Kinetic Mixing in SO(10), Gravitational Waves and PTA data

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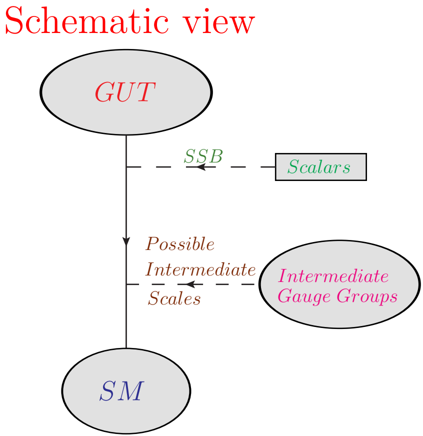
Hearing BSM with Cosmic Sources of Gravitational Waves
@ ICTS - TIFR, Bengaluru

Outline

1 *Introduction*

Grand Unification Beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, is embedded in a larger simple group, $\mathcal{G}$.



GUT Examples

- $SU(5)$ (rank $= 4$): $\bar{5} + 10 \Rightarrow$ SM fermions.

  Georgi, Glashow, PRL **32**, 438 (1974)

- $SO(10)$ (rank $= 5$): $16 \Rightarrow$ SM fermions $\oplus\ \nu_L^C$.

  Fritzsch, Minkowski, Ann. Phys. **93**, 93-266 (1975)

- $E(6)$ (rank $= 6$): $27 \Rightarrow$ SM fermions $\oplus\ \nu_L^C\ \oplus$

  $$\underbrace{(2, \pm\frac{1}{2}, 1) + (1, -\frac{1}{3}, 3) + (1, \frac{1}{3}, \bar{3}) + (1, 0, 1)}_{\text{Exotic fermions}}.$$

  Gursey, Ramond, Sikivie, PLB **60** (1976) 177

  Shafi, PLB **79** (1978) 301

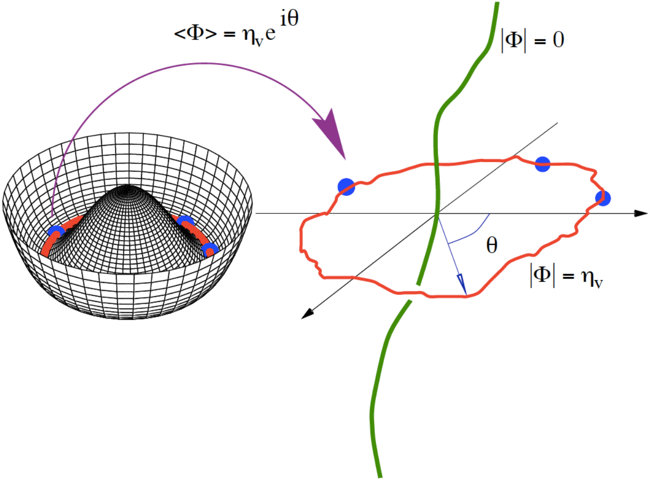
Prediction of topological defects

- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.

- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold $(\mathcal{M} = \mathcal{G}/\mathcal{H})$ implies formation of topological defects.

- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc.
  $SU(N)_X \rightarrow U(1)_X$: Monopoles, $U(1) \rightarrow Z_N$: Strings

Cosmic string



Vachaspati et. al. arXiv:1506.04039

Cosmic string network

- String tension $\mu \simeq \pi v^2$, v is the VEV that form the string.

- Strings inter-commute, form loops, radiate GWs and the evolution of the network enters a 'scaling' regime.

- Scaling energy density $\rho_s \sim \mu/t^2$. Critical density: $\rho_c \sim 1/Gt^2$ in RD and MD.

