

Heavy quarkonia in a collisional anisotropic quark-gluon plasma

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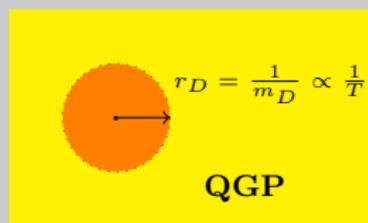
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Hard probes in non-equilibrium QCD matter

Quarkonia in QGP



- In 1986, Matsui and Satz proposed that the quarkonia is suppressed if QGP is formed because the binding potential becomes short-range due to color Debye screening.
- Debye screening makes the potential short-range, and when the screening radius r_D becomes less than the quarkonia radius, it dissociates.
- As the temperature increases, the Debye radius decreases and which leads to sequential suppression of quarkonia, one of the most striking signatures of the QGP.

T. Matsui and H. Satz, Phys. Lett. B 178, 416-422 (1986)

- The in-medium heavy-quark potential is obtained from the Fourier transform of the sum of the static dressed retarded, advanced, and Feynman propagators, i.e.

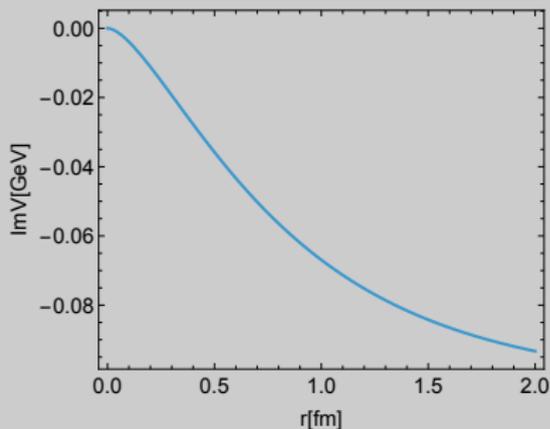
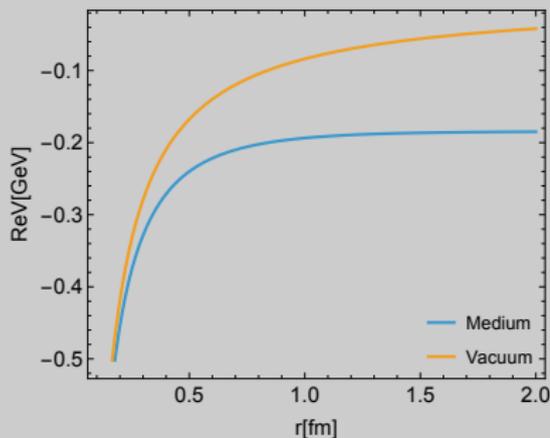
$$V(\mathbf{r}) = -\frac{g^2 C_F}{2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \left[\tilde{\mathcal{D}}_R^{00} + \tilde{\mathcal{D}}_A^{00} + \tilde{\mathcal{D}}_F^{00} \right]_{\omega \rightarrow 0}$$

Here, $\tilde{\mathcal{D}}$ is the one-loop HTL propagator.

- The first two terms involving the retarded and advanced propagators are purely real. The imaginary part of the potential comes solely from the Fourier transform of the Feynman propagator.
- Within HTL resummation [Strickland, Dumitru et. al.]

$$\text{Re} V_{\text{HTL}}(r, T) = -\alpha_s C_F m_D \left(1 + \frac{e^{-m_D r}}{m_D r} \right),$$

$$\text{Im} V_{\text{HTL}}(r, T) = -2\alpha_s C_F T m_D^2 \int \frac{p dp}{(p^2 + m_D^2)^2} \left(1 - \frac{\sin pr}{pr} \right).$$



- Pure Coulombic contribution, no string term is considered in this plot.

