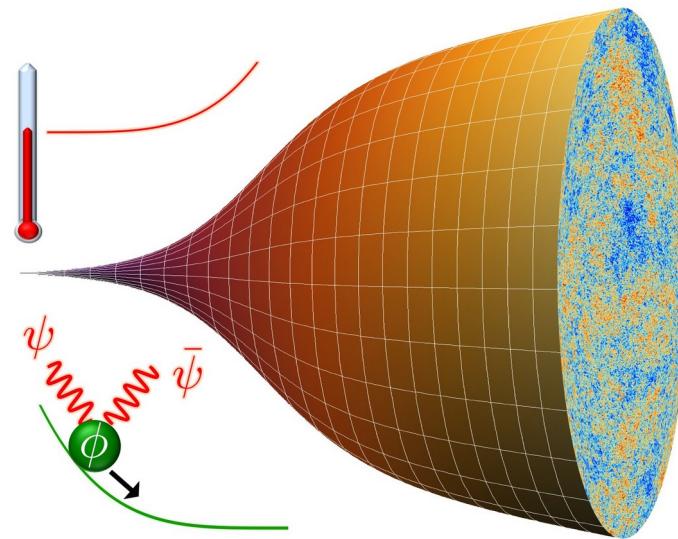


# Warm Inflation & Gravitational Waves



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INTERNATIONAL  
CENTRE *for*  
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# Warm Inflation & Gravitational Waves

Cold inflation/Warm inflation

Model building: dissipative coefficient

Primordial spectrum: PBH & (scalar induced) GW

# Expanding Universe

Flatness problem

$$\Omega_T = 1 \rightarrow \Omega_T(t_{\text{nucl}}) - 1 \approx 10^{-16}$$

Horizon problem

The observable Universe was larger than the **particle horizon** at LSS

Inflation

Early period of accelerated expansion

$$\ddot{a} > 0: P < -\rho/3$$

Primordial spectrum?

Too small sub-horizon  
**(causal)** perturbations

Unwanted relics...

**monopoles**, moduli, gravitinos,...

[Starobinsky '80; Guth '81; Sato' 81; Albrecht, Steinhardt '82; Linde '82]

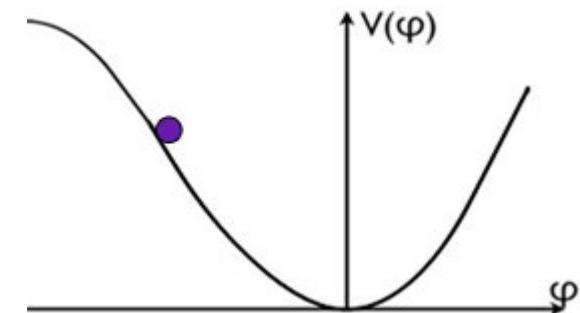
# Slow Roll Inflation

Scalar field rolling down its (flat) potential

$$P = \dot{\varphi}^2/2 - V(\varphi) \approx -V(\varphi) \quad \text{negative pressure}$$

**“Flat” potential**

The curvature and the slope smaller than the (Hubble) expansion rate H



**Kinetic energy << potential energy**  $H^2 \sim V/3m_P^2$  **Hubble parameter** ( $H = \dot{a}/a$ )  
 $(a = \text{scale factor})$

**Slow-roll parameters**

$$\eta_\varphi = m_P^2 \left| \frac{V''}{V} \right| < 1 \quad \epsilon_\varphi = \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2 < 1$$

**curvature**

**slope**



[No. efolds:  $N_e = \ln a/a_i$ ]

**Slow-roll equation**

$$\dot{\varphi} \simeq -V'/3H$$

**Primordial spectrum**

$$P_R \simeq \left( \frac{H}{\dot{\varphi}} \right)^2 \left( \frac{H}{2\pi} \right)^2 \quad n_s = 1 + 2\eta_\varphi - 6\epsilon_\varphi$$

$$r = \frac{P_T}{P_R} \simeq 16\epsilon_\varphi \quad [\text{tensor-to-scalar ratio}]$$

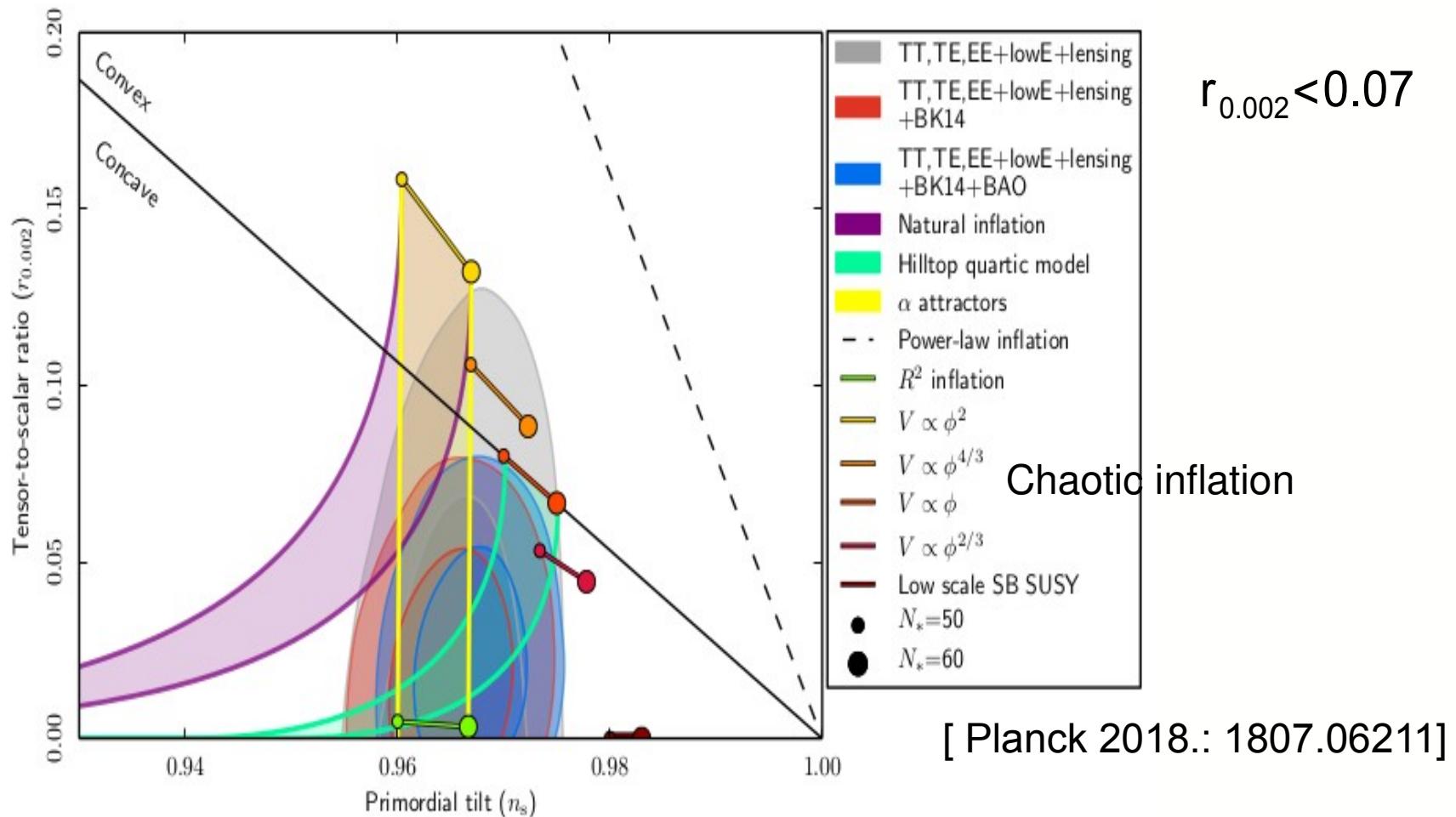
$$V^{1/4} \sim 10^{16} \left( \frac{r}{0.1} \right)^{1/4} \text{Gev}$$

[single field models]

# Primordial spectrum: ~adiabatic, ~scale-invariant, gaussian?, tensors?

**Primordial spectrum:**  $P_R = P_R(k_0) (k/k_0)^{n_s - 1}$      $k_0 = 0.002 \text{ Mpc}^{-1}$

**Tensor-to-scalar Ratio:**  $r = P_T/P_R$      $P_R = 2.2 \times 10^{-9}$

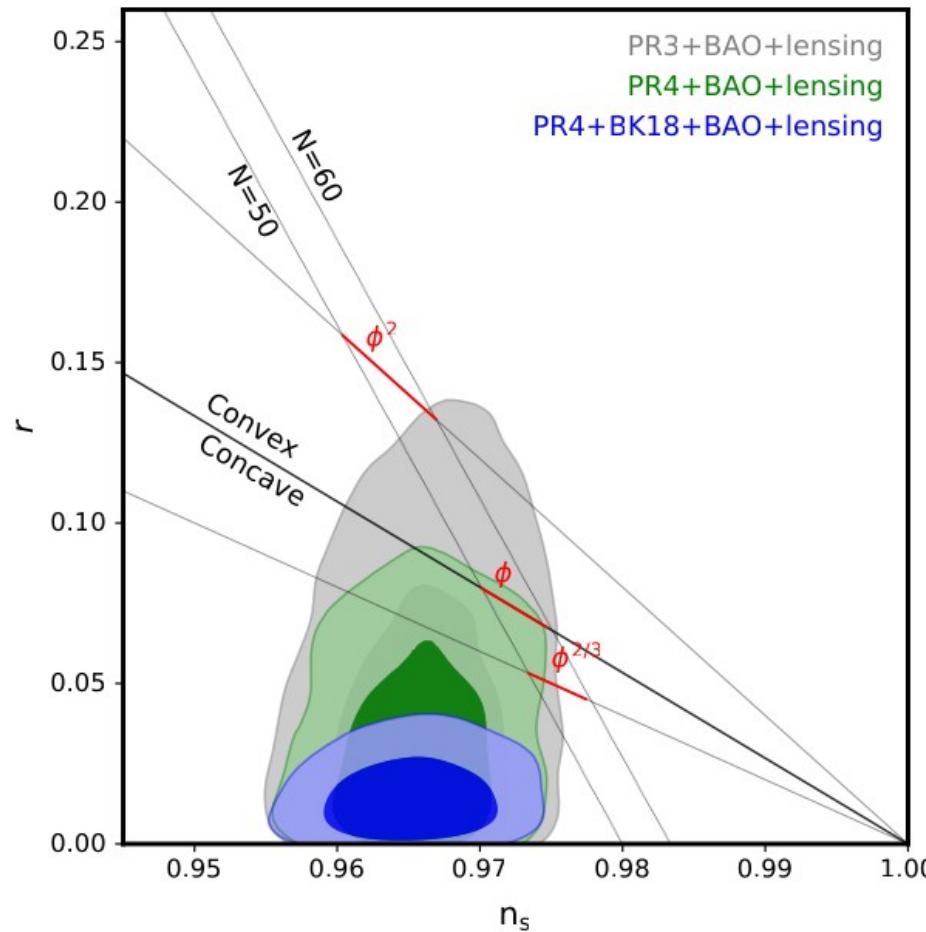


**CMB constraints ~10 efolds of inflation....**

**Primordial spectrum: ~adiabatic, ~scale-invariant, gaussian?, tensors?**

**Primordial spectrum:**  $P_R = P_R(k_0)(k/k_0)^{n_s-1}$      $k_0 = 0.05 \text{ Mpc}^{-1}$

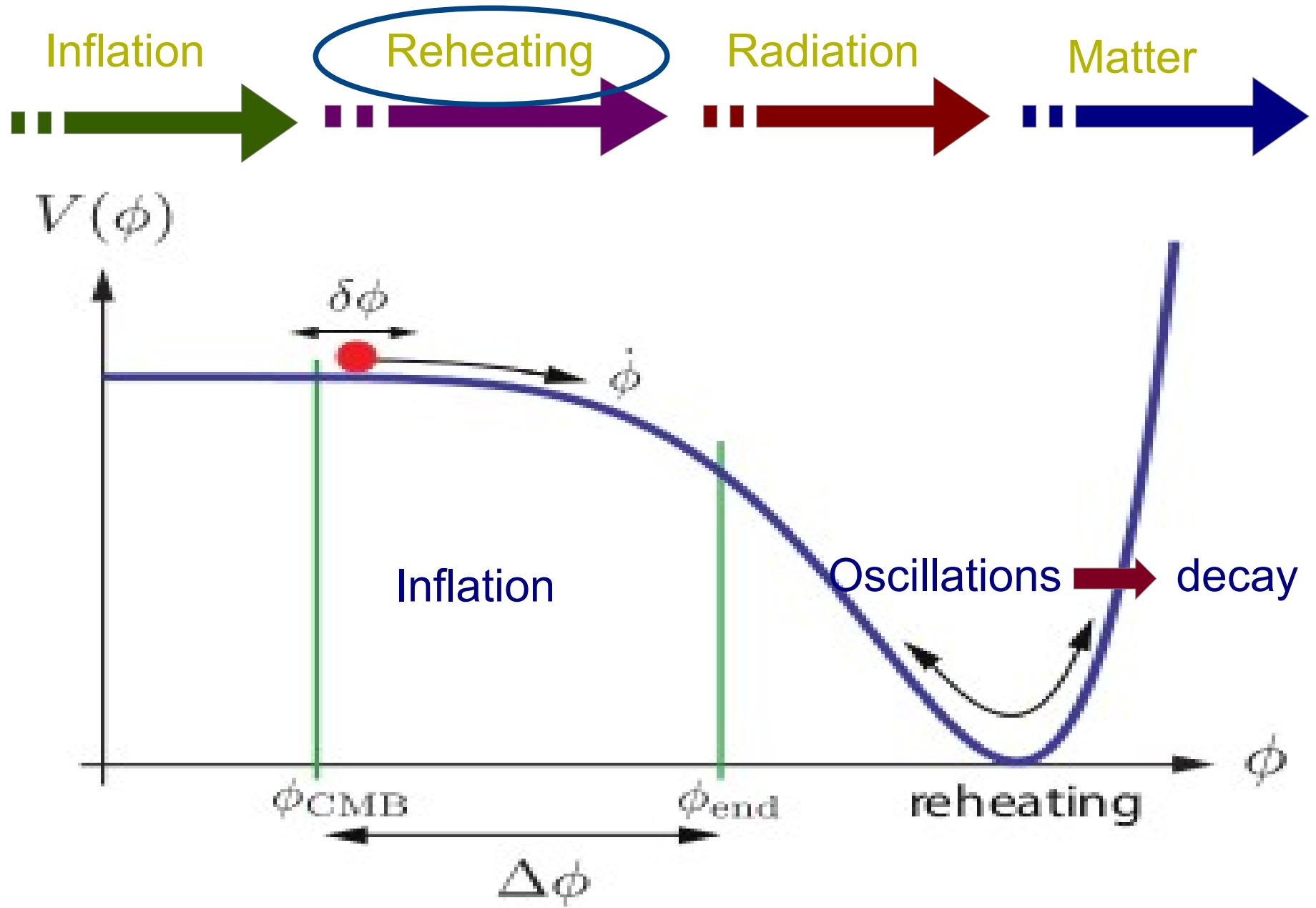
**Tensor-to-scalar Ratio:**  $r = P_T/P_R$      $P_R = 2.2 \times 10^{-9}$