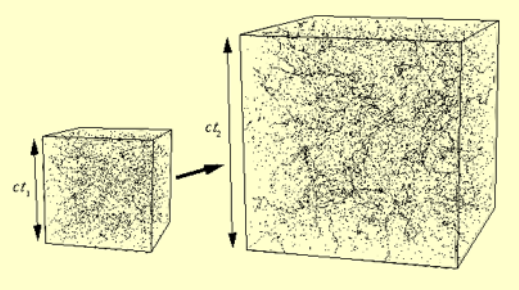


Image source: ctc.cam.ac.uk

Kibble, NPB 252 (1985) 227; Vachaspati, Vilenkin PRD 31 (1985) 3052; Bennett, Bouchet, PRL 60 (1988) 257 ...

Strings and gravitational waves

- Loops of initial length $\boxed{l_i = \alpha t_i}$ ($\alpha \simeq 0.1$) decay via emission of gravity waves. Blanco-Pillado, Olum, Shlaer, Phys. Rev. D **89** (2014) 023512

$$\frac{dE^{(k)}_{\rm GW}}{dt} = \Gamma_k G\mu^2; \quad \Gamma_k \propto k^{-n} \quad \text{with } n = \begin{cases} 4/3 & \text{cusps} \\ 5/3 & \text{kinks} \\ 2 & \text{kink-kink collisions.} \end{cases}$$

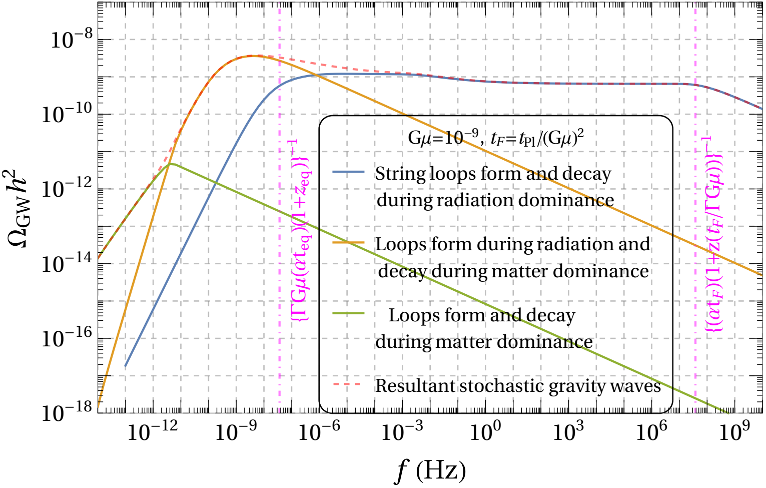
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by Vilenkin, Shellard, 1994, CUP

$$f = \underbrace{\frac{a(\tilde{t})}{a(t_0)}}_{\text{Redshift}} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad \Gamma = \sum \Gamma_k \sim 50$$

Redshift

$\frac{dl}{dt} = -\Gamma G\mu \Rightarrow$ Loop size at time $\tilde{t}$

Stochastic gravitational wave background



Observational constraints from defects

- Stable domain walls contradict standard cosmology.

  Zeldovich, Kobzarev, Okun, Zh. Eksp. Teor. Fiz. **67**, 3-11 (1974)

- Upper bound on comoving monopole number density: $Y_M = n_M/s \gtrsim 10^{-27}$. MACRO: EPJC 25 511, IceCube: PRL 128 (2022) 051101, ANTARES: JHEAp 34 (2022) 1, ...

- CMB constraint on stable strings: $G\mu \lesssim 10^{-7}$.

  PhysRevD.93.123503, ...

- LIGO-VIRGO O3 data constraint on "undiluted" strings: $G\mu \lesssim 10^{-7}$ around decaHz frequencies. PhysRevLett.126.241102

- PTA experiments put a constraint on stable cosmic strings : $G\mu \lesssim 10^{-10}$ around the nanoHertz frequencies.

  arXiv:2306.16219,...