Potential with string term [Isotropy system]

- With the vacuum potential $V_{\text{vac}} = -\frac{\alpha_s C_F}{r} + \sigma r$,

$$\text{Re}V = -\alpha_s C_F m_D \left(1 + \frac{e^{-m_D r}}{m_D r} \right) + \frac{2\sigma}{m_D} \left(1 + \frac{e^{-m_D r} - 1}{m_D r} \right)$$
$$\text{Im}V = -2Tm_D^2 \int \frac{p dp}{(p^2 + m_D^2)^2} \left(\alpha_s C_F + \frac{2\sigma}{p^2} \right) \left(1 - \frac{\sin pr}{pr} \right).$$

[Ref. BK Patra, L. Thakur et. al]

Anisotropic plasma

- The medium produced in heavy-ion collisions possesses a significant degree of momentum-space anisotropy.
- The momentum-space anisotropy can be introduced via a specific form for the distribution function for the light quarks and gluons $f_{\text{aniso}}(\mathbf{p}) \equiv f_{\text{iso}} \left(\frac{1}{\lambda} \sqrt{\mathbf{p}^2 + \xi(\mathbf{p} \cdot \mathbf{n})^2} \right)$
- The self-energies and dressed propagators can be expanded in terms of the tensor basis as

$$\begin{aligned}\Pi_{R,A,F}^{\mu\nu} &= \alpha_{R,A,F} A^{\mu\nu} + \beta_{R,A,F} B^{\mu\nu} + \gamma_{R,A,F} C^{\mu\nu} + \delta_{R,A,F} D^{\mu\nu} \\ \tilde{D}_{R,A,F}^{\mu\nu} &= \alpha'_{R,A,F} A^{\mu\nu} + \beta'_{R,A,F} B^{\mu\nu} + \gamma'_{R,A,F} C^{\mu\nu} + \delta'_{R,A,F} D^{\mu\nu}\end{aligned}$$

- Tensor structure:

$$A^{\mu\nu} = -\eta^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} + \frac{\tilde{m}^\mu \tilde{m}^\nu}{\tilde{m}^2}$$

$$B^{\mu\nu} = -\frac{p^2}{(m \cdot p)^2} \frac{\tilde{m}^\mu \tilde{m}^\nu}{\tilde{m}^2}$$

$$C^{\mu\nu} = \frac{\tilde{m}^2 p^2}{\tilde{m}^2 p^2 + (n \cdot p)^2} \left[\tilde{n}^\mu \tilde{n}^\nu - \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} (\tilde{m}^\mu \tilde{n}^\nu + \tilde{m}^\nu \tilde{n}^\mu) + \frac{(\tilde{m} \cdot \tilde{n})^2}{\tilde{m}^4} \tilde{m}^\mu \tilde{m}^\nu \right]$$

$$D^{\mu\nu} = \frac{p^2}{m \cdot p} \left[2 \frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \tilde{m}^\mu \tilde{m}^\nu - (\tilde{n}^\mu \tilde{m}^\nu + \tilde{m}^\mu \tilde{n}^\nu) \right].$$

- Here, m^μ is the heat-bath four-velocity, which in the local rest frame is given by $m^\mu = (1, 0, 0, 0)$, and

$$\tilde{m}^\mu = m^\mu - \frac{m \cdot p}{p^2} p^\mu,$$

is the component of m^μ orthogonal to p^μ .

- The direction of anisotropic in momentum space is $n^\mu = (0, \mathbf{n})$,

Within HTL approximation, the R/A/F self-energy can be obtained calculating the Feynman diagram as

- Retarded/Advance propagators:

$$\Pi_{R/A}^{\mu\nu}(p, \xi) = m_D^2 \int \frac{d\Omega}{4\pi} v^\mu \frac{v^l + \xi(\mathbf{v} \cdot \mathbf{n})n^l}{(1 + \xi(\mathbf{v} \cdot \mathbf{n})^2)^2} \left[-\eta^{\nu l} + \frac{v^\nu p^l}{p \cdot v \pm i\epsilon} \right]$$