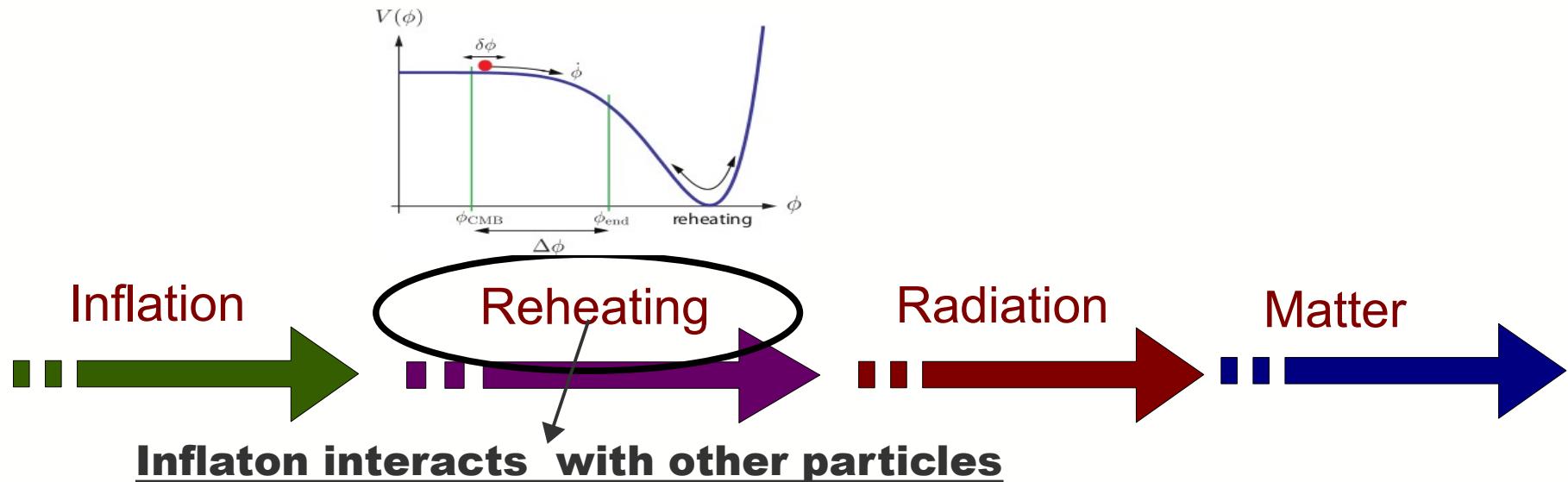


$r < 0.032$

$H_I < 4.4 \times 10^{13} \text{ GeV}$

[ Tristam et al.: 2112.07961]





[Gravitational reheating inconsistent with BBN: limit on GW energy density]

[Figueroa & Tanin 1811.04093]

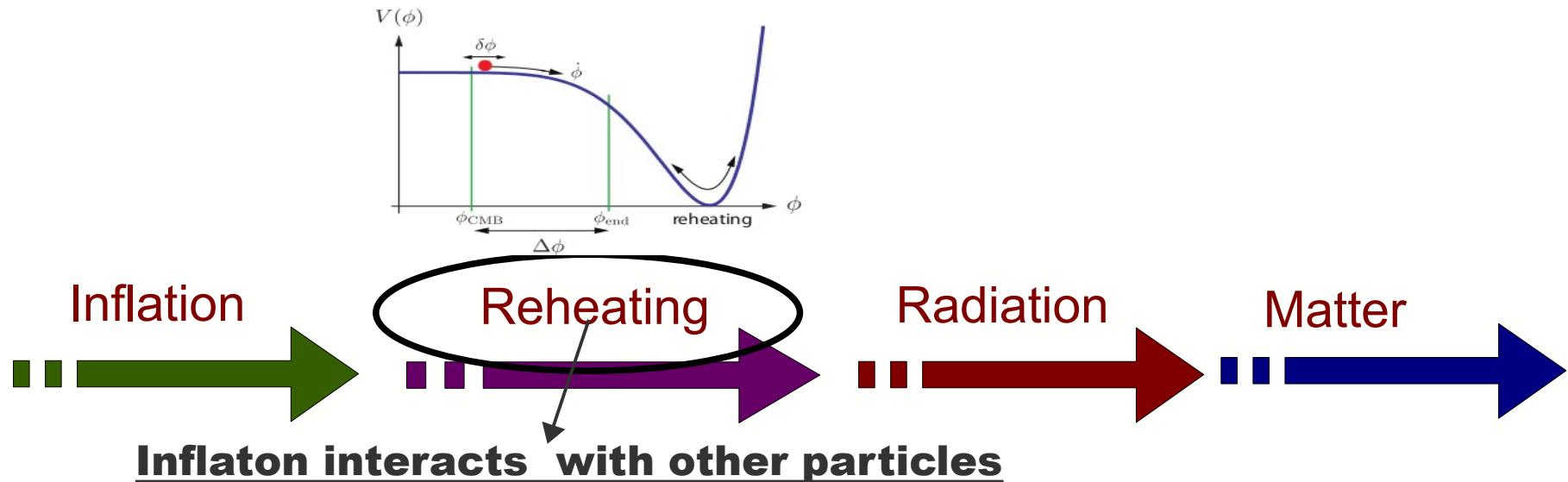
"Cold" inflation: Interactions negligible during Inflation → Radiation

Inflation & Particle production (non-thermal): "Dark photon"

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}$$

[Anber & Sorbo PRD81 2010]

- Non-Gaussian Primordial spectrum
- Gravitational Waves
- Dark Matter...



[Gravitational reheating inconsistent with BBN: limit on GW energy density]

[Figueroa & Tanin 1811.04093]

"Cold" inflation: Interactions negligible during Inflation

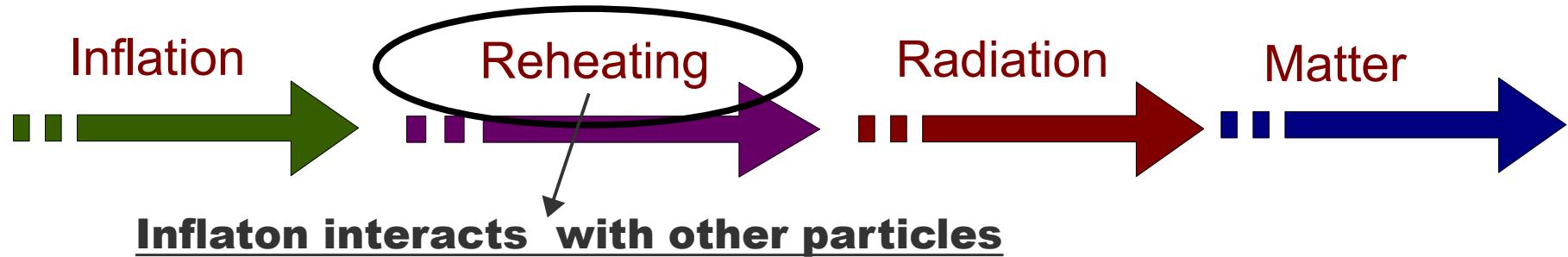


"Warm" inflation: Inflaton "decay" into light dof  
(through a mediator)



$$[T > H, \Gamma_\chi > H]$$

[Berera PRD55 '97; Berera, Moss & Ramos RPP72 '07]

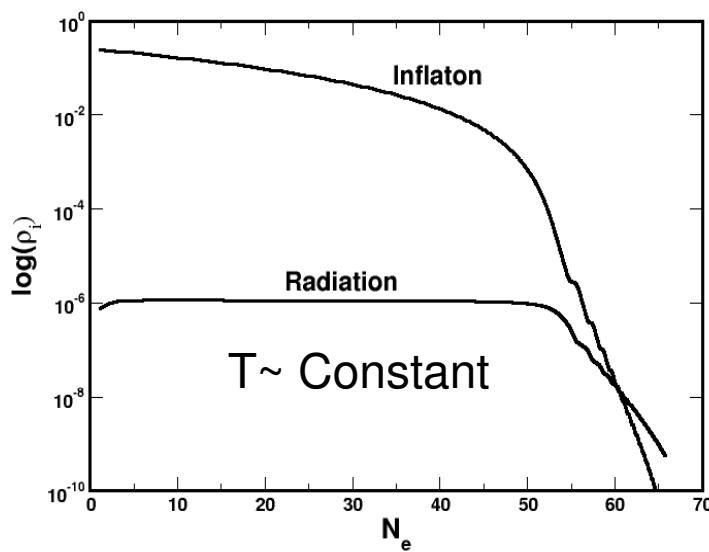


Interactions with the cosmic plasma induce dissipation

$$\ddot{\varphi} + (3H + Y)\dot{\varphi} + V_{\varphi} = 0$$

▲ “Decay” into light dof= extra friction

“Warm” inflation:



A (small) fraction of the vacuum energy is converted into radiation during inflation

$$\dot{\rho}_R + 4H\rho_R = Y\dot{\varphi}^2 \quad \text{“Source term”}$$

Slow-roll:

$$\left\{ \begin{array}{l} (3H + Y)\dot{\varphi} \approx -V_{\varphi} \\ 4H\rho_R \approx Y\dot{\varphi}^2 \end{array} \right.$$

Extra friction term:  $Q = Y/(3H)$ ,  $Y(T, \phi)$

- $Q \ll 1, T \ll H$   $\rightarrow$  Standard Cold Inflation
- $Q < 1, T > H$   $\rightarrow$  Weak Dissipative Regime
- $Q > 1, T > H$   $\rightarrow$  Strong Dissipative Regime

} Standard slow-roll

Slow-roll :  $3H(1+Q)\dot{\phi} \approx -V_\phi(\phi, T)$ ,  $\rho_r \approx \frac{3}{4}Q\dot{\phi}^2$

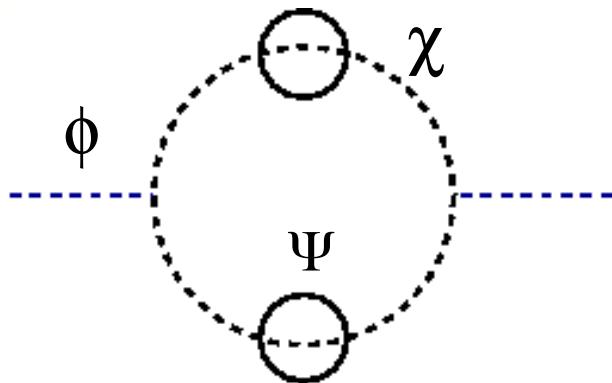
$$|\eta_\phi| < (1+Q), \quad \epsilon_\phi < (1+Q), \quad \delta_T = T V_{T\phi} / V_\phi < 1 \quad \text{Thermal corrections}$$

- $Q$  varies during inflation (WDR  $\rightarrow$  SDR)
- Dissipation induces thermal inflaton fluctuations (**different primordial spectrum**)

- Inflation lasts longer due to extra friction  $\rightarrow H_{60}^{\text{warm}} < H_{60}^{\text{cold}} \rightarrow r < 0.1$
- Slow-roll parameters can be larger than 1, when  $Q > 1$  (strong dissipative regime)

[ no “eta” problem in sugra models]

## Interactions & (T-dependent) Dissipative coefficient



$$L = -\frac{1}{2} m_\varphi^2 \varphi^2 - \frac{g^2}{2} \varphi^2 \chi^2 + h \chi \psi \bar{\psi} + \dots$$

light fermions, scalars  
mediator

- “Heavy” ( $m > T$ ) mediator: [low-T regime]  $Y \simeq C_\varphi \frac{T^3}{\varphi^2}$

[ $T > H$ ]

Thermal corrections under control (inflaton coupled to heavy fields), but getting 50-60 efolds of inflation typically requires large no. of fields  $C_\varphi \sim 10^6$

[BG, Berera, Ramos & Rosa 2012; BG, Berera & Kronberg 2015; R. Ayra et al. 2018]

- “Light” ( $m < T$ ) mediator: [high-T regime]  $Y \simeq C_\varphi T, C_\varphi/T$

[ $T > H$ ]

Thermal corrections may spoil inflation!  $\Delta V_T = -\frac{\pi^2}{90} g_R T^4 + \frac{g^2 \varphi^2}{12} T^2 + \dots$

[Berera, Gleiser & Ramos PRD'98; Yokoyama & Linde PRD '98]

**Solution:** use symmetries to protect the inflaton mass

- “Little Warm Inflation” :

Inflaton a PNGB of a broken U(1) symmetry + pair of fermions/scalars + exchange symmetry

$$\varphi_1 = \frac{M}{\sqrt{2}} e^{\varphi/M}, \quad \varphi_2 = \frac{M}{\sqrt{2}} e^{-\varphi/M}$$

$$\varphi_1 \longleftrightarrow i\varphi_2 \quad \Psi_{1L,R} \longleftrightarrow \Psi_{2L,R}$$

$$L = \cdots - g M \cos(\varphi/M) \bar{\psi}_1 \psi_1 - g M \sin(\varphi/M) \bar{\psi}_2 \psi_2 - h \sigma \sum_{i=1,2} (\bar{\psi}_i \psi_\sigma + \bar{\psi}_\sigma \psi_i) + \cdots$$

Fermion masses are bounded!! [  $g M < T < M$  ]

$$\Delta V_T = -\frac{\pi^2}{90} g_R T^4 + \underbrace{\frac{g^2 M^2}{12} T^2}_{\text{No thermal mass for the inflaton}} + \frac{g^4(\varphi) M^4}{16\pi^2} \left( \log \frac{\mu^2}{T^2} - c_f \right)$$

No thermal mass for the inflaton

### Inflaton + pair of fermions

$$Y \simeq C(h) \frac{g^2}{h^2} T$$

Linear T

### Inflaton + pair of scalars

$$Y \simeq \frac{4g^2}{h^2} \frac{g^2 M^2}{T} F[m_\chi/T]$$

Inverse T

- “Minimal Warm Inflation” :

[Berghaus, Graham, Kaplan 1910.07525]

Axion-like inflation (PNGB of a broken gauge symmetry) + gauge production [SU(N)]

$$L = \dots - \frac{1}{2g^2} \text{Tr } G_{\mu\nu} G^{\mu\nu} - \frac{\phi}{16\pi^2 M} \text{Tr } G_{\mu\nu} \tilde{G}^{\mu\nu} - \bar{\Psi} (\gamma^{\mu} D_{\mu} - m_f) \Psi$$

Gauge production → dissipation

[Mishra, Mohanty & Nautiyal, 1106.3039; Visinelli 1107.3523]

[Ferrerira & Notari 1711.07483; Kamali 1901.01897]

- Axion mass protected by shift symmetry at perturbative level; non-perturbative corrections ~ axion mass, negligible

- Dissipative coefficient

Sphaleron rate       $\Gamma_{\text{sph}} \simeq N_c^5 \alpha^5 T^4$

$$Y \simeq \frac{\Gamma_{\text{sph}}}{2TM^2} \frac{Cm_f^2/T}{Cm_f^2/T + C_R \Gamma_{\text{sph}}} \quad \left\{ \begin{array}{l} Y \sim T^3 \quad (m_f \rightarrow \infty) \\ Y \sim T \quad (m_f \text{ light}) \end{array} \right.$$