2 *SO(10) Symmetry Breaking, Monopoles and Strings*

SO(10) symmetry breaking, monopoles and strings

$$SO(10) \xrightarrow[M_U]{\langle 210\rangle} SU(4)_c \times SU(2)_L \times SU(2)_R$$
$$\xrightarrow[M_U]{\langle(15,1,3)\in 210\rangle} SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\xrightarrow[M_R]{\langle(1,1,1,-\frac{1}{2})\in 16\rangle} SU(3)_c \times SU(2)_L \times U(1)_Y.$$

- Symmetry breaking $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$ produces 'Red' monopoles with magnetic fluxes

$$X \equiv B - L + 2T_c^8/3 = diag(1, 1, -1, -1).$$

- $SU(2)_R \to U(1)_R$ generates 'Blue' monopoles with fluxes

$$T_R^3 = diag(1, -1).$$

Lazarides, Shafi, JHEP 10 (2019) 193

Strings connecting monopoles

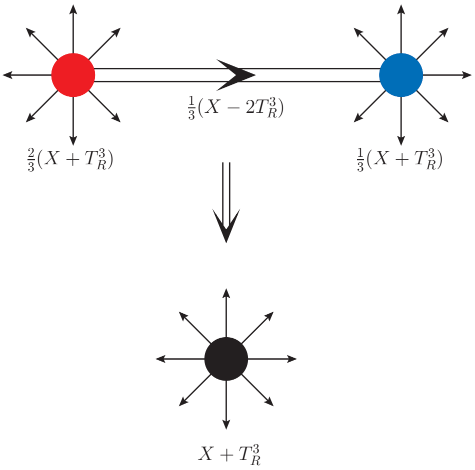
- $U(1)_R \times U(1)_{B-L} \to U(1)_Y$ generates topologically unstable strings.

- These strings connects
  1. a blue monopole to a red monopole.
  2. a monopole to its anti-monopole.
  3. ends on itself forming a loop.

- Red and blue monopoles combined to form stable Schwinger monopoles.

Lazarides, Shafi, JHEP 10 (2019) 193

Lazarides, **RM**, Shafi, JCAP 05 (2024)



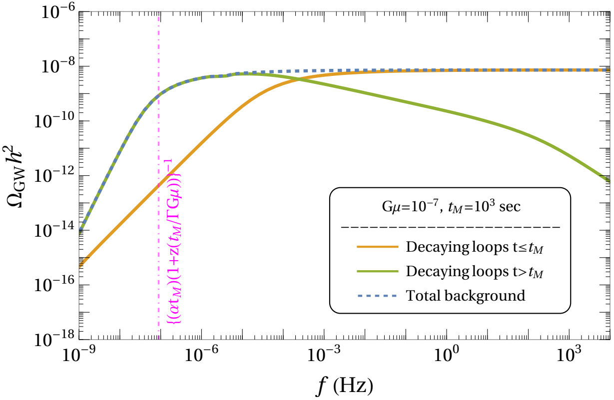
3 *GWB from Quasi-stable Strings and PTA Data*

Formation of quasi-stable strings

- Magnetic monopoles, created prior to the cosmic strings, experience "partial" inflation.
- The strings make random walks with step of the order of the horizon, and form a network of stable strings before the horizon reentry of the monopoles.
- The strings inter-commute and form loops which decay into gravitational waves. $\boxed{f = \frac{1}{1+z(t)} \frac{2k}{l(t)},}$ $k \in Z^+$.
- As monopoles reenter the horizon at a cosmic time t_M, we obtain monopoles connected by string segments which eventually decay.