- Symmetry propagator:

$$\Pi_F^{ij}(p, \xi) = -\frac{i\lambda m_D^2}{|\mathbf{p}|} \int d\Omega \frac{v^i v^j}{(1 + \xi(\mathbf{v} \cdot \mathbf{n})^2)^{3/2}} \delta\left(\mathbf{v} \cdot \hat{\mathbf{p}} - \frac{\omega}{|\mathbf{p}|}\right).$$

- Note that Fluctuation-Dissipation Theorem (FDT) is strictly broken for an anisotropic plasma as $\frac{4i\lambda}{\omega} \text{Im}\Pi_R^{ij} \neq \Pi_F^{ij}$.

Quarkonium potential

- The real part of the potential

$$\begin{aligned}\text{Re}V(\mathbf{r}, \xi) &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \tilde{\mathcal{D}}_R^{00}(\omega = 0, \mathbf{p}, \xi) \\ &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \frac{\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathbf{p}^2 + m_\beta^2) - m_\delta^4}\end{aligned}$$

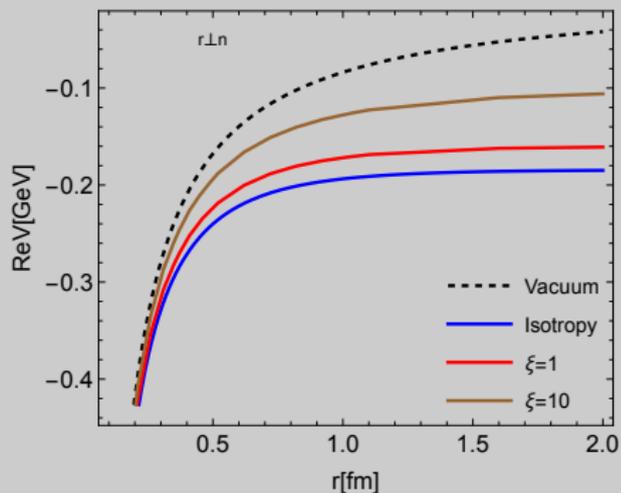
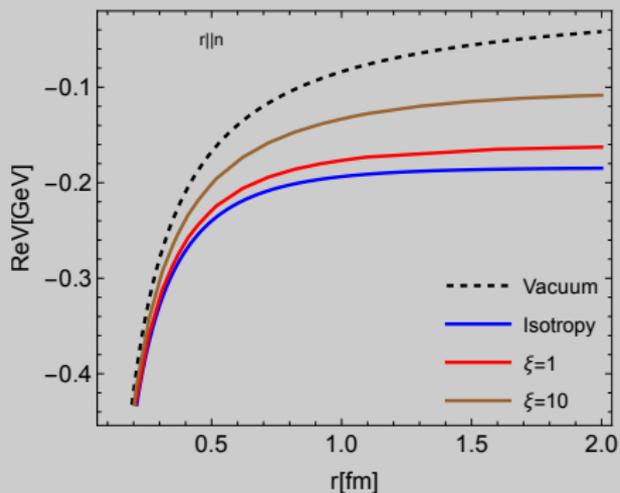
with

$$\alpha_{R/A} \approx m_\alpha^2 \pm i\omega m_{\alpha,1} + \mathcal{O}(\omega^2)$$

$$\beta_{R/A} \approx -\frac{\omega^2}{p^2} [m_\beta^2 \pm i\omega m_{\beta,1}] + \mathcal{O}(\omega^4)$$

$$\gamma_{R/A} \approx m_\gamma^2 \pm i\omega m_{\gamma,1} + \mathcal{O}(\omega^2)$$

$$\delta_{R/A} \approx -\frac{\hat{\omega}}{\tilde{n}} [\pm i m_\delta^2 + \omega m_{\delta,1}] + \mathcal{O}(\omega^3)$$



- $\text{Re}V$ is less screened when $\mathbf{r} \perp \mathbf{n}$ [though the difference is not visible]