- MWI + hybrid inflation → SDR ( $Q \gg 1$ ) at horizon crossing
- Warm inflation is an attractor

- ◆ Sphalerons induce (constant) dissipation even with  $T=0$  (vacuum decay  $Y \sim \text{Constant}$ )



[Laine & Procacci 2102.09913]

$$Y \simeq \frac{d_A \alpha^2}{f_a^2} \left( \underbrace{\kappa (\alpha N_c T)^3 F[\omega/T]}_{\text{Thermal}} + \underbrace{(1+n_B(\omega)) \frac{\pi \omega^3}{(4\pi)^4}}_{\text{Vacuum}} \right) \quad [\omega \simeq m]$$

Thermal  $[\omega \simeq m < \alpha^2 T]$

$$Y_{IR} \sim \frac{\alpha^5 T^3}{f_a^2}$$

Vacuum  $[\omega \simeq m \gg \alpha^2 T]$

$$Y_{UV} \sim \frac{\alpha^2 \omega^3}{f_a^2} \quad [\omega \simeq m]$$

$$F[\omega/T] \simeq \frac{1 + \frac{\omega^2}{(c_{IR} \alpha^2 N_c^2 T)^2}}{1 + \frac{\omega^2}{(c_M \alpha N_c T)^2}}, \quad d_A = N_c^2 - 1, \quad \kappa \simeq 1.5, \quad c_{IR} \simeq 106, \quad c_M \simeq 5.1$$

# Fluctuations & primordial spectrum: coupled system

Metric:  $ds^2 = -(1+2\alpha)dt^2 - 2a\partial_i\beta dx^i dt + a^2[\delta_{ij}(1+2\phi) + 2\partial_i\partial_j\gamma]dx^i dx^j$

Scalar metric eom : Einstein Eqs

Field eom:

$$\delta \ddot{\varphi}_k + (3H + Y)\delta \dot{\varphi}_k + \dot{\varphi}\delta Y + \left(\frac{k^2}{a^2} + V_{\varphi\varphi}\right)\delta \varphi_k \simeq (2YT)^{1/2} \hat{\xi}_k + \xi_q + [\text{metric}]$$

fluctuation force  $\xi$

$$P_{\delta\varphi} \simeq \left(\frac{H}{2\pi}\right)^2$$

$\uparrow T \rightarrow 0$

Radiation fluid:

$$\delta \dot{\rho}_r + 4H\delta \rho_r \simeq \frac{k^2}{a^2} \Psi_r + \dot{\varphi}^2 \delta Y + 2\dot{\varphi}Y\delta\dot{\varphi} + [\text{metric}] \quad \text{Energy density}$$

$$\dot{\Psi}_r + 3H\Psi_r = -\delta \rho_r^{GI}/3 - \dot{\varphi}Y\delta\dot{\varphi} + [\text{metric}] \quad \text{Momentum density}$$

[Ramos & da Silva, 1302.3544; BG, Berera, Moss & Ramos, 1401.1149]

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[Ramos & da Silva, 1302.3544; BG, Berera, Moss & Ramos, 1401.1149]

Tensors:  $\ddot{h}_{ij} + 3hH_{ij} + \frac{k^2}{a^2}h_{ij} \simeq \frac{2}{m_P^2 a^2} \Pi_{ij}^{\text{TT}}$  [Qui & Sorbo, 2107.09754]

$$P_h = 8\left(\frac{H}{2\pi m_P}\right)^2 + P_h(T) \simeq 8\left(\frac{H}{2\pi m_P}\right)^2$$

[ $P_h(T) \sim \frac{T^5}{H m_P^4}$ ]

## Fluctuations & primordial spectrum: coupled system

Field EOM:

$$\delta \ddot{\varphi}_k + (3H + Y) \delta \dot{\varphi}_k + \dot{\varphi} \delta Y + \left( \frac{k^2}{a^2} + V_{\varphi\varphi} \right) \delta \varphi_k \simeq (2 YT)^{1/2} \hat{\xi}_k + \dots$$

fluctuation force  $\hat{\xi}$

$$Y \sim T^c$$

$$\frac{\delta Y}{Y} = c \frac{\delta T}{T}$$



Coupled system  
inflaton-radiation

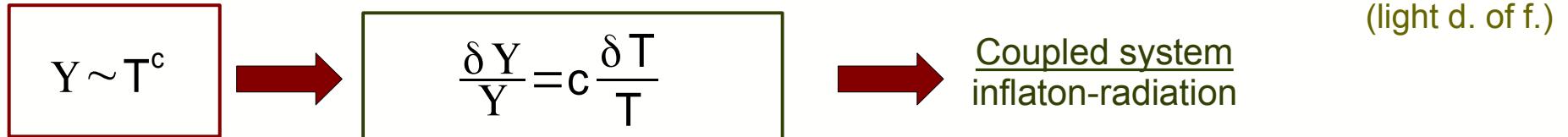
(light d. of f.)

# Fluctuations & primordial spectrum: coupled system

Field EOM:

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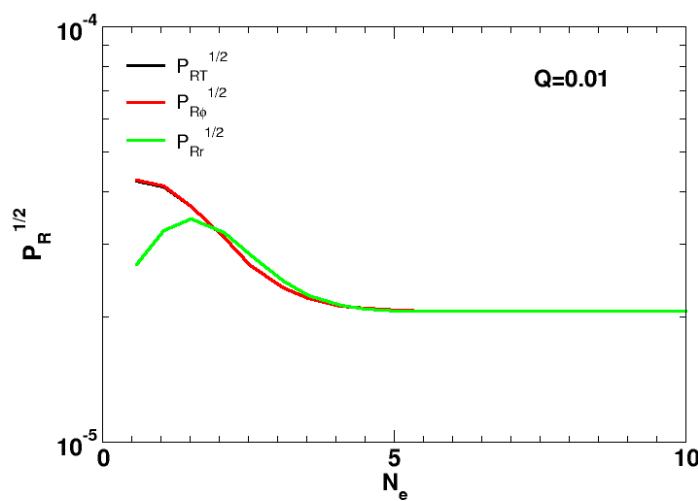
fluctuation force  $\hat{\xi}$



- Weak dissipative regime ( $Q=Y/H \ll 1$ ) : field decoupled from radiation

$$\delta \ddot{\varphi}_k + (3H + Y) \delta \dot{\varphi}_k + \left( \frac{k^2}{a^2} + V_{\varphi\varphi} \right) \delta \varphi_k \simeq (2 YT)^{1/2} \hat{\xi}_k$$

$Q=0.01, c=1$



[ $R$  is constant after horizon crossing (freeze-out)]

$$P_R = \frac{h_\varphi}{h_T} P_{R\varphi} + \frac{h_r}{h_T} P_{Rr} \simeq P_{Rr} \simeq P_{R\varphi}, \quad (h_i = \rho_i + p_i)$$

$$P_R \simeq (P_R)_{Q=0} \left( 1 + 2N + \frac{T}{H} \frac{4\pi Q}{\sqrt{1 + 4\pi Q/3}} \right)$$

Dissipative processes may maintain a non-trivial distribution of inflaton particles:  $N \simeq n_{BE} = (e^{k/aT} - 1)^{-1}$

# Fluctuations & primordial spectrum: coupled system

Field EOM:

$$\delta \ddot{\phi}_k + (3H + Y) \delta \dot{\phi}_k + \dot{\phi} \delta Y + \left( \frac{k^2}{a^2} + V_{\phi\phi} \right) \delta \phi_k \simeq (2YT)^{1/2} \hat{\xi}_k + \dots$$

fluctuation force  $\hat{\xi}$

$$Y \sim T^c$$

$$\frac{\delta Y}{Y} = c \frac{\delta T}{T}$$



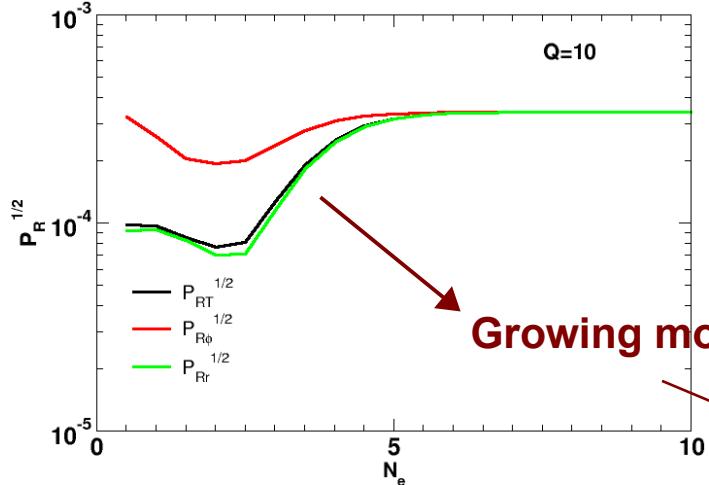
Coupled system  
inflaton-radiation

(light d. of f.)

- Strong dissipative regime ( $Q=Y/H > 1$ )

[ $R$  is constant after horizon crossing (freeze-out)]

$$P_R = \frac{h_\phi}{h_T} P_{R_\phi} + \frac{h_r}{h_T} P_{R_r} \simeq P_{R_r} \simeq P_{R_\phi}, \quad (h_i = \rho_i + p_i)$$



$$G[Q] \sim Q^\alpha$$

When  $c > 0, \alpha > 0$



Blue tilted spectrum  
**Amplification of the primordial spectrum**

Numerical integration

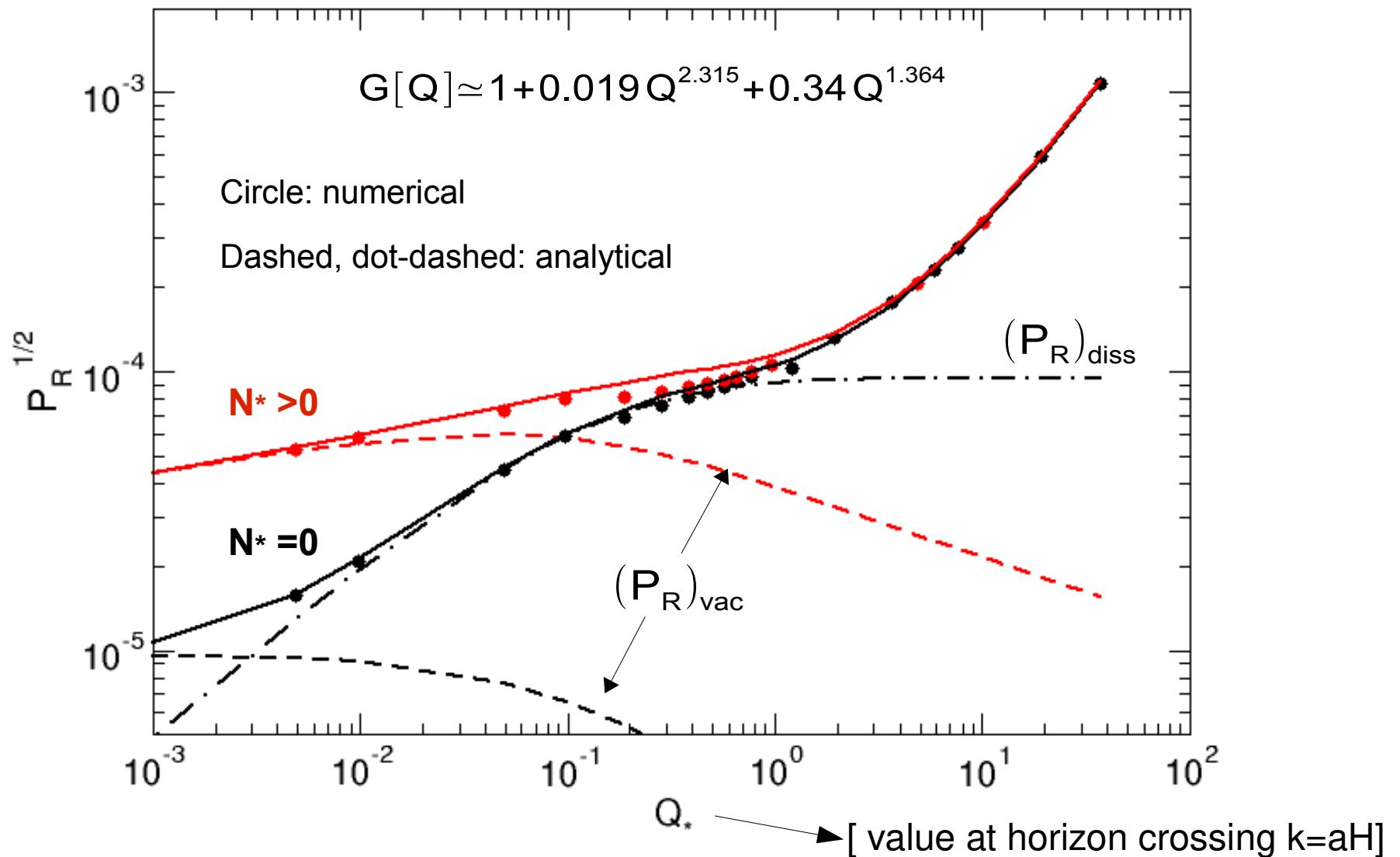
$$P_R \simeq \left( \frac{H^2}{2\pi \dot{\phi}} \right)^2 \left( 1 + 2N + \frac{T}{H} \frac{2\pi Q}{\sqrt{1+4\pi Q/3}} \right) G[Q]$$

**Cold inflation,  $Q=0, T/H=0$**

[Moss & Graham 0905.3500; BG, Berera & Ramos, 1106.0701]

## Primordial spectrum: an example, WLI

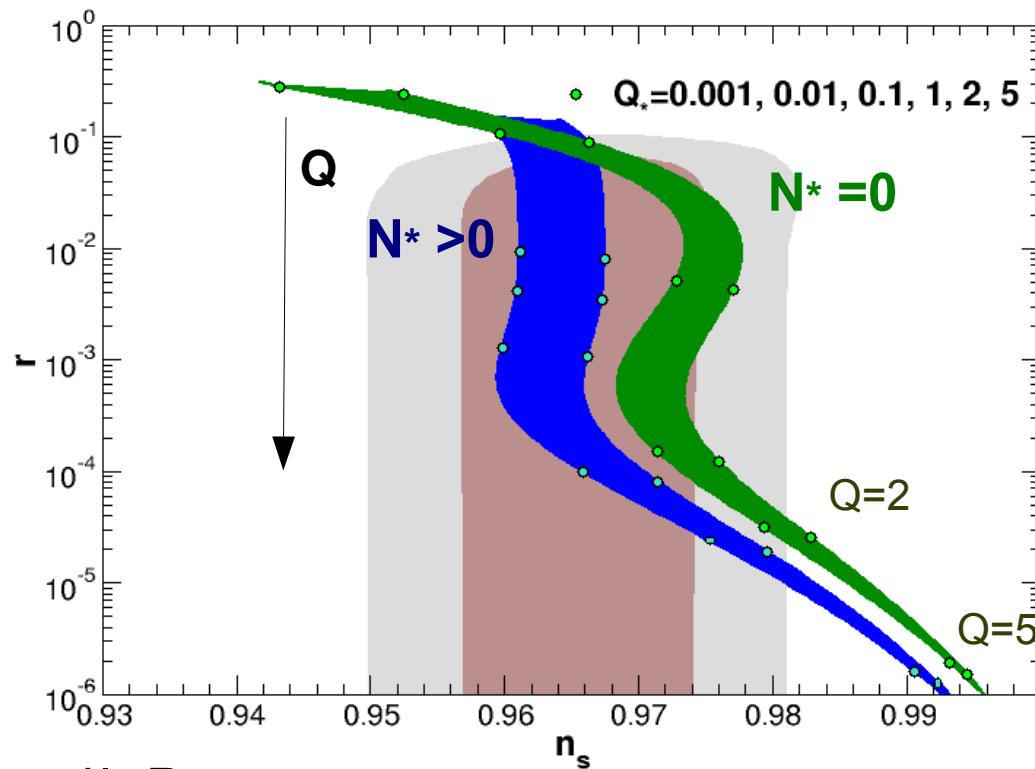
$$P_R \simeq ((P_R)_{\text{vac}} + (P_R)_{\text{diss}}) G[Q]$$



