{avg environ}) \left(1 + \frac{h\sigma \cos(\omega t_0)}{\omega} \right) & , \omega \gg \omega_r \end{cases}$$

where resonance frequency, ω_r is growth rate (or inverse pop size)

Impact of dominance (Mendel 1865)



(Hossain et al. 2019)

One type only: Dominant/Recessive

$$h = 1/0$$

Both types: Intermediate dominance

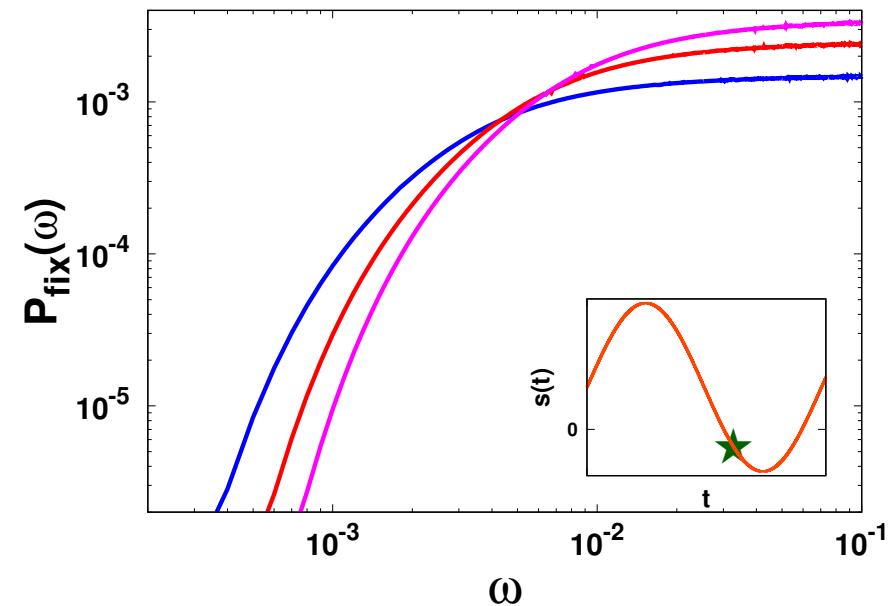
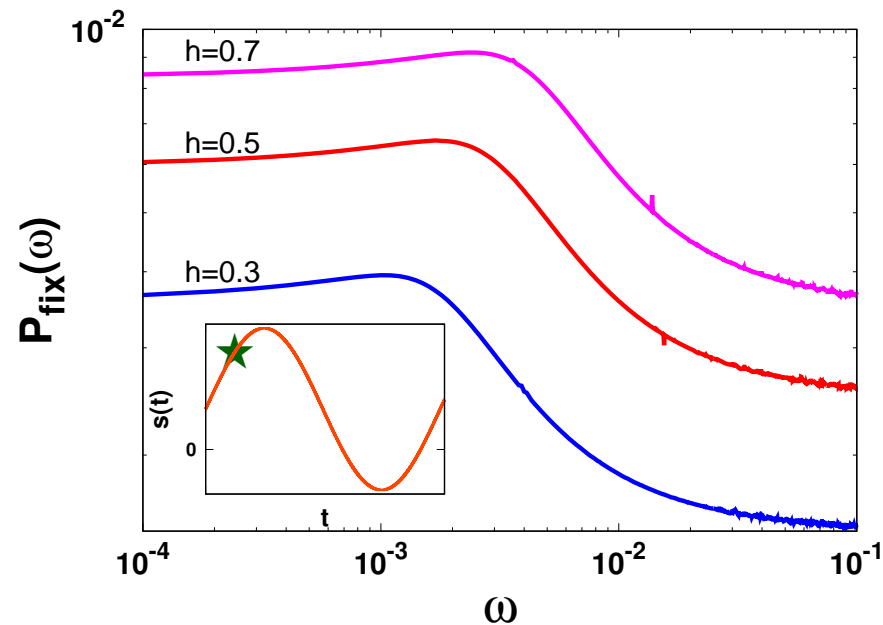
$$0 < h < 1$$



Haldane's sieve (Haldane 1927, Turner 1981)

Dominant beneficial mutants have higher fixation prob than recessives

→ infer direction of evolution (dominants are younger)



Haldane's sieve does not always operate in changing environments

(Devi & Jain 2020)

Summary

- Discussed a stochastic model of biological evolution
- In a periodically changing environment,
 - chances of survival can be substantially different from that in static environment
 - Haldane's sieve does not always operate
- Fixation time in changing environments? (Kaushik & Jain, in prep)