Cosmic Strings

an introduction

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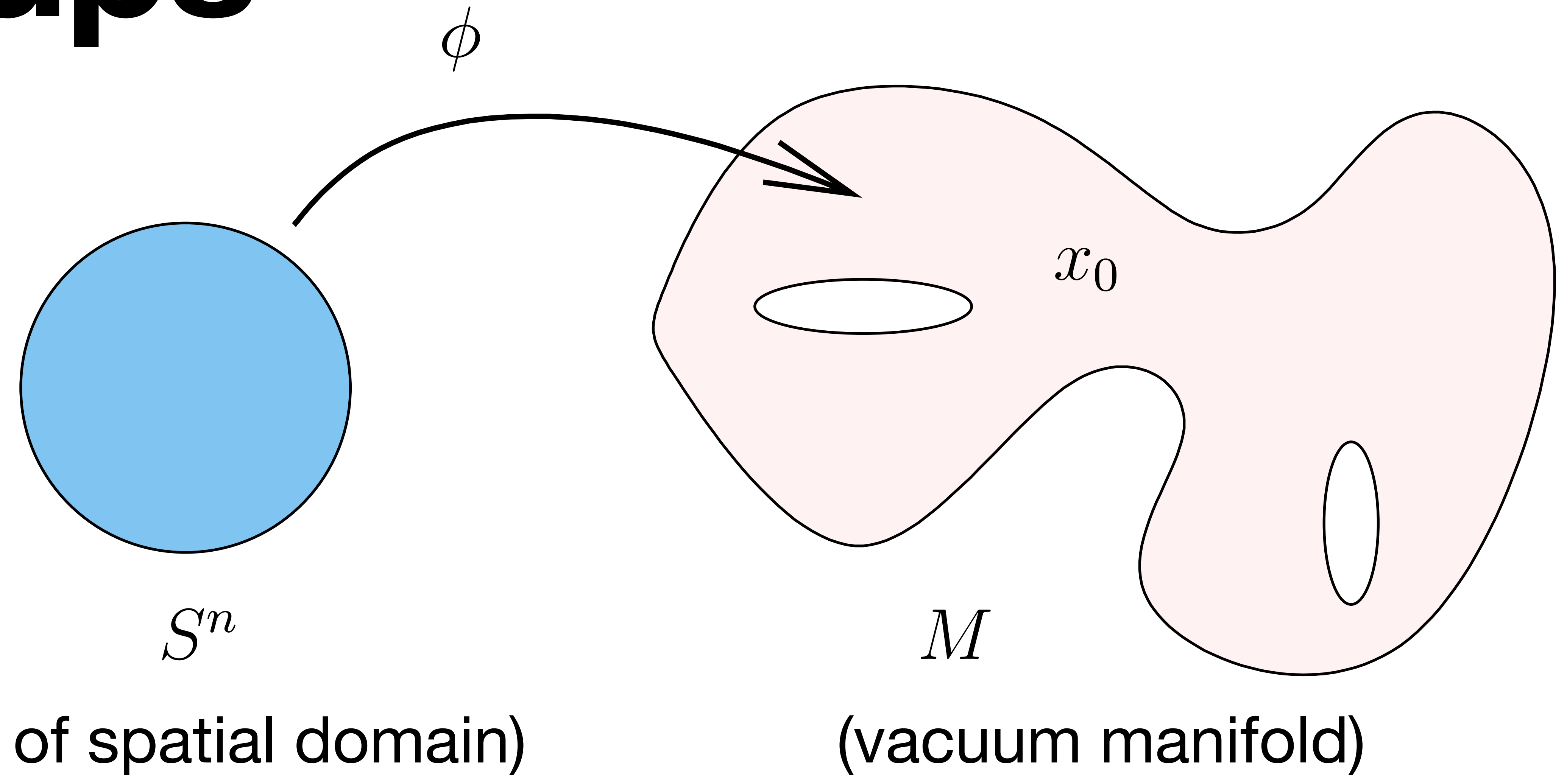
Reviews:

- Solitons and Particles, C. Rebbi & G. Soliani, 1985.
- Vortices and Monopoles, J. Preskill, Les Houches Lectures, 1986.
- Cosmic Strings, M. Hindmarsh & T.W.B. Kibble, hep-ph/9411342, 1994.
- Cosmic Strings and Other Topological Defects, A. Vilenkin & E.P.S. Shellard, 2000.

(Please see Reviews for further references.)