Chaotic model:  $V(\varphi) = \lambda \varphi^4/4$ ,  $\lambda = 10^{-14}$ ,  $N_e = 50$ ,  $Y \propto T$

# Primordial spectrum: quartic chaotic model LWI

$$V(\varphi) = \frac{\lambda}{4} \varphi^4, \quad Y \sim T, \quad N_e = 50 - 60$$



$$n_s - 1 = \frac{d \ln P_R}{d N_e} = (n_s - 1)_N + (n_s - 1)_{\text{diss}} + (n_s - 1)_G, \quad (n_s - 1)_G > 0$$

$$r \simeq \frac{16 \epsilon_\phi}{(1+2N+\Delta_Q) G[Q]} \leq 16 \epsilon_\phi$$

**Quartic:**

$$N \neq 0, Q < 1: \quad n_s \simeq 1 - 2/N_e, \quad r \simeq 16 \epsilon_\phi \left( \frac{H}{T} \right) \ll 0.1$$

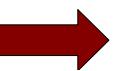
## Primordial spectrum: PBHs & GW

- CMB constraints the primordial spectrum at scales that leave the horizon 50-60 e-folds before the end:

$$P_R(k_{\text{CMB}}) \simeq 2.2 \times 10^{-9}$$

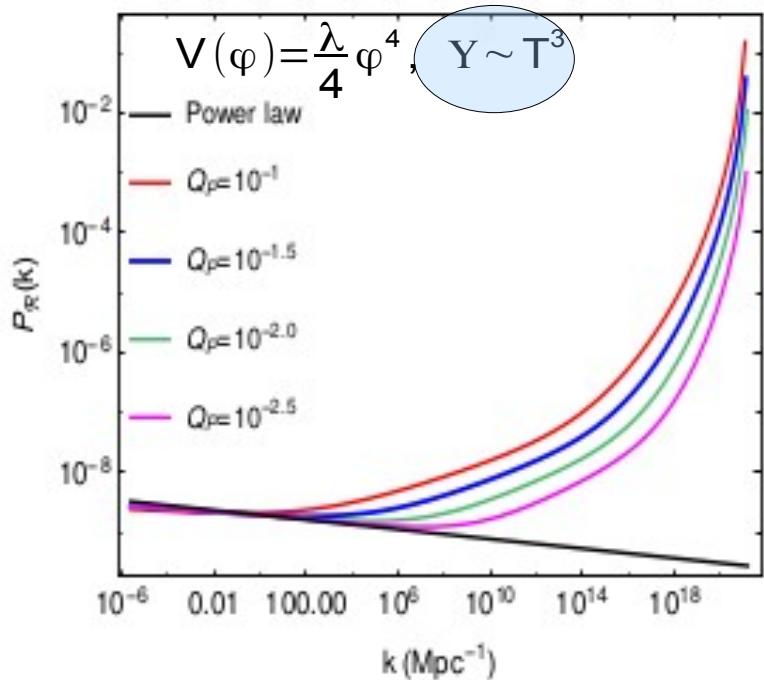
- Dissipative effects amplify the spectrum at smaller scales (near the end)

$$P_R(k) \sim O(0.01 - 0.1)$$



PBHs?

GW?



$P_R \sim O(0.01-0.1)$  will lead to PBH formation on re-entry

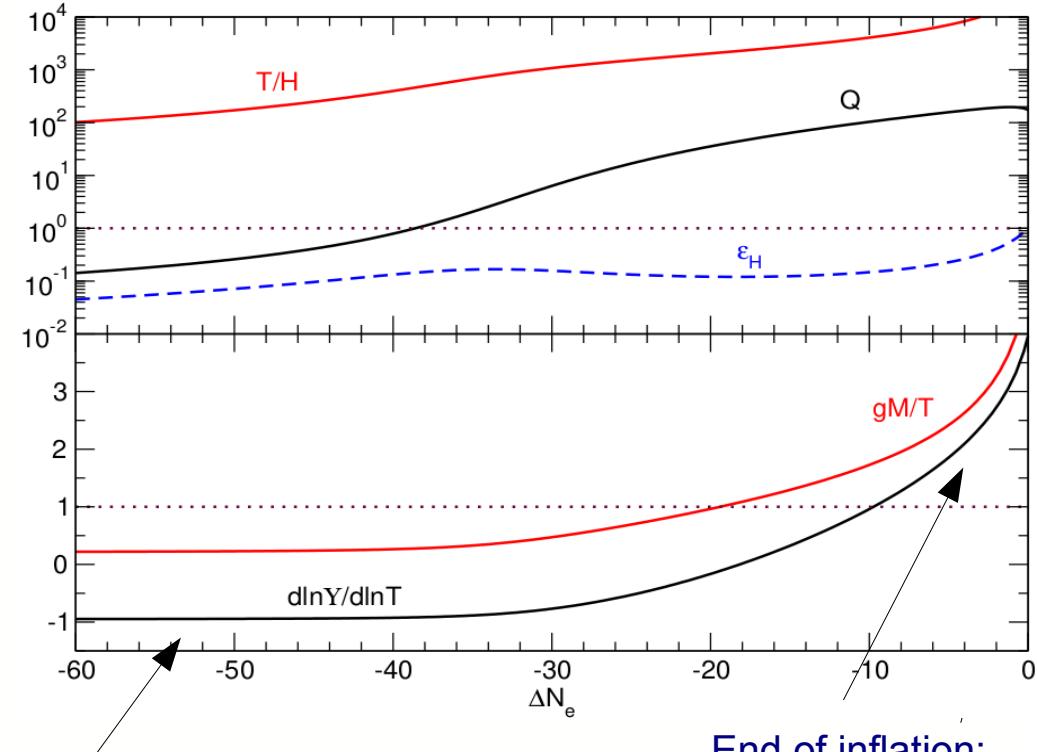
$$M_{\text{PBH}}(k) \simeq \gamma \frac{4 \pi m_p^2}{H_M} \quad \begin{array}{l} \text{H on re-entry, close to the end of} \\ \text{Inflation, during radiation} \end{array}$$

$$M_{\text{PBH}} \sim [5 \times 10^4 \text{ g}, 10^6 \text{ g}] \quad \begin{array}{l} \text{Light, evaporating black holes} \end{array}$$

# Scalar Warm Little Inflaton Model + quartic chaotic

$$V = \lambda \varphi^4 / 4$$

$[M = 10^{-4} m_P, g = 1, h = 2.5, \lambda = 10^{-14}]$



CMB scales: no growing mode

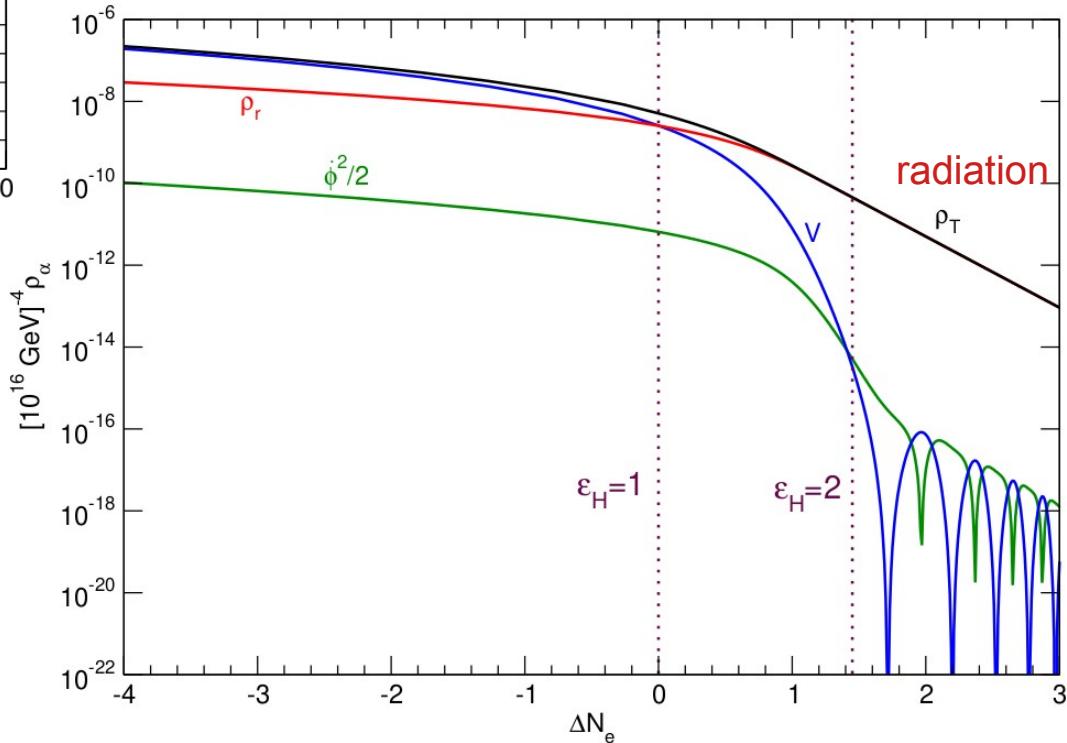
$$Y \simeq M^2/T$$

Primordial spectrum compatible  
with observations

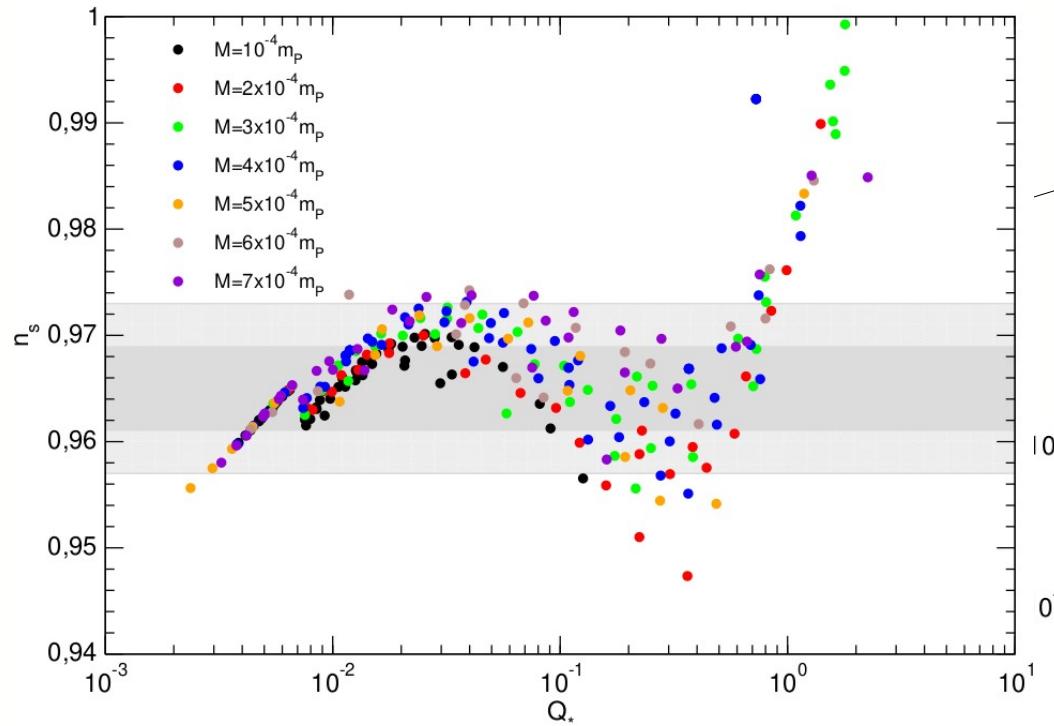
[BG & Subías 2105.08045]

$$Y \simeq \frac{4g^2}{h^2} \frac{g^2 M^2}{T} F[m_x/T]$$

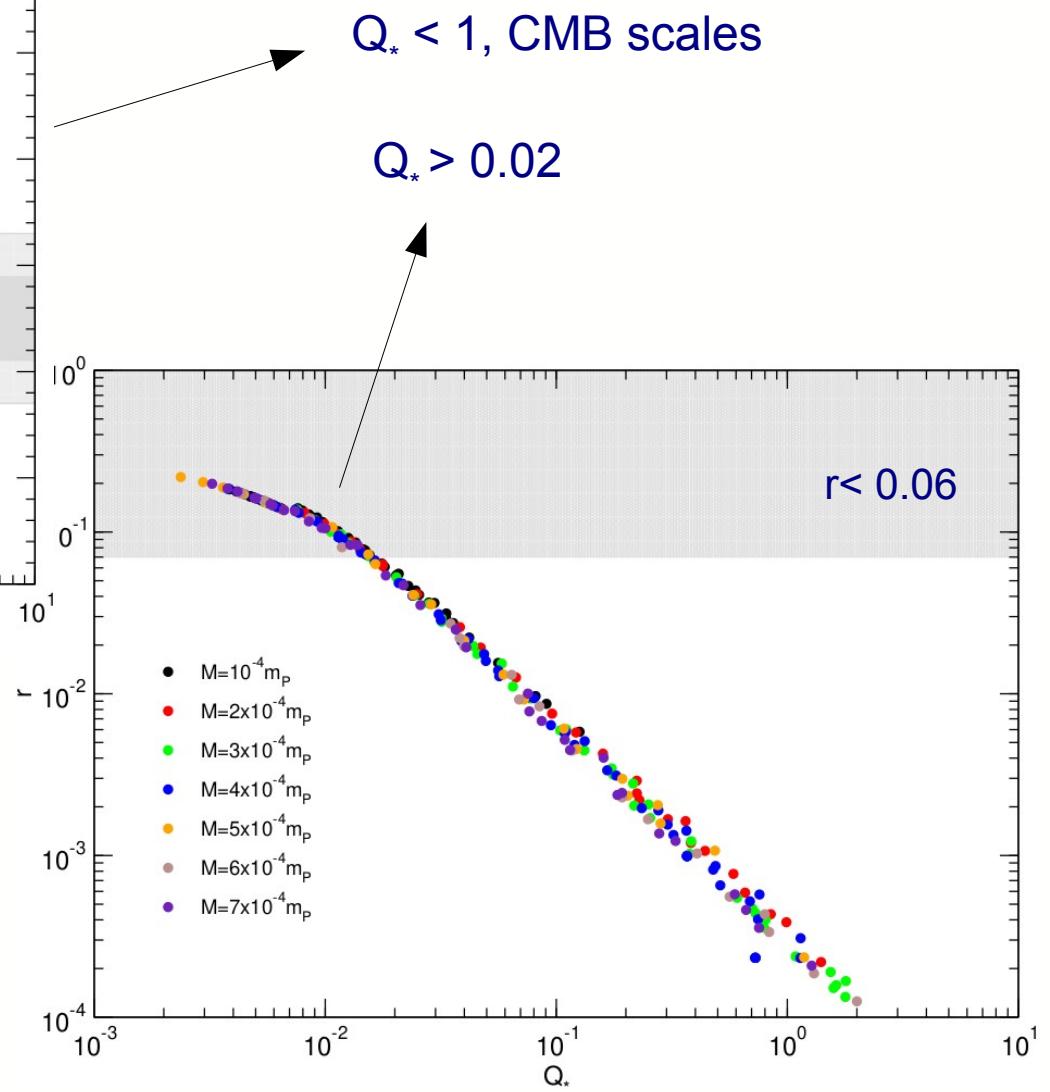
$$\left\{ \begin{array}{lll} M^2/T & \text{"High-T"} & gM/T \ll 1 \\ T^3/M^2 & \text{"Low-T"} & gM/T \gg 1 \end{array} \right.$$