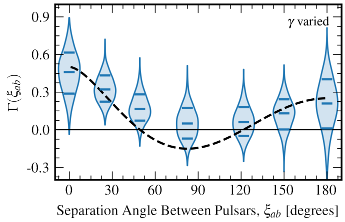
Lazarides, **RM**, Shafi, JCAP 08 (2022) 042

Gravitational waves from quasi-stable strings



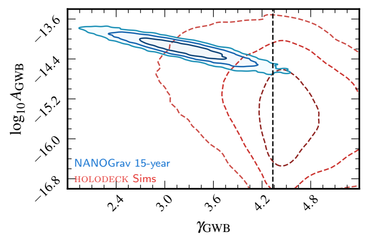
- Large string loops and segments ($> 2t_M$) are absent.
- Gravitational wave spectrum in the low frequency region $f \lesssim 1/t_M(1 + z(t_M/\Gamma G\mu))$ is suppressed.

Evidence of GWB in PTA

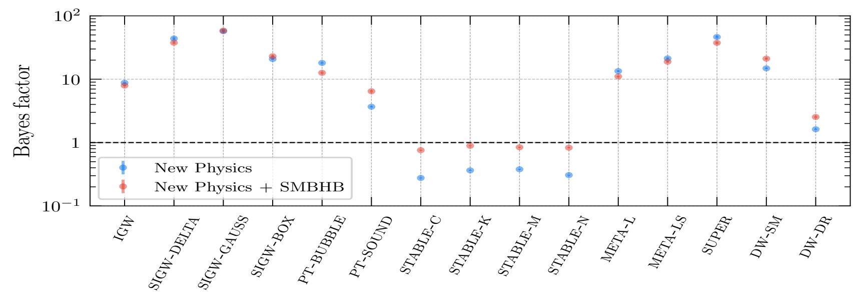


NG15:2306.16219,...

Mergers of SMBHBs?

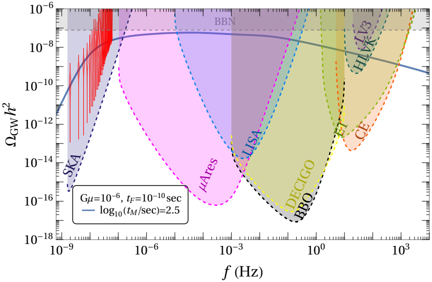


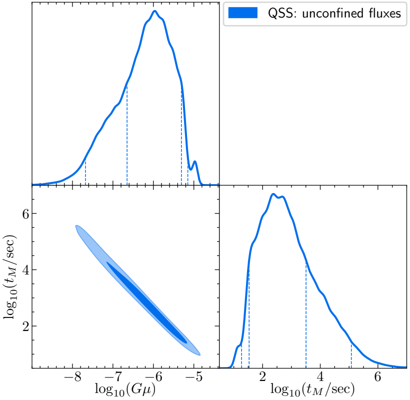
Cosmological Origin? New Physics?



Gravitational waves and NANOGrav 15 year data

Parameters	Bayesian Credible Intervals	
	68%	95%
$\log_{10}(G\mu)$	$[-6.65, -5.31]$	$[-7.68, -5.15]$
$\log_{10}(t_M/\mathrm{sec})$	$[1.53, 3.51]$	$[1.27, 5.08]$





RM, Shafi, JHEP 10 (2024) 157

4 *Unification Solutions*

Dim-5 operators and kinetic mixing of $U(1)_{B-L} \times U(1)_R$

- $F^p_{\mu\nu} = \partial_\mu A^p_\nu - \partial_\nu A^p_\mu$ ($p = X, R$; $X \equiv B - L$) mixes as:

$$-\frac{1}{4} F^X_{\mu\nu} F^{X\mu\nu} - \frac{1}{4} F^R_{\mu\nu} F^{R\mu\nu} - \frac{\epsilon}{2} F^X_{\mu\nu} F^{R\mu\nu},$$

ϵ is the mixing parameter. Holdom, PLB 166 (1986)