- The imaginary part of the potential

$$\text{Im}V(\mathbf{r}, \xi) \equiv -\frac{g^2 C_F}{2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \tilde{\mathcal{D}}_F^{00} \Big|_{\omega \rightarrow 0}.$$

with

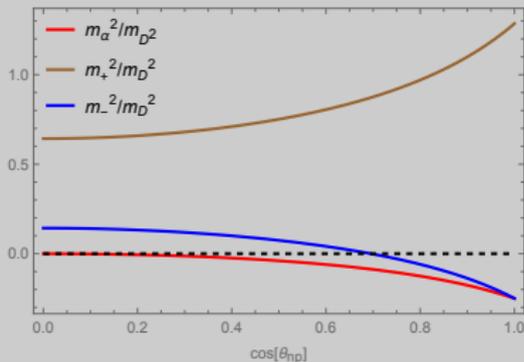
$$\begin{aligned} \tilde{\mathcal{D}}_F^{00} \Big|_{\omega \rightarrow 0} &= \frac{4i\lambda m_D^2}{\varsigma p \left[(\mathbf{p}^2 + m_\beta^2) (\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2) - m_\delta^4 \right]^2} \\ &\times \left[\frac{m_\delta^4 - \varsigma (\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)^2}{1 + \varsigma} E(-\varsigma) - m_\delta^4 K(-\varsigma) \right]. \end{aligned}$$

Here, $\varsigma = \xi p_x^2 / p^2$; K and E are complete elliptic integrals of first and second kind.

- The denominator can be expressed as

$$(\mathbf{p}^2 + m_\beta^2) (\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2) - m_\delta^4 = (\mathbf{p}^2 + m_+^2) (\mathbf{p}^2 + m_-^2).$$

- One can show that m_+^2 is positive for all ξ and angles of propagation; however, m_-^2 can be negative for some propagation angles.



- Because of the second order pole in the previous equation, the imaginary part of the potential contains a pinch singularity which is related to the presence of the (chromo-)Weibel instability in momentum-space anisotropic plasmas.
- The existence of the chromo-Weibel instability makes it difficult to understand the late time dynamics of momentum-anisotropic plasmas.

Weibel instability

- Anisotropic momentum distributions cause kinetic instabilities. For the particular distribution with $\hat{\mathbf{n}} \parallel \hat{\mathbf{p}}$ and $\xi > 0$ there exists a magnetic instability, the so called filamentation or Weibel instability.
- Its existence is due to a surplus of particles with momentum perpendicular (or close to perpendicular) to \mathbf{p} .
- These particles are trapped in the direction of \mathbf{p} by the background magnetic field and cause currents, which generate magnetic fields that add to the original one. Hence they contribute to instability, while all other particles have a stabilizing effect.
- In the isotropic case the stabilizing and destabilizing contributions cancel, such that no instability arises

Ref: B Schenke, M. Strickland, C. Greiner, and MH. Thoma, PRD 2006

Collisional plasma

- The instabilities are found to have a significant effect on the system's evolution, leading to a faster isotropization and equilibration.
- The equilibration due to instabilities only happens indirectly, because the instabilities driven isotropization is a mean-field reversible process, which does not produce entropy.
- However, parton momentum distributions are influenced by the isotropization, which speeds up the equilibration. Collisions, being responsible for the dissipation are needed to reach the equilibrium state of maximum entropy.
- In the perturbation theory, the collisions among the hard particles that enter at higher orders in g is neglected. With the coupling $\alpha_s \sim 0.2 - 0.4$, higher order terms will be important.
- Hence, collisions can not simply be neglected and their effect on the system's evolution, particularly on the collective modes.