Homotopy groups



Topological defects exist if, when the spatial boundary is contracted to a point, its image in the vacuum manifold cannot be contracted to a point.

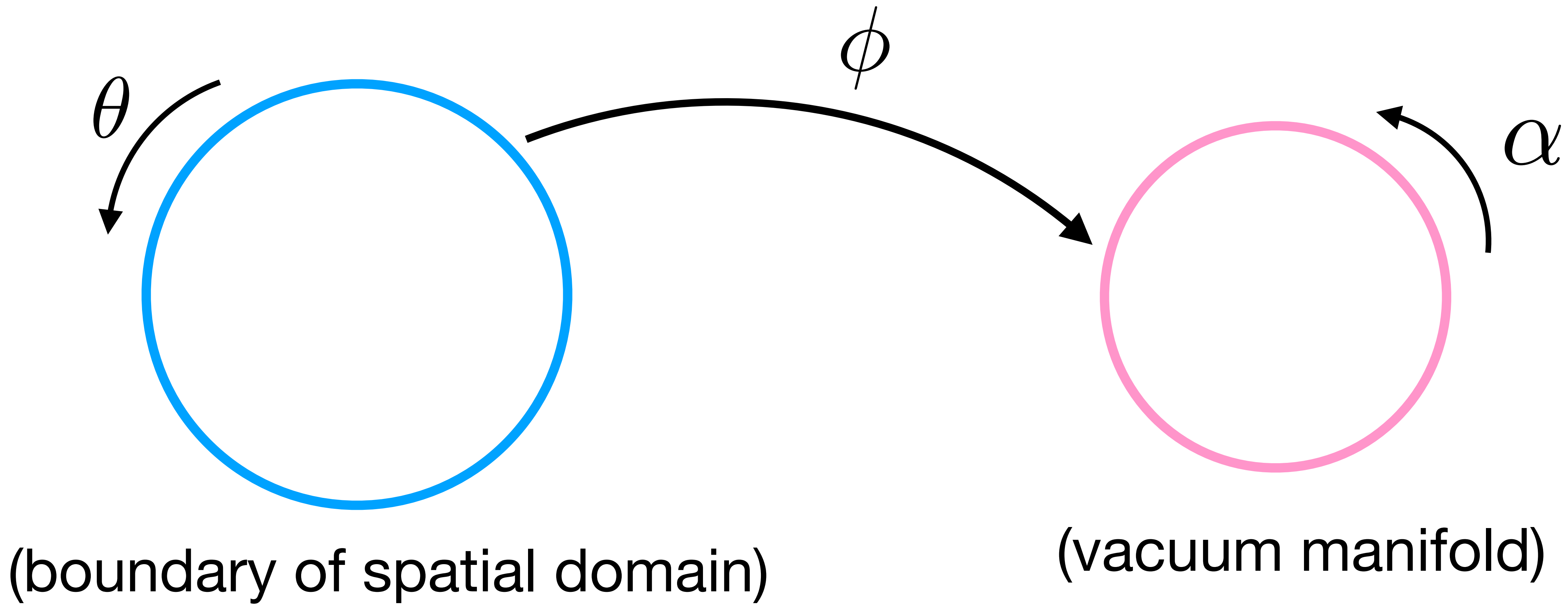
Mathematically: $\pi_n(M) \neq 1$

Symmetry breaking: $G \rightarrow H \qquad M \cong G/H \qquad \pi_n(G/H) \neq 1$

Strings case: n=1.

(Mathematicians have calculated homotopy groups for lots of manifolds.)

Simple example in 2 spatial dimensions:

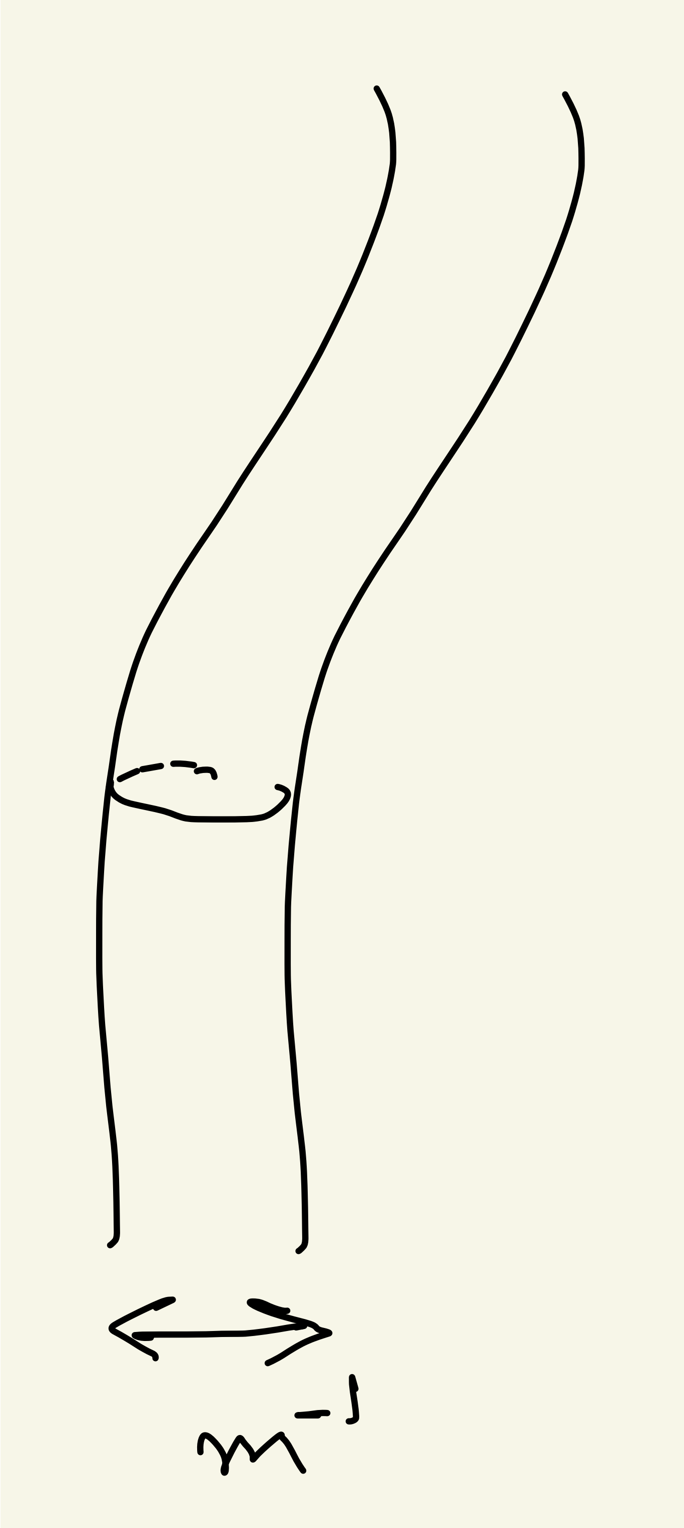


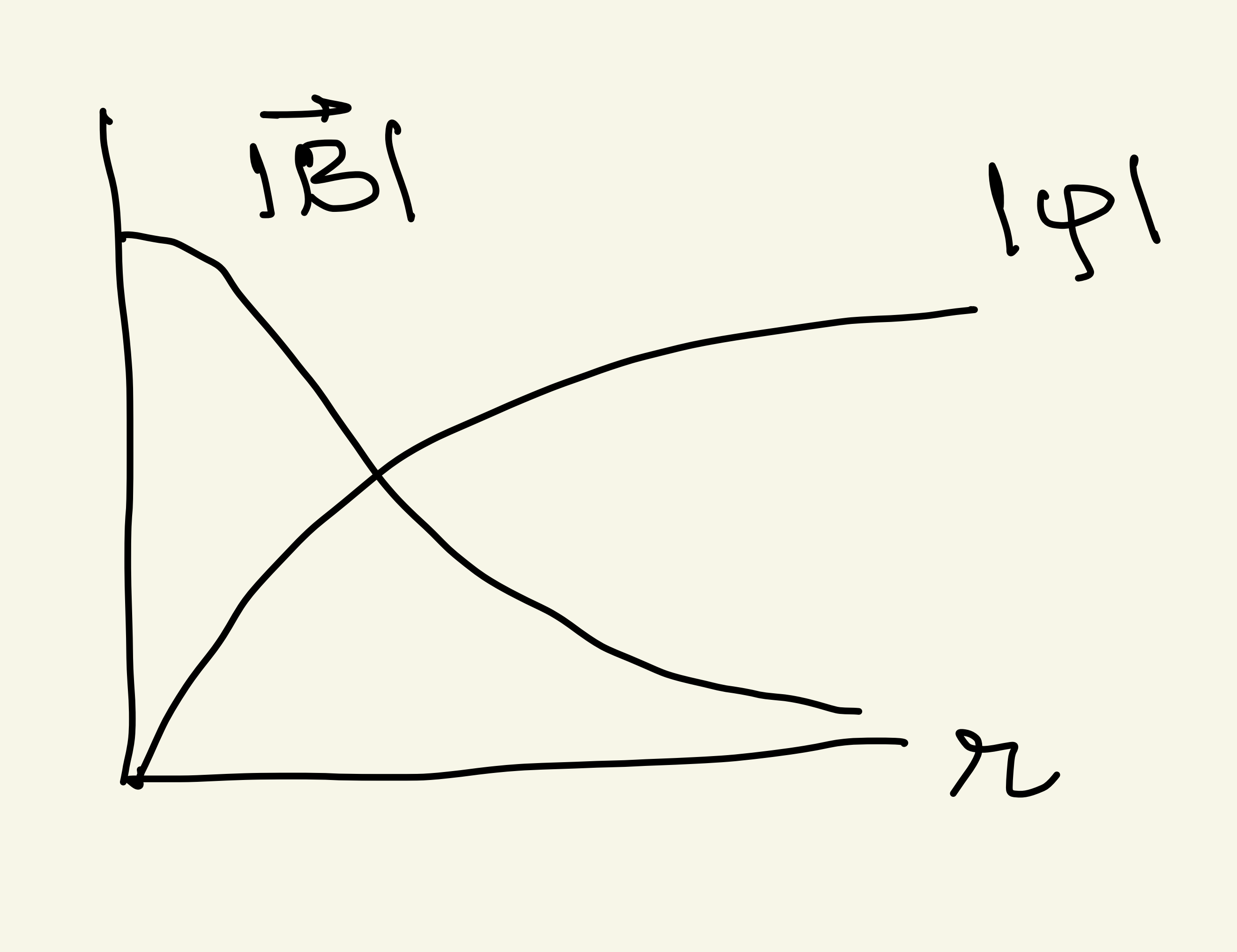
Example of topologically non-trivial mapping: $\alpha(\theta) = \theta$