- Dim-5 operator suppressed by the cut-off scale Λ:

$$\frac{\mathcal{C}}{\Lambda} G^{a\mu\nu} \Phi_{ai} G^i_{\mu\nu}, \quad \Phi_{ai} \equiv (15, 1, 3) \in 210_H$$

induces $\boxed{\epsilon \sim \mathcal{O}(M_U/\Lambda)}$ at the GUT scale as Φ_{ai} gets a VEV $\sim M_U$. **RM**, Shafi, JHEP 10 (2024) 157

- The canonical form is obtained through change of basis $A^p \to B^p$:

$$\begin{pmatrix} A^X_\mu \\ A^R_\mu \end{pmatrix} = \begin{pmatrix} 1 & \frac{-\epsilon}{\sqrt{1-\epsilon^2}} \\ 0 & \frac{1}{\sqrt{1-\epsilon^2}} \end{pmatrix} \begin{pmatrix} B^X_\mu \\ B^R_\mu \end{pmatrix}.$$

Abelian mixing and matching of gauge couplings

- In the covariant derivative part, the diagonal gauge coupling $diag[g_X, g_R] \to G$:

$$G = \begin{pmatrix} g_{XX} & g_{XR} \\ 0 & g_{RR} \end{pmatrix},$$

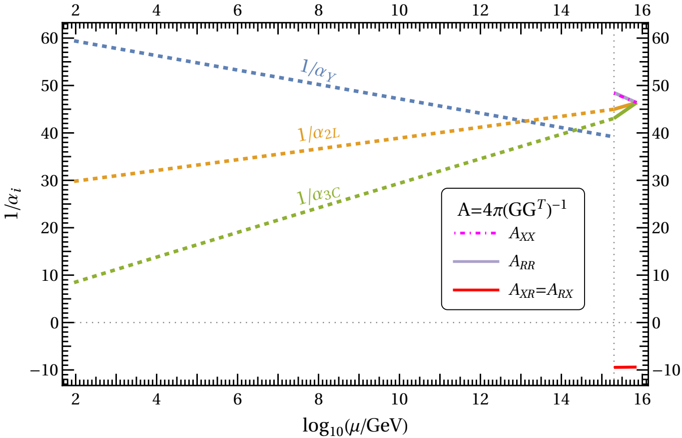
$$\text{with} \quad g_{XX} = g_X, \quad g_{XR} = -\frac{\epsilon g_X}{\sqrt{1-\epsilon^2}}, \quad g_{RR} = \frac{g_R}{\sqrt{1-\epsilon^2}}.$$

- At the GUT scale M_U: $g_X = g_R \equiv g_U$.
- We have the matching condition at the breaking scale M_R given by

$$\frac{1}{\alpha_Y(M_R)} = 4\pi P(GG^T)^{-1}P^T \quad \text{with } P = (\sqrt{\frac{2}{5}}, \sqrt{\frac{3}{5}})$$

$$\Rightarrow \frac{1}{g_Y^2} = \frac{3g_{XX}^2 - 2\sqrt{6}g_{XX}g_{XR} + 2\left(g_{XR}^2 + g_{RR}^2\right)}{5g_{XX}^2 g_{RR}^2}.$$

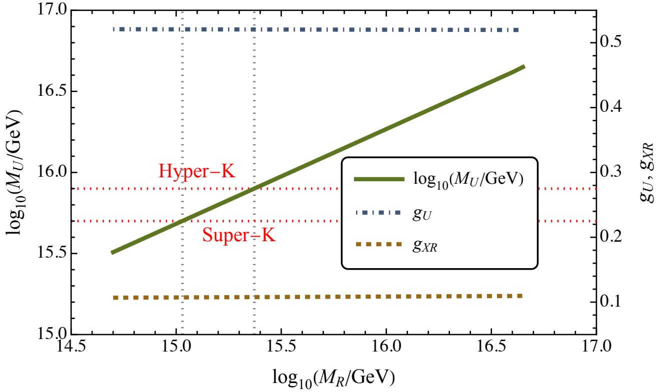
Unification solution



- The unification occurs for $\boxed{\epsilon \sim M_U/\Lambda \sim 0.1}$

RM, Shafi, JHEP 10 (2024) 157

Unification solution



- The unification occurs for $\epsilon \sim M_U/\Lambda \sim 0.1$.
- String tension ($\mu \sim \pi M_R^2$): $\boxed{\log_{10}(G\mu) \in [-7.7, -4.7]}$
- Seesaw scale $\sim \epsilon M_R^2/M_U \sim 10^{14}$ GeV.