## Spectral index & tensor-to-scalar ratio

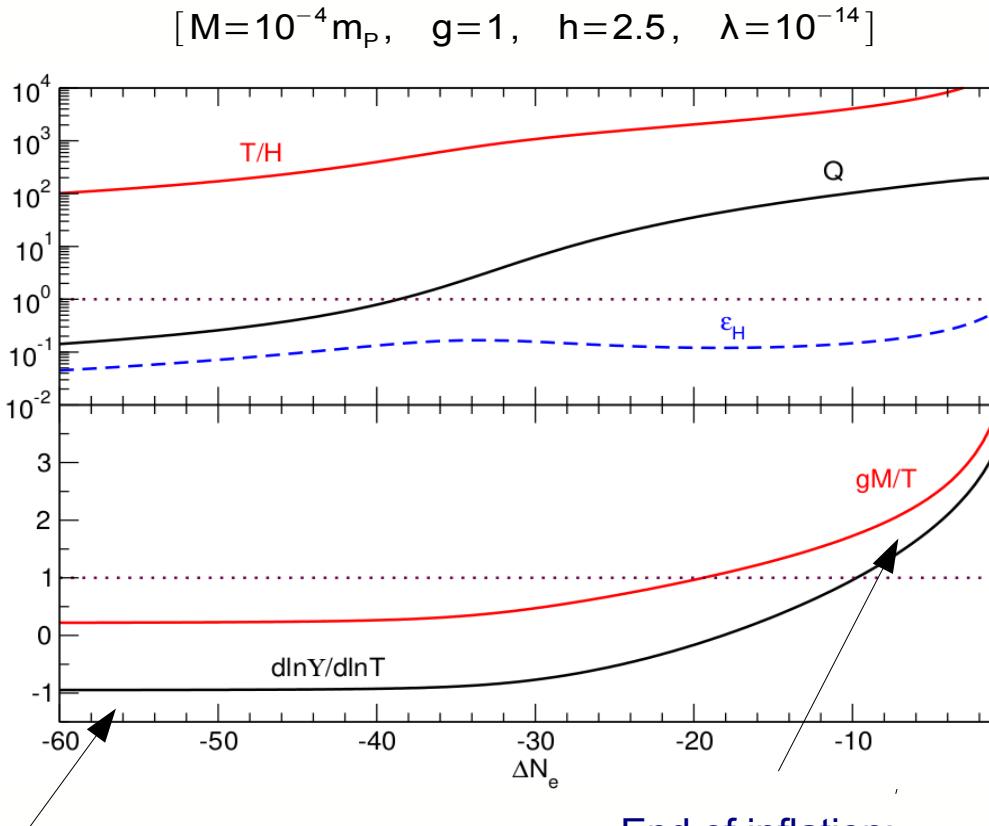


$[M \sim O(10^{-4})m_P, g \sim 0.5 - 1, h \sim O(1)]$



# Scalar Warm Little Inflaton Model + quartic chaotic

$$V = \lambda \varphi^4 / 4$$



CMB scales: no growing mode

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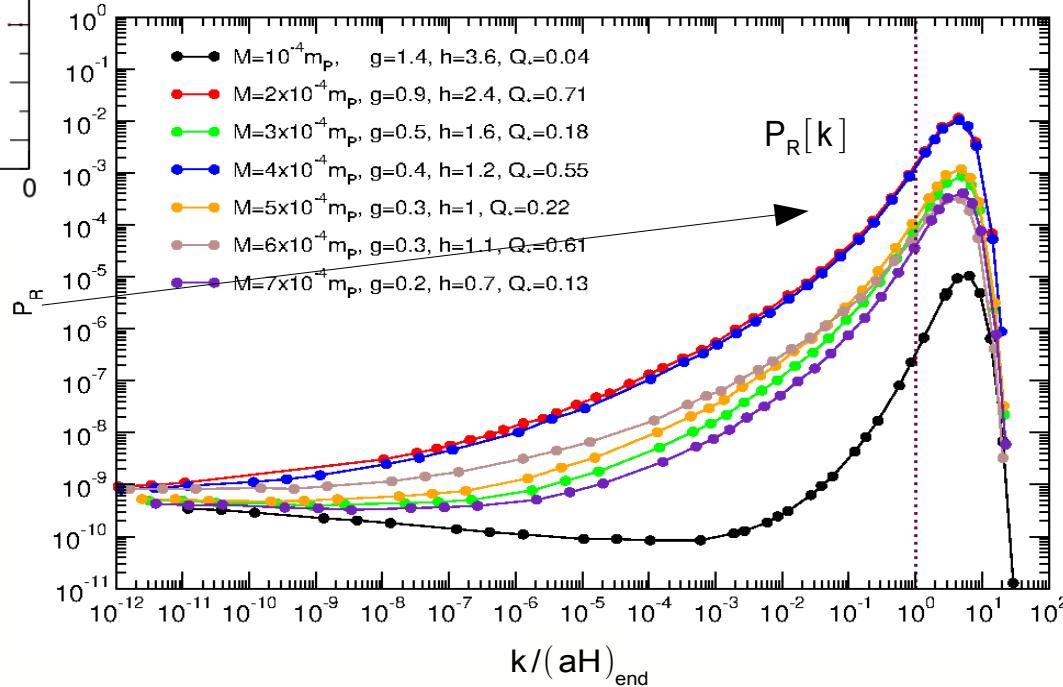
Primordial spectrum compatible  
with observations

[BG & Subías 2105.08045]

$$Y \simeq \frac{4 g^2}{h^2} \frac{g^2 M^2}{T} F[m_x/T]$$

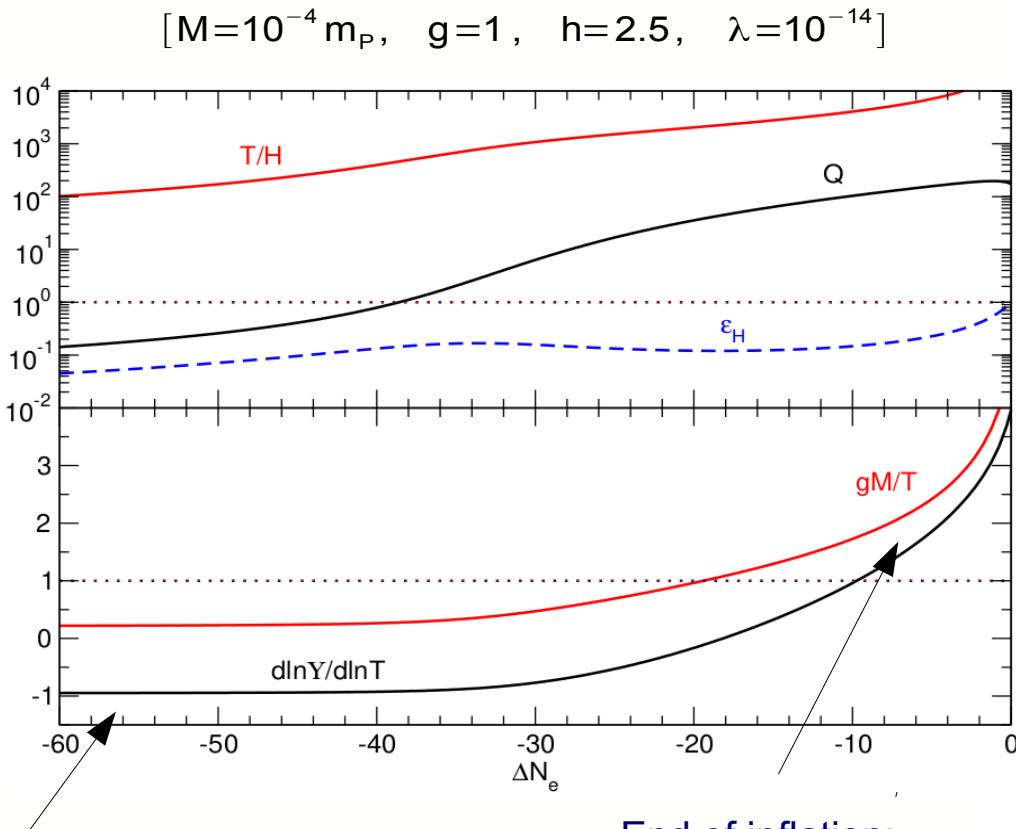
$M^2/T$  “High-T”  
 $T^3/M^2$  “Low-T”

Numerical integration upto  $\epsilon_H = 2$



# Scalar Warm Little Inflaton Model + quartic chaotic

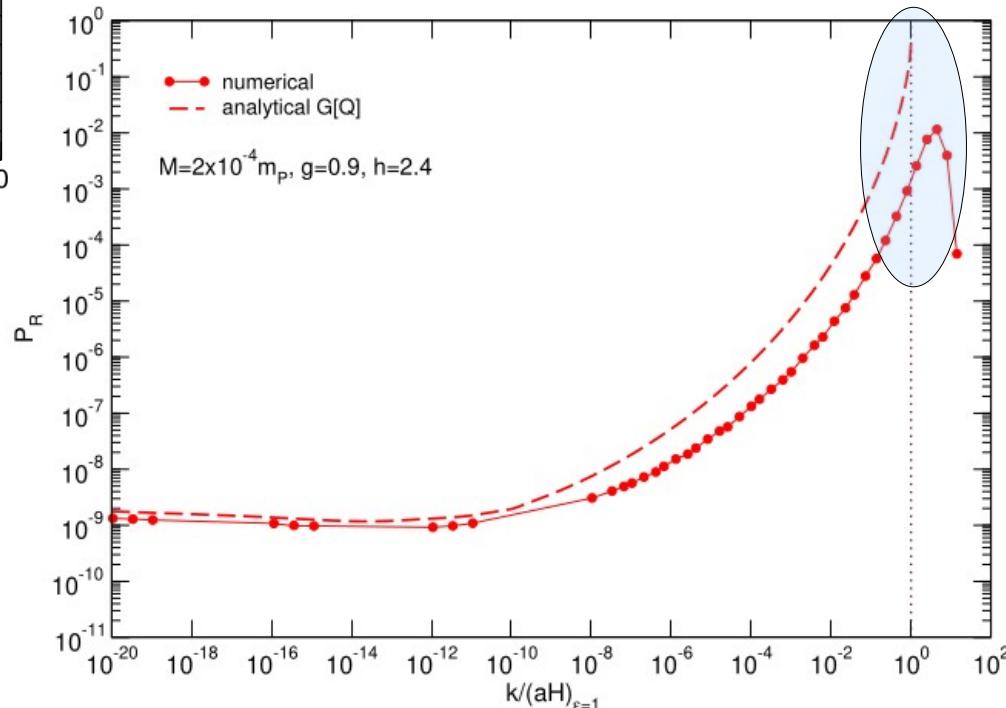
$$V = \lambda \varphi^4 / 4$$



$$Y \simeq \frac{4g^2}{h^2} \frac{g^2 M^2}{T} F[m_\chi/T]$$

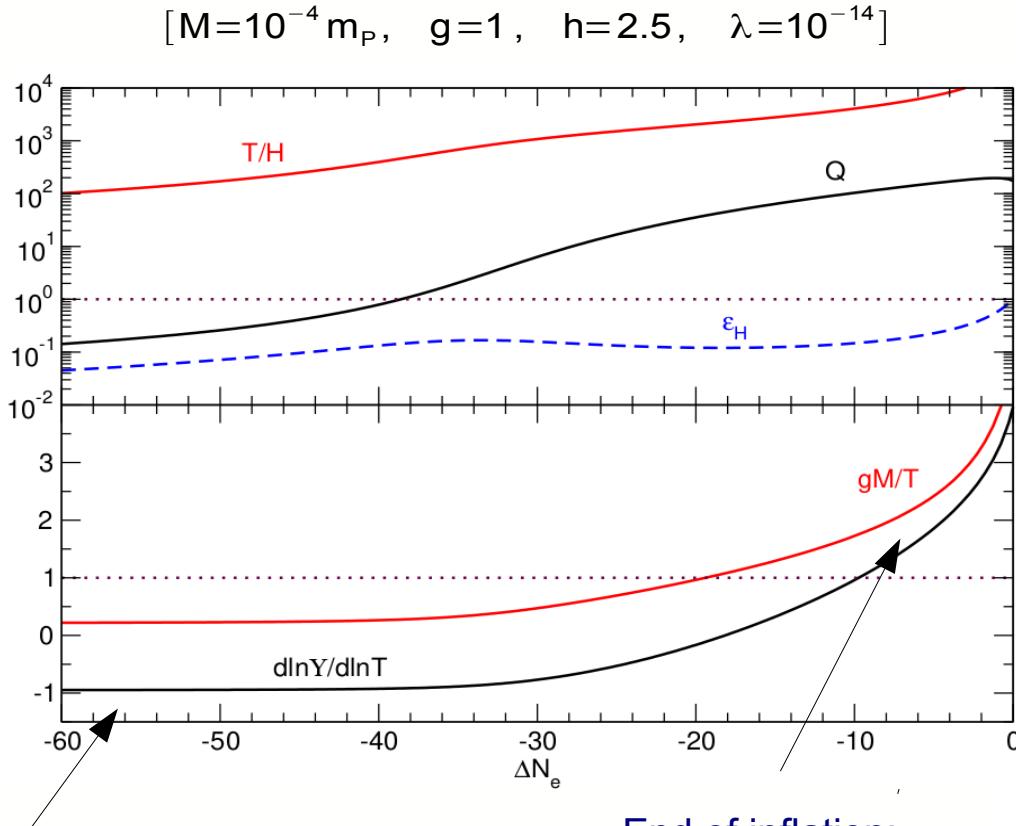
$$\left\{ \begin{array}{ll} M^2/T & \text{"High-T"} \\ T^3/M^2 & \text{"Low-T"} \end{array} \right.$$

## Numerical versus analytical $G[Q]$ at CMB scales



# Scalar Warm Little Inflaton Model + quartic chaotic

$$V = \lambda \varphi^4 / 4$$



CMB scales: no growing mode

$$Y \simeq M^2/T$$

Primordial spectrum compatible with observations

End of inflation:  
amplification of the  
primordial spectrum

$$Y \simeq T^3/M^2$$

[BG & Subías 2105.08045]