String solutions

Explicit example: Abelian-Higgs model

$$S_{\rm AH} = \int d^4x \left[|D_\mu \phi|^2 - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 \right] \qquad D_\mu = \partial_\mu + ieA_\mu$$





Vacuum manifold is a circle.

$$\phi = |\phi| e^{i\theta}$$

Energy per unit length (=tension):

$$\mu \sim \eta^2$$

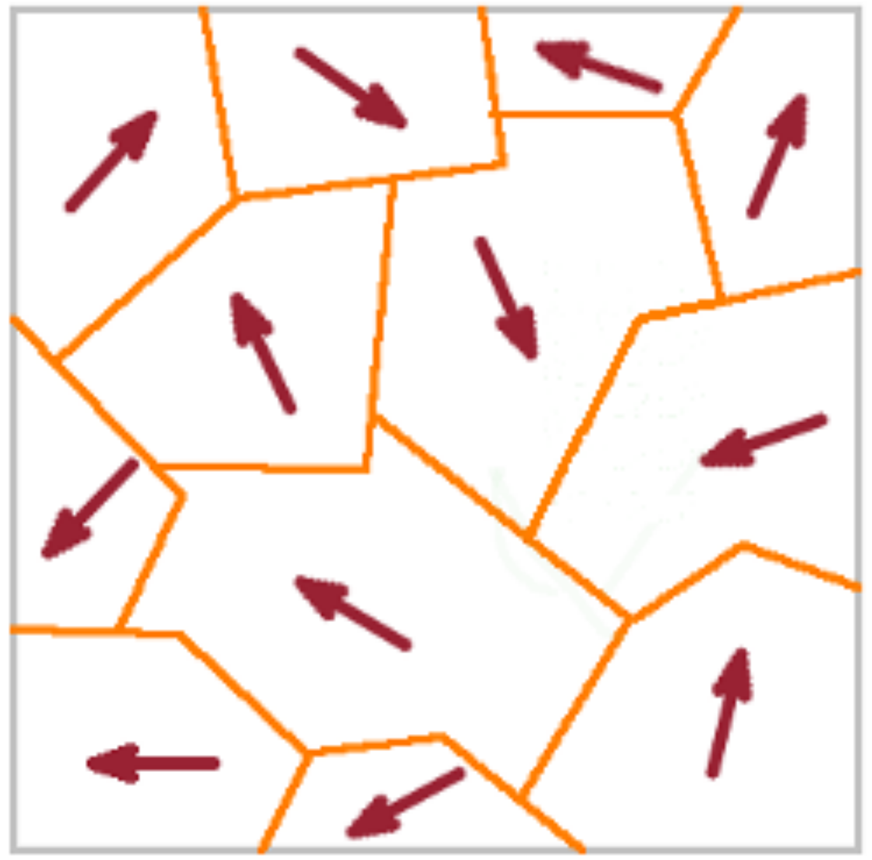
GUTs: $G\mu \sim 10^{-6} - 10^{-10}$

Cosmology

Symmetry breaking implies domain formation.