Transport equations with collisions

- In the kinetic theory, the distribution of hard particles in the QCD plasma is described by the gauge covariant Wigner functions $W^i(\mathbf{p}, X)$
- Expanding the Wigner function around the color neutral background fields $W^i(\mathbf{p})$, namely, $W^i(\mathbf{p}, X) = W^i(\mathbf{p}) + \delta W^i(\mathbf{p}, X)$, the linearized kinetic equations read

$$V \cdot \partial_X \delta f_a^i(\mathbf{p}, X) + g \theta_i V_\mu F_a^{\mu\nu} \partial_\nu^{(P)} f^i(\mathbf{p}) = C_a^i(\mathbf{p}, X),$$

where $V = (1, \mathbf{v})$ with $\mathbf{v} \equiv \mathbf{p}/p$ and $\theta_q = \theta_g = 1$, $\theta_{\bar{q}} = -1$.

- Here, the weak-coupling limit is taken, and higher order terms of coupling g are neglected. $\Rightarrow F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathcal{O}(g)$.

- The distribution function $f^i(\mathbf{k})$ and its color fluctuations $\delta f_a^i(\mathbf{k}, X)$ are related via the Wigner function as

$$f^{q/\bar{q}}(\mathbf{k}) = \frac{1}{N_c} \text{Tr}[W^{q/\bar{q}}(\mathbf{k}, X)], \quad f^g(\mathbf{k}) = \frac{1}{N_c^2 - 1} \text{Tr}[W^g(\mathbf{k}, X)],$$

and

$$\delta f_a^{q/\bar{q}}(\mathbf{k}, X) = 2 \text{Tr}[t_a \delta W^{q/\bar{q}}(\mathbf{k}, X)], \quad \delta f_a^g(\mathbf{k}, X) = \frac{1}{N_c} \text{Tr}[T_a \delta W^g(\mathbf{k}, X)]$$

- The collision term with BGK kernel is

$$C_a^i(p, X) = -\nu \left[f_a^i(p, X) - \frac{N_a^i(X)}{N_{\text{eq}}^i} f_{\text{eq}}^i(|\mathbf{p}|) \right],$$

- Solving for $\delta f_a^i(p, X)$ and Fourier-transforming leads to the result for the linearized induced current by each particle species J_{ind}^μ as

$$\begin{aligned}
J_{\text{ind } a}^{\mu}(K) &= g \int_{\mathbf{k}} V^{\mu} \{2N_c \delta f_a^g(k, X) + N_f [\delta f_a^g(k, X) - \delta f_a^{\bar{g}}(k, X)]\} \\
&= g^2 \int_{\mathbf{p}} V^{\mu} \partial_{(p)}^{\beta} f(\mathbf{p}) \mathcal{M}_{\gamma\beta}(K, V) D^{-1}(K, \mathbf{v}, \nu) A_a^{\gamma} + 2N_c g \nu \mathcal{S}^g(K, \nu) \\
&+ g^2 (i\nu) \int \frac{d\Omega}{4\pi} V^{\mu} D^{-1}(K, \mathbf{v}, \nu) \int_{\mathbf{p}'} \partial_{(p')}^{\beta} f(\mathbf{p}') \mathcal{M}_{\gamma\beta}(K, V') D^{-1}(K, \mathbf{v}', \nu) \\
&+ 2N_c g^2 (i\nu^2) \int \frac{d\Omega}{4\pi} V^{\mu} D^{-1}(K, \mathbf{v}, \nu) \mathcal{S}^g(K, \nu) \mathcal{W}^{-1}(K, \nu)
\end{aligned}$$

with

$$\mathcal{M}_{\gamma\beta}(K, V) = g_{\gamma\beta}(\omega - \mathbf{k} \cdot \mathbf{v}) - V_{\gamma} K_{\beta},$$

$$D(K, \mathbf{v}, \nu) = \omega + i\nu - \mathbf{k} \cdot \mathbf{v}$$

$$\mathcal{W} = 1 - \frac{i\hat{\nu}}{2} \ln \frac{\omega + i\nu + p}{\omega + i\nu - p}.$$