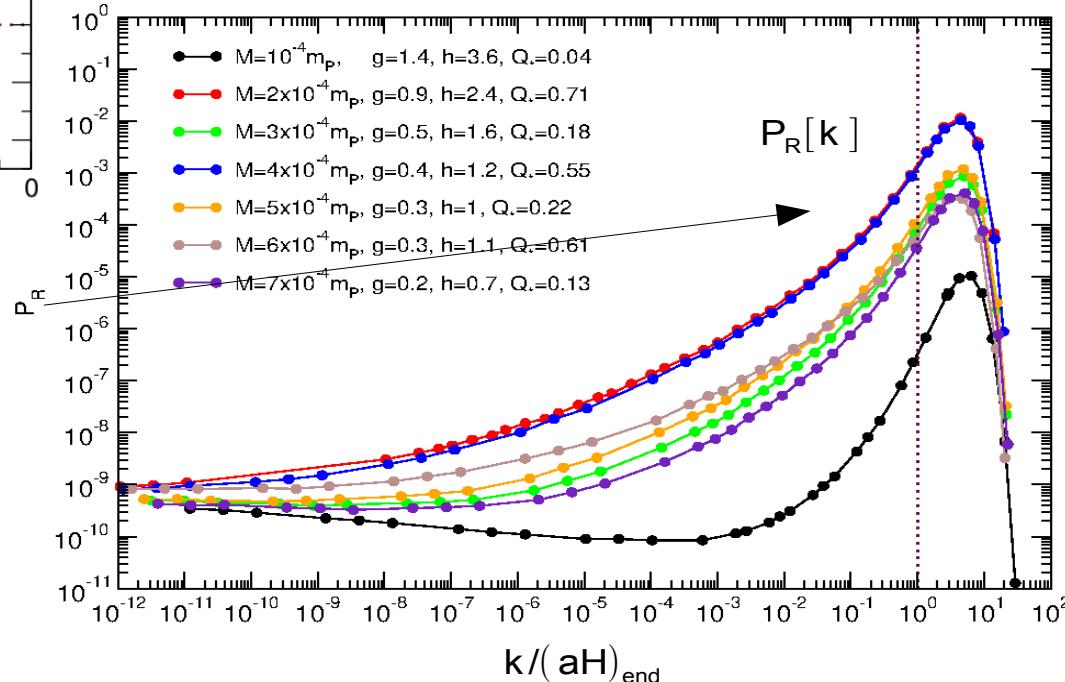
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$$\left\{ \begin{array}{ll} M^2/T & \text{"High-T"} \\ T^3/M^2 & \text{"Low-T"} \end{array} \right.$$

**PBHs?**

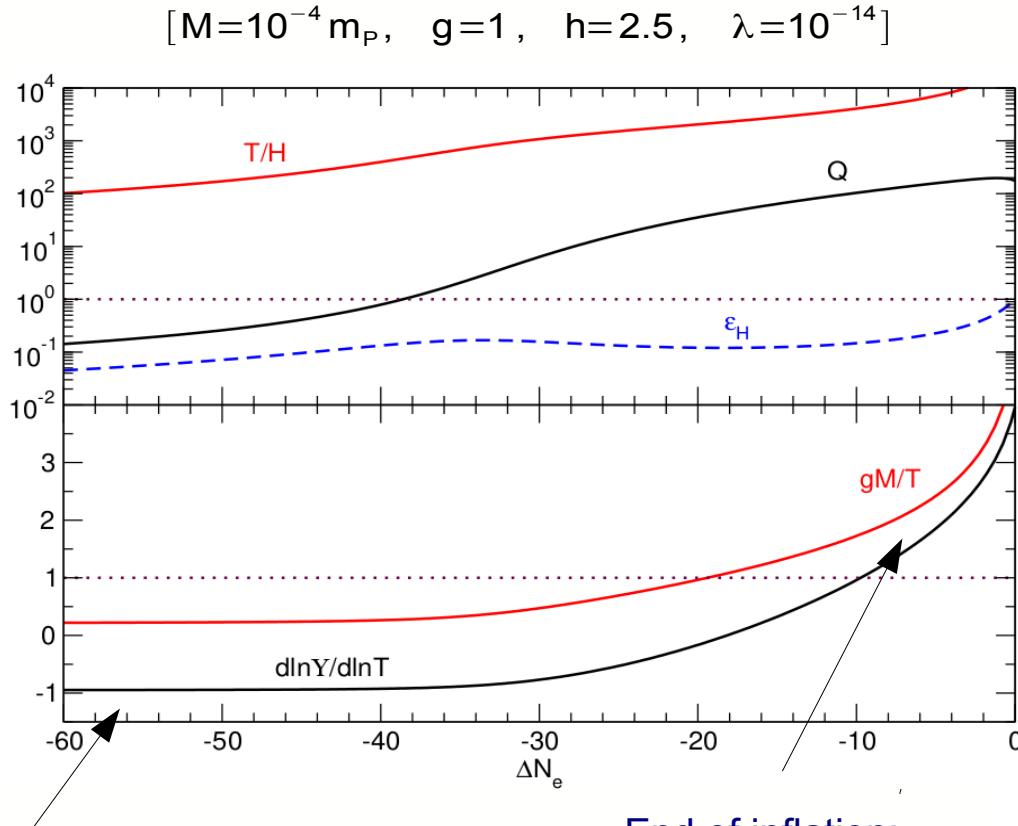
[R. Arya 1910.05238]

Light, evaporating PBHs:  $M_{\text{PBH}} \sim O(10^3-10^6) \text{ g}$



# Scalar Warm Little Inflaton Model + quartic chaotic

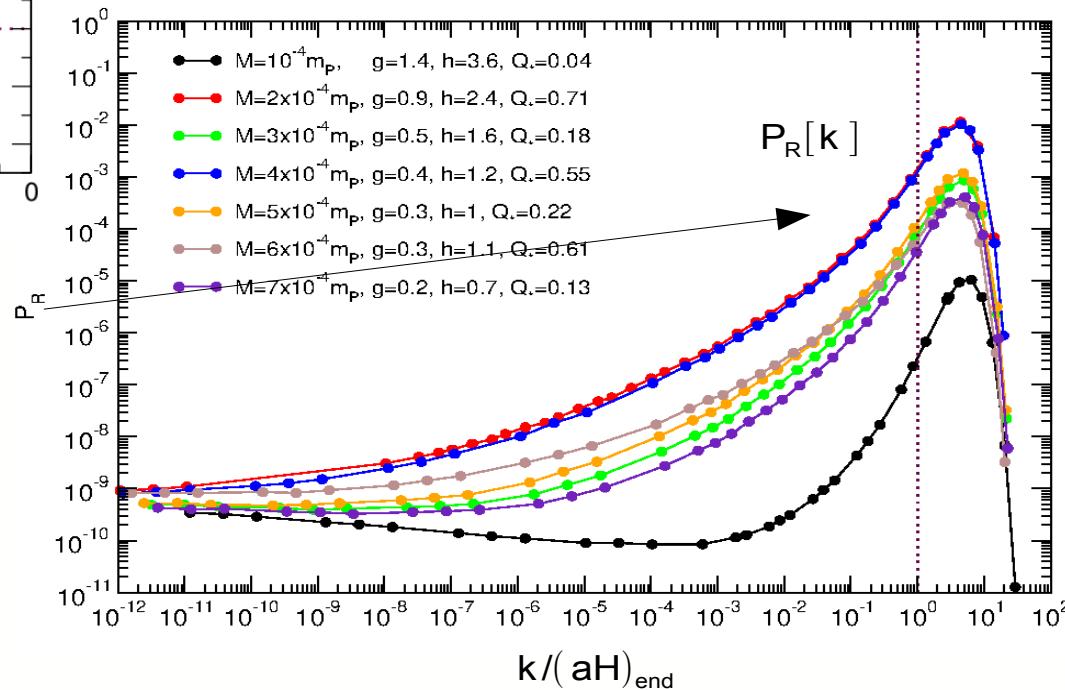
$$V = \lambda \phi^4 / 4$$



$$Y \simeq \frac{4g^2 g^2 M^2}{h^2 T} F[m_\chi/T]$$

$$\left\{ \begin{array}{ll} M^2/T & \text{"High-T"} \\ T^7/M^6 & \text{"Low-T"} \end{array} \right.$$

2<sup>nd</sup> order source  
of primordial tensors?



## Induced 2<sup>nd</sup> order GW

Although at linear order scalar, vector and tensor perturbations decouple,  
large scalar fluctuations source tensors at second order

$$\ddot{h}_k + 3H\dot{h}_k + \frac{k^2}{a^2}h_k = S_k[\Phi_k]$$

Primordial spectrum

Gravitational potential:  $\Phi_k = T[k\tau]\phi_k, \quad \langle\phi_k\phi_q\rangle = \delta(\vec{k}+\vec{q}) \frac{2\pi^2}{k^3} \left(\frac{3+3w}{5+3w}\right)^2 P_\zeta(k)$  [w=1/3 radiation]

GW spectral density:

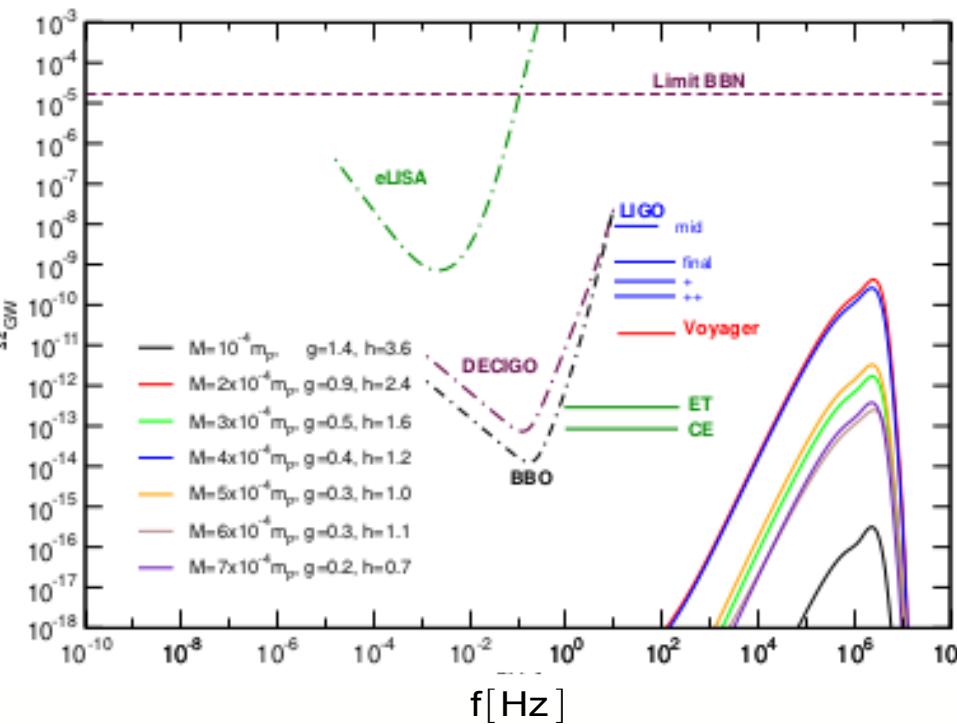
$$\Omega_{GW,0}(k) \simeq 0.4 \Omega_{r,0} \times \frac{1}{24} \left(\frac{k}{aH}\right)^2 P_h(k, \tau_c)$$

[Kohri & Terada 1804.08577]

Today:  $f \sim 10^5 - 10^6$  Hz

$$\Omega_{GW,0}^{\max} \simeq 10^{-9}$$

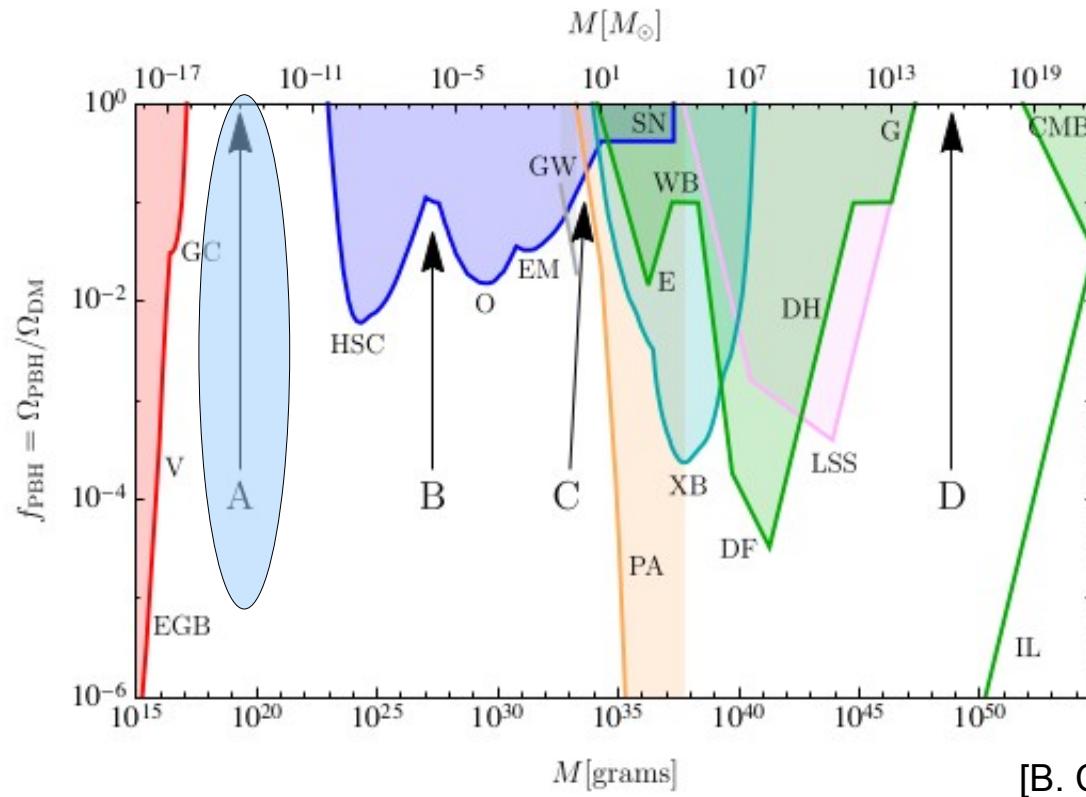
$$[\Omega_{GW,0} \propto f^3 \ln^2 f / f_P]$$



Consistent with all constraints (CMB, PBHs)

[BG & Subías 2105.08045]