RM, Shafi, JHEP 10 (2024) 157

5 *Summary*

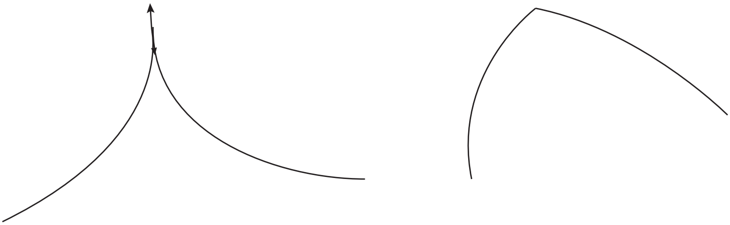
Summary

- Realistic grand unified theory of $SO(10)$ predicts formation of superheavy strings which connects monopoles.
- Quasistable strings can be formed if the monopoles experience partial inflation.
- The GWs from superheavy QSS with $G\mu \sim 10^{-6}$ can explain the evidence of GWs in recent PTA data with a monopole horizon reentry time $t_M \sim 10^2$ sec.
- Realistic grand unified theory of $SO(10)$ predicts the formation of such a quasistable string network.

Thank You

Back up slides

Cusp and Kink



Pulsar Timing Arrays

- Pulsars are rapidly spinning neutron stars with a strong magnetic field ⇒ Radiate beam of radio waves.
- Repeating pulses are observed as the radio beam intersects the observers periodically.
- Millisecond pulsar (MSP) produces exceedingly stable and regular pulse profile ⇒ "Perfect Clock".

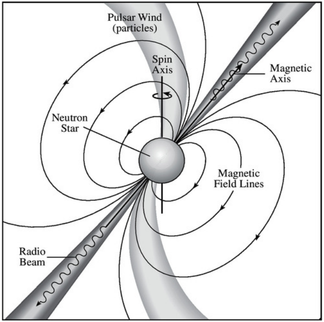
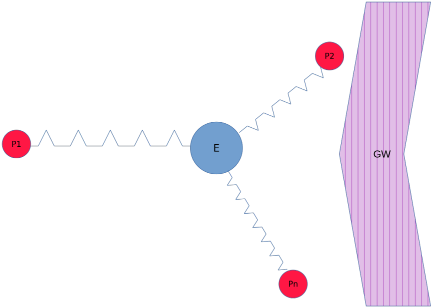


Image source: K.R. Lang, NASA's Cosmos

Pulsar Timing Arrays



- Measurement of the time of arrival (ToA) of pulses can reveal tiny distortion of spacetime fabric due to gravity waves (GWs) $\Rightarrow$ Pulsar timing!

Image source: Wikipedia

- Difference between observed ToA and the expected ToA from timing model gives time residual.
- Time residual contains information about other signals like GWs.

Pulsar Timing Arrays

- Impossible to distinguish between GWs signal and other source of signal in the timing residual of a single pulsar.
- Need correlations between the timing residuals of different pulsars $\Rightarrow$ Pulsar Timing Array (PTA).
- Gravity waves generate unique quadrupolar correlations between timing residuals of pulsar pairs.
- Correlations depend on the angular separations between the pulsar pairs and follow the Hellings and Downs correlation curve. **APJ. 265, L39 (1983)**