Gluon Self-energy in a collisional anisotropic plasma

$$\begin{aligned}
 \Pi_{ab}^{\mu\nu}(K) &= \frac{\delta J_{\text{ind } a}^{\mu}(K)}{\delta A_{\nu}^b(K)} \\
 &= g^2 \delta_{ab} \int_{\mathbf{p}} V^{\mu} \partial_l^{(\mathbf{p})} f(\mathbf{p}) \frac{g^{l\nu}(\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v}) - \hat{K}^l V^{\nu}}{\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v} + i\hat{\nu}} \\
 &\quad + g^2 \delta_{ab} (i\hat{\nu}) \int \frac{d\Omega}{4\pi} \frac{V^{\mu}}{\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v} + i\hat{\nu}} \int_{\mathbf{p}'} \partial_l^{(\mathbf{p}')} f(\mathbf{p}') \\
 &\quad \times \frac{g^{l\nu}(\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v}') - \hat{K}^l V^{\nu}}{\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v}' + i\hat{\nu}} \mathcal{W}^{-1}(\hat{\omega}, \hat{\nu})
 \end{aligned}$$

Note that the gluon self-energy is not symmetric in Lorentz indices due to the appearance of the BGK-type collision term. The same is also found in a magnetized plasma due to the lack of time-reversal symmetry.

Tensor structure

- Gluon self-energy in a collisional anisotropic plasma can be decomposed as

$$\Pi_{R,A,F}^{\mu\nu} = \alpha_{R,A,F} A^{\mu\nu} + \beta_{R,A,F} B^{\mu\nu} + \gamma_{R,A,F} C^{\mu\nu} + \delta_{R,A,F} D^{\mu\nu} + \rho_{R,A,F} E^{\mu\nu}$$

- A, B, C are the same as of collisionless case, but

$$D^{\mu\nu} = \frac{p^2}{m \cdot p} \left[\frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \tilde{m}^\mu \tilde{m}^\nu - \tilde{n}^\mu \tilde{m}^\nu \right],$$
$$E^{\mu\nu} = \frac{p^2}{m \cdot p} \left[\frac{\tilde{m} \cdot \tilde{n}}{\tilde{m}^2} \tilde{m}^\mu \tilde{m}^\nu - \tilde{m}^\mu \tilde{n}^\nu \right].$$

Feynman self-energy

When the BGK collision kernel is active, the plasma fluctuations are governed by the Boltzmann-Langevin equation. The collisions do two things:

- 1 They grant the hard particles a finite lifetime, broadening the sharp energy-conservation delta function into a Lorentzian:

$$2\pi\delta(v \cdot K) \rightarrow \frac{2\nu}{(\omega - \mathbf{k} \cdot \mathbf{v})^2 + \nu^2} = \frac{2}{k} \frac{\hat{v}}{(\hat{\omega} - \hat{\mathbf{k}} \cdot \mathbf{v})^2 + \hat{v}^2}.$$

- 2 Because the density fluctuation δn is dressed by \mathcal{W}^{-1} in the transport equations, the current-current correlator $\langle j^i j^j \rangle$ that defines Π_F must be dressed by \mathcal{W}^{-1} on both the emission and absorption sides to maintain gauge invariance, yielding a $|\mathcal{W}|^{-2}$ correction.

$$\Pi_{F_1}^{ij} = -\frac{2i\lambda m_D^2}{p} \int \frac{d\Omega}{4\pi} \frac{v^i v^j}{(1 + \xi(\mathbf{v} \cdot \hat{\mathbf{n}})^2)^{3/2}} \frac{2\hat{\nu}}{(\hat{\omega} - \hat{\mathbf{p}} \cdot \mathbf{v})^2 + \hat{\nu}^2}$$

$$\left(\Pi_{F,2}^{ij}\right) = -\frac{2i\lambda m_D^2}{p} \int \frac{d\Omega'}{4\pi} \frac{v'^i (i\hat{\nu})}{\hat{\omega} - \hat{\mathbf{p}} \cdot \mathbf{v}' + i\hat{\nu}}$$

$$\times \int \frac{d\Omega}{4\pi} \frac{v^j}{(1 + \xi(\mathbf{v} \cdot \hat{\mathbf{n}})^2)^{3/2}} \frac{2\hat{\nu}}{(\hat{\omega} - \hat{\mathbf{p}} \cdot \mathbf{v})^2 + \hat{\nu}^2} \times |\mathcal{W}^{-1}(\hat{\omega}, \hat{\nu})|^2$$