# PBHs as DM



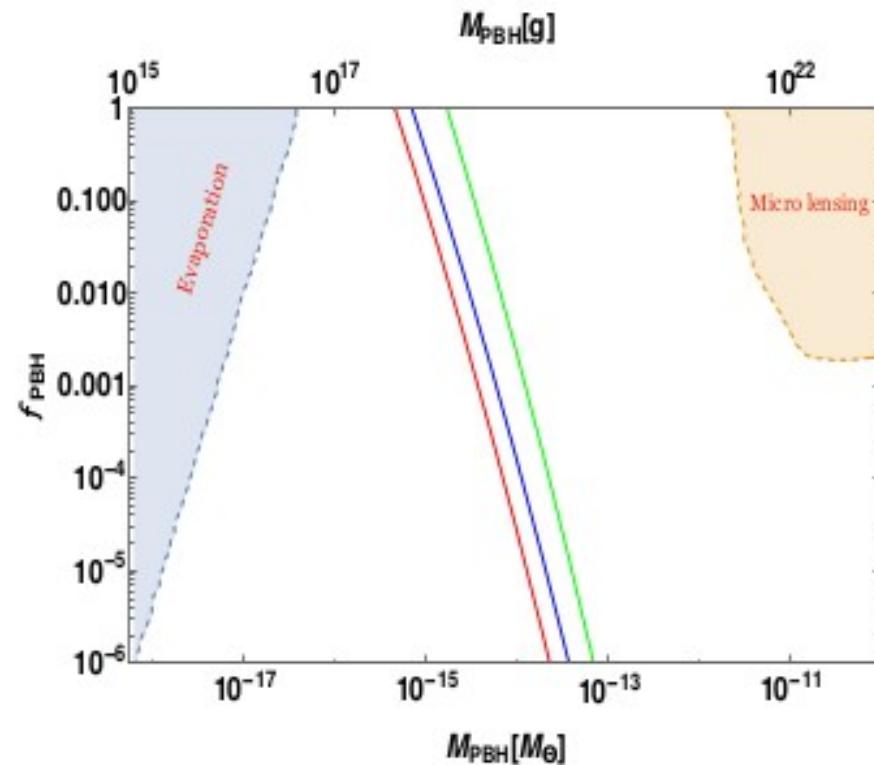
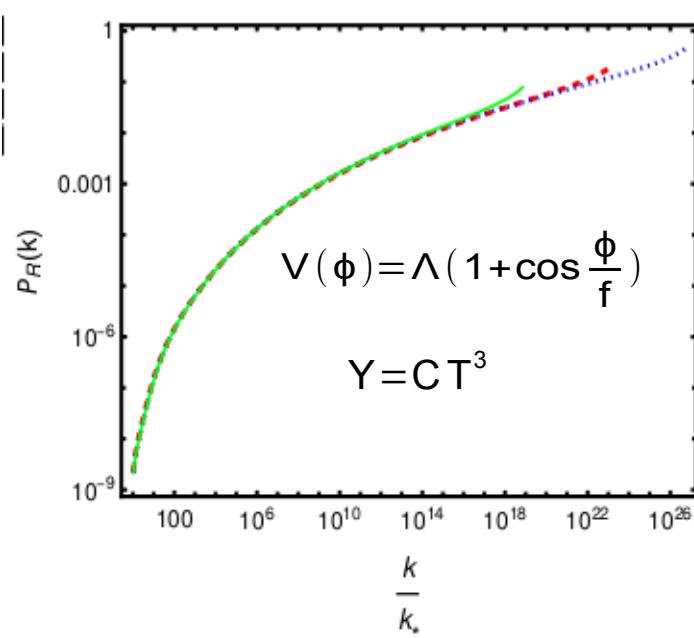
$$M_{\text{PBH}}(k) \sim \frac{4\pi m_p^2}{H_M} \sim 10^{-16} - 10^{-11} M_{\text{sun}} \rightarrow f_{\text{PBH}} \sim \mathcal{O}(1)$$

- We need  $P_R(k) \sim \mathcal{O}(0.01 - 0.1)$  20 - 30 efolds before the end

# PBHs as DM

➤ Natural WI + T<sup>3</sup> dissipation: the spectrum is amplified ~ 20 efolds before the end

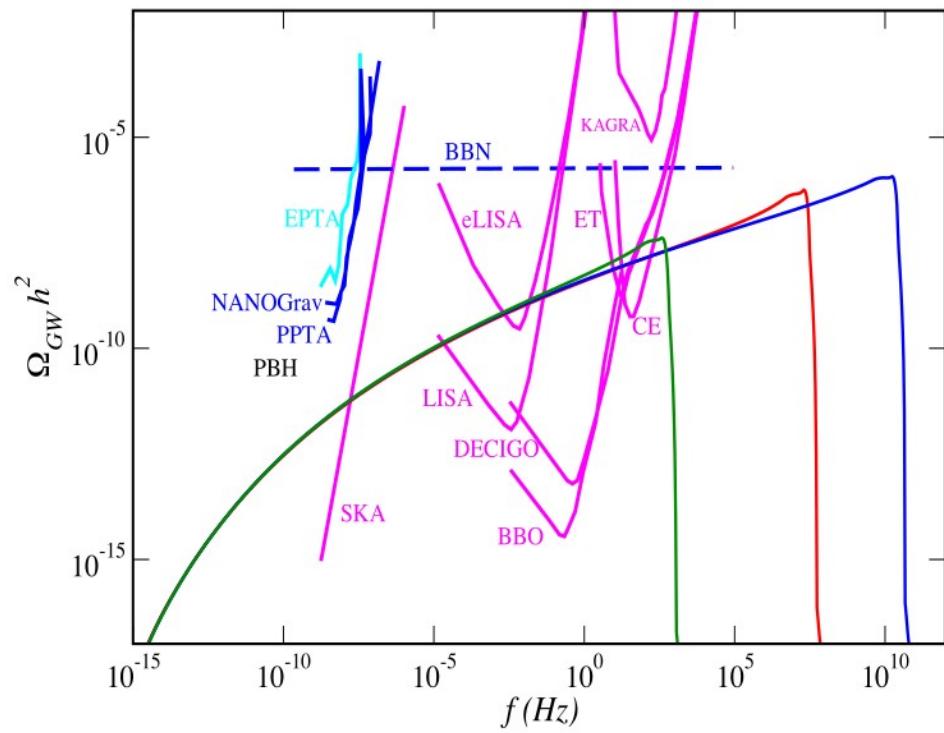
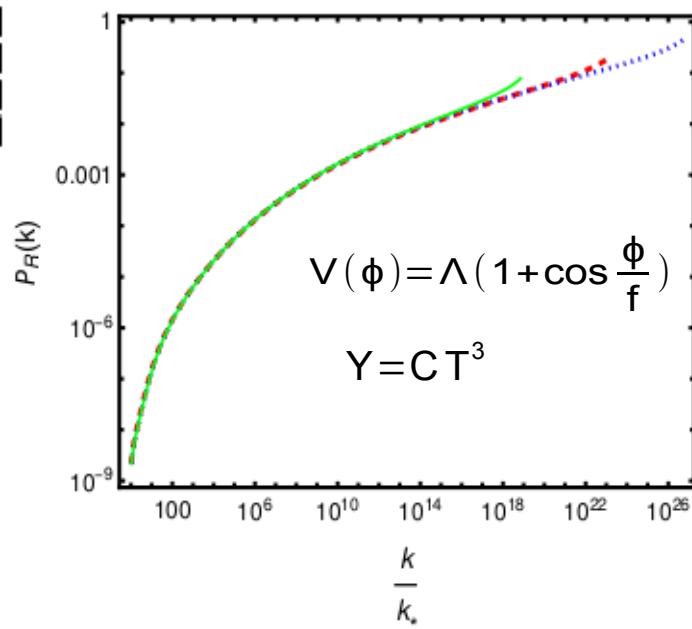
[Correa, Gangopadhyay, Jaman, Mathews, 2207.10394]



$$M_{\text{PBH}}(k) \sim \frac{4\pi m_p^2}{H_M} \sim 10^{-16} M_{\text{sun}} \rightarrow f_{\text{PBH}} \sim O(1)$$

# PBHs as DM

- Natural WI + T<sup>3</sup> dissipation: the spectrum is amplified ~ 20 efolds before the end  
[Correa, Gangopadhyay, Jaman, Mathews, 2306.09641]



# PBHs as DM

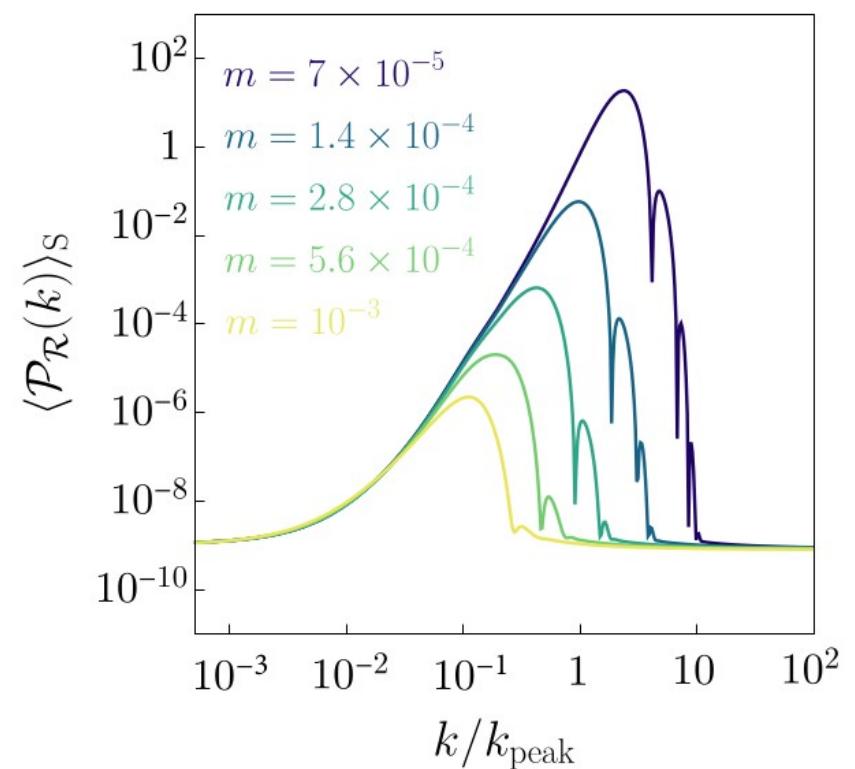
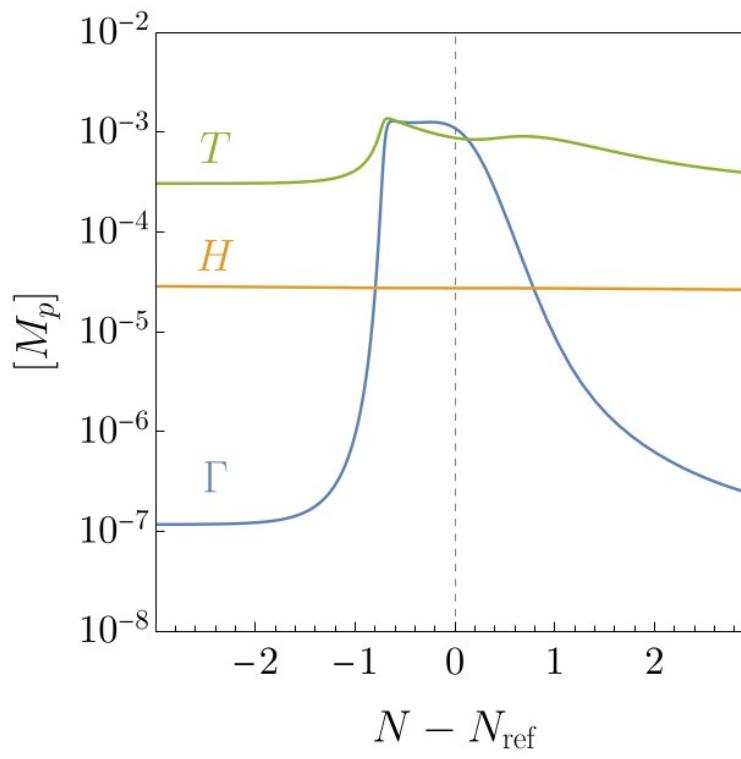
- Transient dissipation : peak in  $Q$  → peak in the spectrum  $\sim 20\text{-}30$  efolds before the end

[Ballesteros, García, Pérez-Rodríguez, Pierre, Rey 2208.14978]

$$Y(\phi, T) = \frac{T^3}{m^2 + M^2 \tanh^2((\phi - \phi_c)/\Lambda)}$$

[Non-minimal kinetic term]

$$V = \lambda \varphi^4 / 4$$



$$[M = 10^{-2} m_P, \Lambda = 0.1 m_P, \lambda = 2.5 \times 10^{-15}]$$

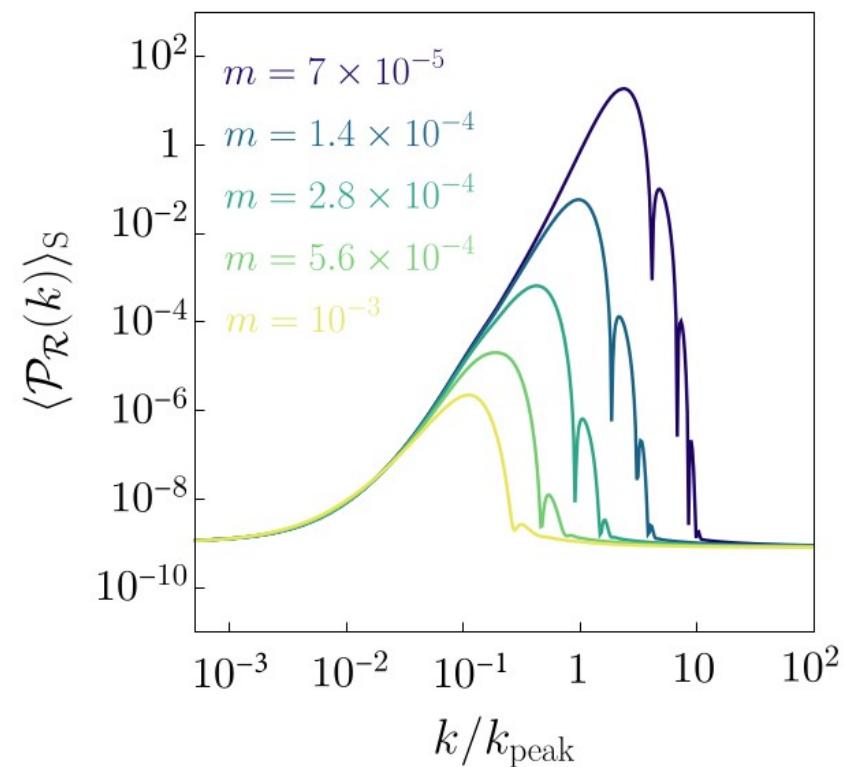
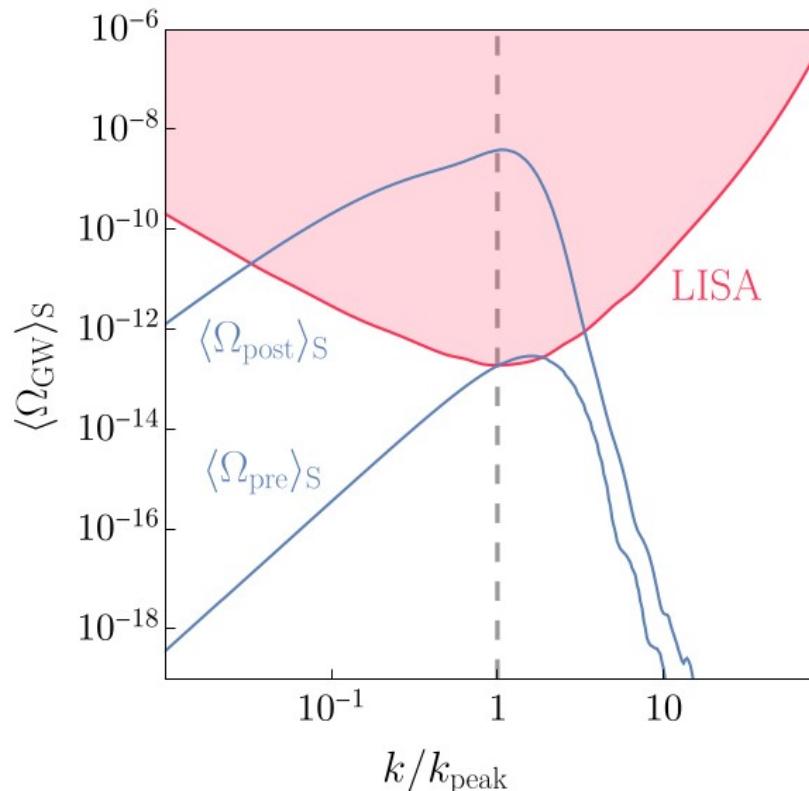
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# Summary

- Dissipative effects due to decaying fields can be relevant during inflation, and modify the inflationary predictions

$\lambda\phi^4$  compatible with data (WDR)

Other models (hybrid, running inflaton....) compatible with data in the SDR ( $Y \gg H$ )

- Thermal corrections to inflation potential under control with symmetries:

LWI: Inflaton a PNGB of a broken U(1) symmetry + pair of fermions/scalars + exchange sym.