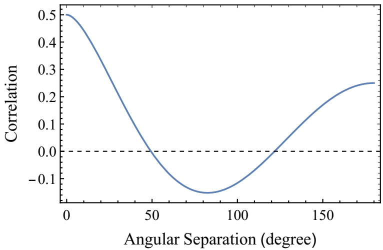
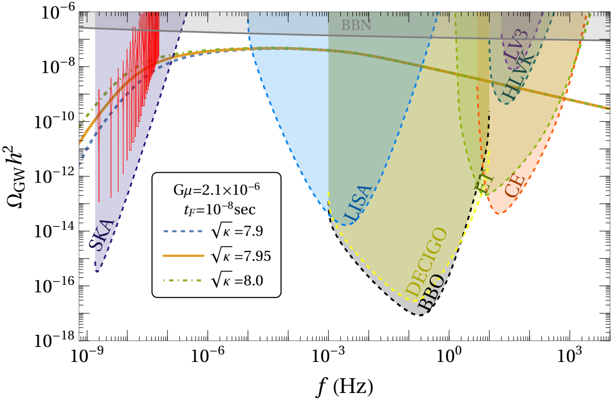


Figure: Hellings and Downs curve.

Metastable Strings: PTA and LIGO-VIRGO



- The GWs from MSS explain PTA data at nanoHertz frequency, but violate the bound from LIGO-VIRGO third observing run!
- An early matter domination or partial inflation can reduce the spectra at high f.

Lazarides, **RM**, Moursy, Shafi, JCAP 03 (2024) 006
RM, Park, JCAP 01 (2024) 015

Formation of Metastable Strings (MSS)

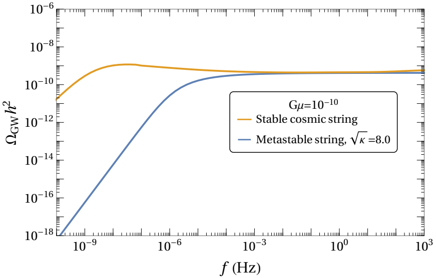
- Magnetic monopoles, created prior to the cosmic strings, experience inflation.
- The lifetime of decay of the strings via quantum mechanical tunneling is much smaller than the age of Universe.
- The strings form a network of stable strings before the time $t_s = 1/\sqrt{\Gamma_d}$.
- The strings network disappear at a time $t_e \sim 1/\sqrt{\Gamma_d \Gamma G\mu}$.

Leblond, Shlaer, Siemens, PRD **79** (2009) 123519

Buchmuller, Domcke, Schmitz, JCAP **12** (2021) 006

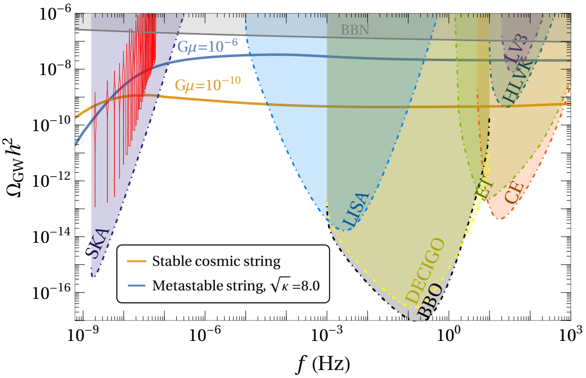
Metastable Strings and GWs

- The strings inter-commute and form loops which decay into gravitational waves.
- String loops larger than αt_s are absent.



- Gravitational wave spectrum in the low frequency region, $f \lesssim 1/\Gamma G\mu t_e(1+z_e)$, becomes suppressed.

PTA and Observational Prospects of Strings



- Pulsar Timing Arrays (PTAs)
  1. found evidence of a stochastic background which can be explained by superheavy "metastable" strings $G\mu \sim 10^{-6} - 10^{-5}$.
  2. put a contraint "undiluted stable" cosmic strings : $G\mu \lesssim 10^{-10}$.

  in the nanoHertz frequencies. arXiv:2306.16219,...