With Feynman self-energy in hand, we can obtain the resummed Feynman gluon propagator as

$$\begin{aligned} \tilde{\mathcal{D}}_F^{00}(p) &= (1 + 2f_B(p)) \operatorname{sgn}(\omega) \left[\tilde{\mathcal{D}}_R^{00}(p) - \tilde{\mathcal{D}}_A^{00}(p) \right] \\ &\quad + \tilde{\mathcal{D}}_R^{00}(p) \left\{ \Pi_F(p) - (1 + 2f_B(p)) \operatorname{sgn}(\omega) \left[\Pi_R^{00}(p) - \Pi_A^{00}(p) \right] \right\} \tilde{\mathcal{D}}_A^{00}(p) \end{aligned}$$

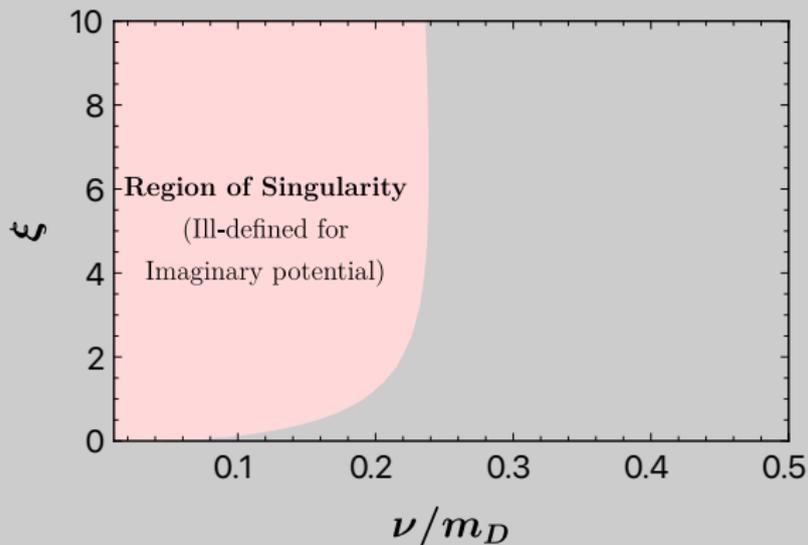
- $(1 + 2f_B(p))$ terms get canceled

-

$$\lim_{\omega \rightarrow 0} \left(\tilde{\mathcal{D}}_R \Pi_F \tilde{\mathcal{D}}_A \right)^{00} = \beta'_R \beta_F \beta'_A \frac{p^6}{\omega^6} + \delta'_R (\alpha_F + \gamma_F) \delta'_{A\perp} p^2 \frac{p^2}{\omega^2}$$

- With α, β and γ , one can obtain the imaginary part of the $Q\bar{Q}$ potential.

Region of stability[incomplete]



Summary

- The real part of the heavy-quarkonium potential can be obtained using the HTL resummed gluon propagator in real time formalism.
- Using FDT, the imaginary part of the gluon self-energy can be obtained and it gives the imaginary part of the $Q\bar{Q}$ potential
- In an anisotropic system, FDT is explicitly broken and Feynman self-energy is obtained diagrammatically. However, $\text{Im}V$ is ill-defined in an anisotropic plasma because of the pinch-singularity.
- By taking into account the effect of collisions between the plasma constituents with Bhatnagar-Gross-Krook kernel gives us a region in ξ - ν plane in which the $\text{Im}V$ is well-defined.