MWI: Inflaton a PNGB of a broken U(1) symmetry + gauge field production

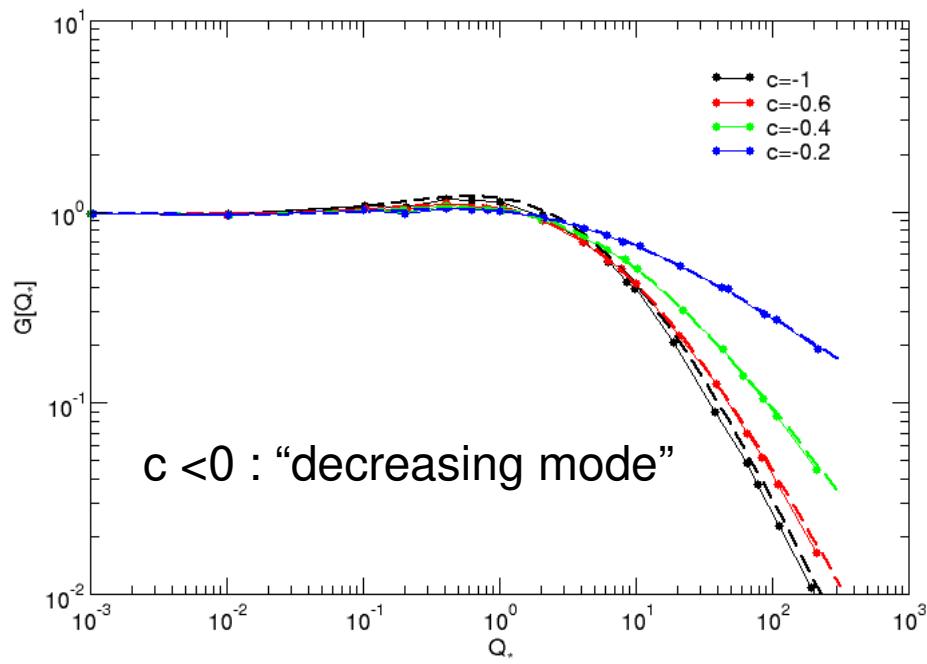
- Warm inflation: amplification of the spectrum 

PBHs, 2<sup>nd</sup> order GW, ....

Thank you!

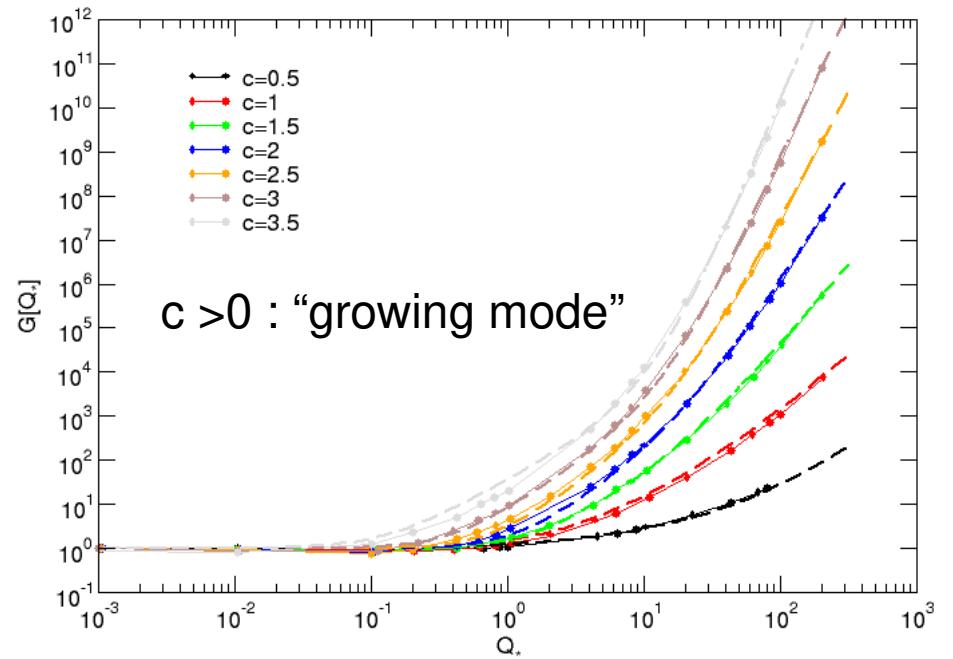
# Backup slides

# Primordial spectrum: growing/decreasing mode $Y \propto T^c$



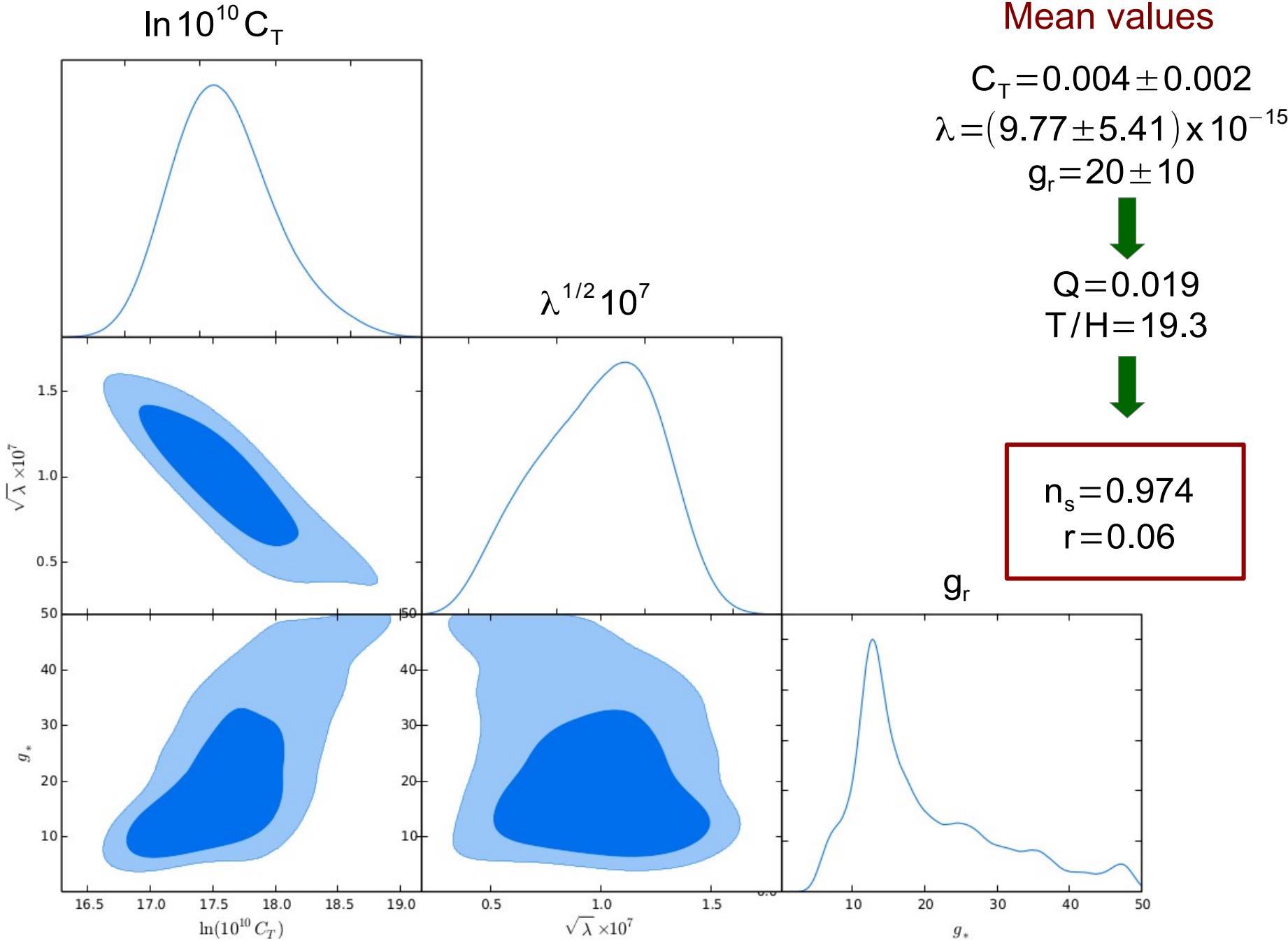
$c < 0$  : "decreasing mode"

$$G[Q] = \frac{P_R[\text{num.}]}{((P_R)_{\text{vac}} + (P_R)_{\text{diss}})}$$



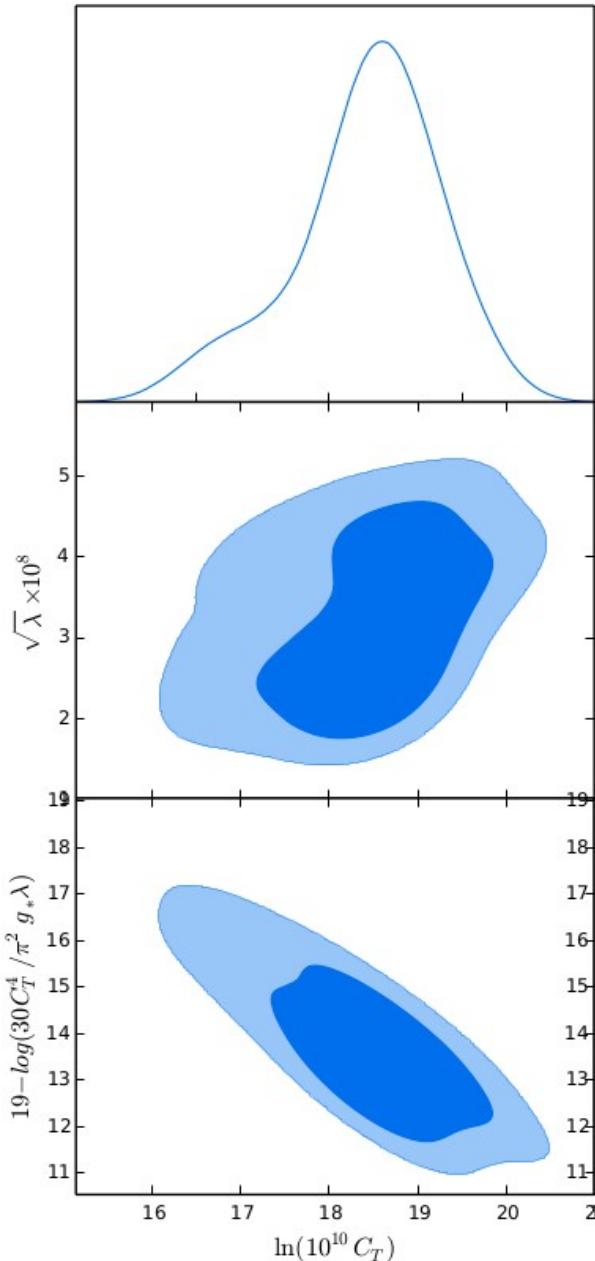
$c > 0$  : "growing mode"

# Little warm inflation & CMB data: non thermal inflaton

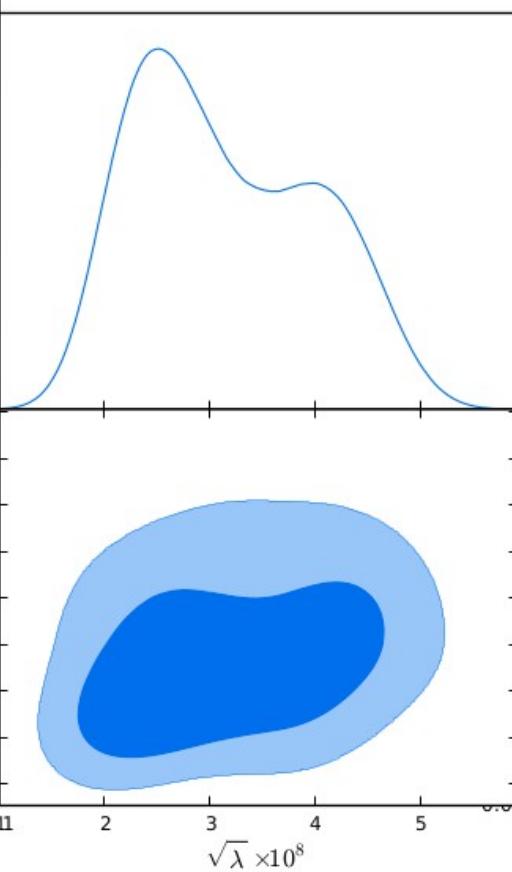


# Little warm inflation & CMB data: thermal inflaton

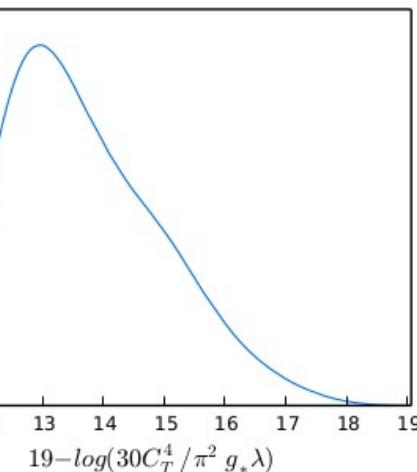
$\ln 10^{10} C_T$



$\lambda^{1/2} 10^7$



$g_r$



Mean values

$$C_T = 0.010 \pm 0.008$$

$$\lambda = (9.74 \pm 6.78) \times 10^{-16}$$

$$g_r = 140 \pm 488$$

$$Q = 0.14$$

$$T/H = 40.7$$

$$n_s = 0.965$$

$$r = 0.006$$

# Warm inflation & Non-gaussianity : T dependent diss. coefficient

- **Bispectrum:**  $B_R(k_1, k_2, k_3) = \sum_{\text{cyc}} \langle R_1(k_1) R_1(k_2) R_2(k_3) \rangle = A_B(k) \bar{B}(k_1, k_2, k_3)$  shape
- $f_{NL} = \frac{18}{5} \frac{A_B(k)}{P_R(k)^2}$  Non-linear parameter