Kibble; Zurek; …

Mukhopadhyay, TV & Zahariade



Basis for topological defect formation in cosmology and condensed matter systems.

E.g. He-3.

If strings exist in the theory, they are bound to exist in cosmology unless special arrangements are made to eliminate them (*e.g.* inflation).

Formation

TV & Vilenkin, 1984.

Throw phases using a uniform distribution on the vacuum manifold on a lattice and construct string network.

Loops (closed strings) with spectrum:

$$dn \sim \frac{dR}{R^4}$$

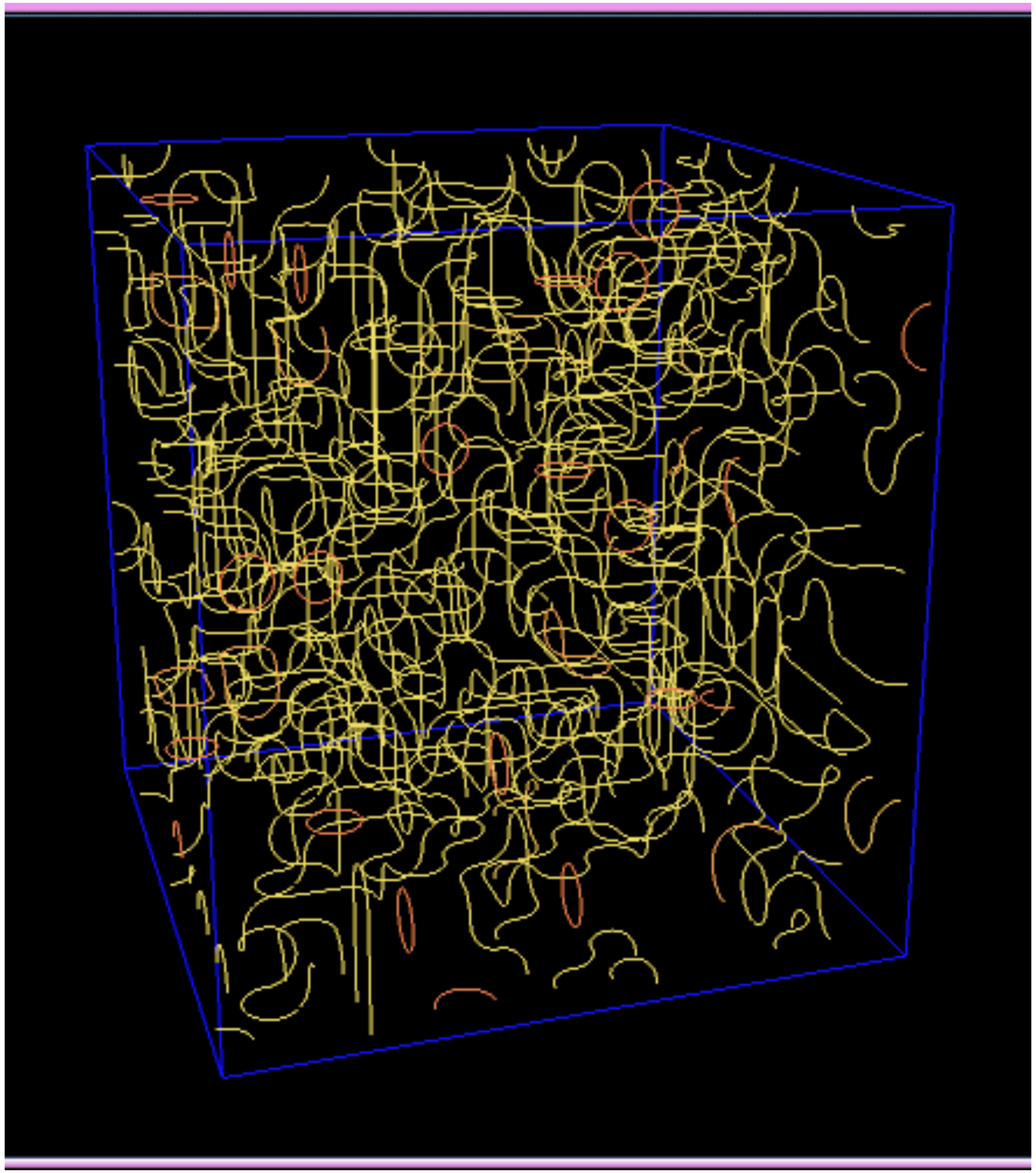
Scale invariant spectrum

$$l \sim R^2$$

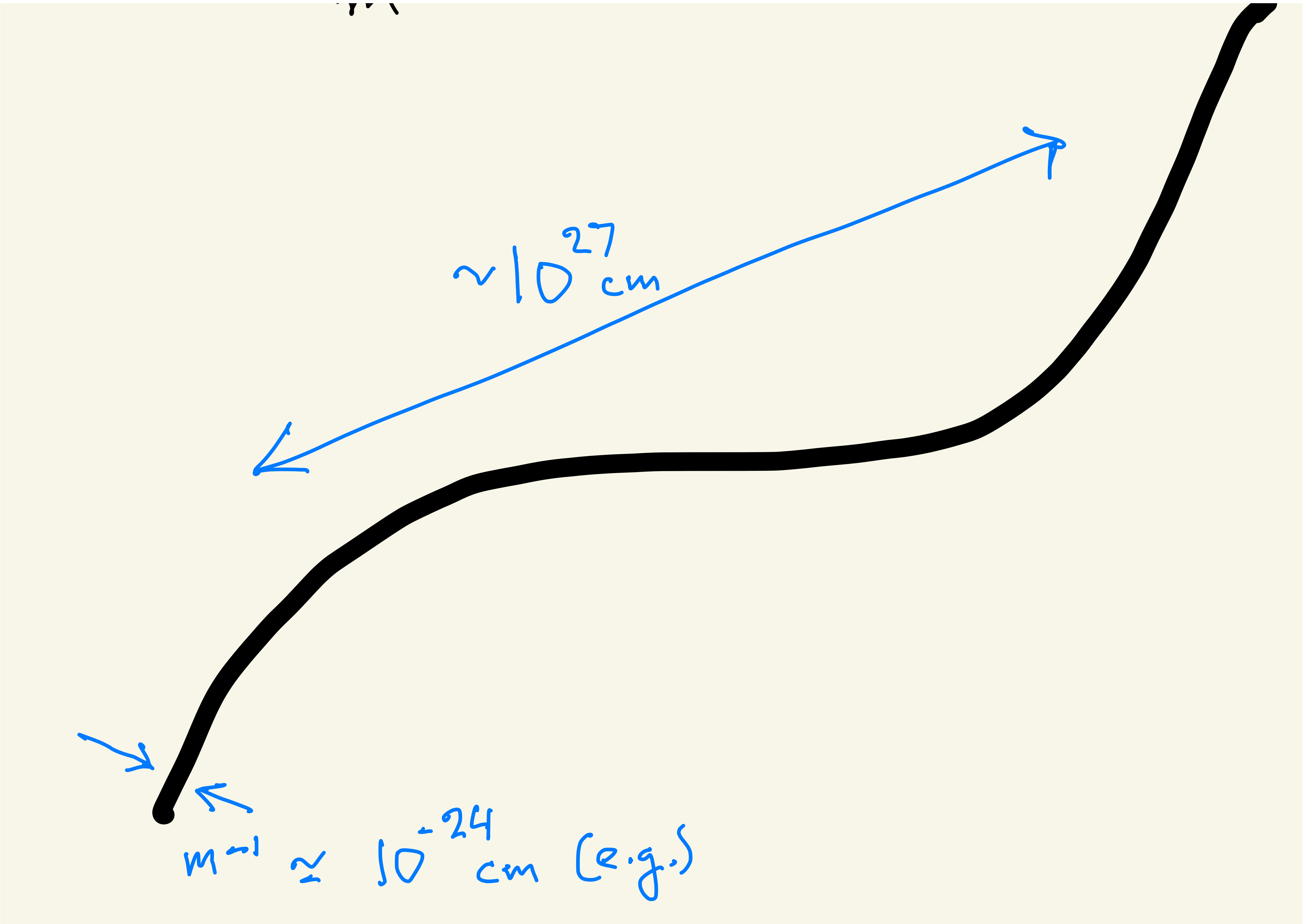
Brownian walk

About 80% of the string is in infinite strings.
(Recall similar result for domain walls.)

Formation simulation



Equation of motion



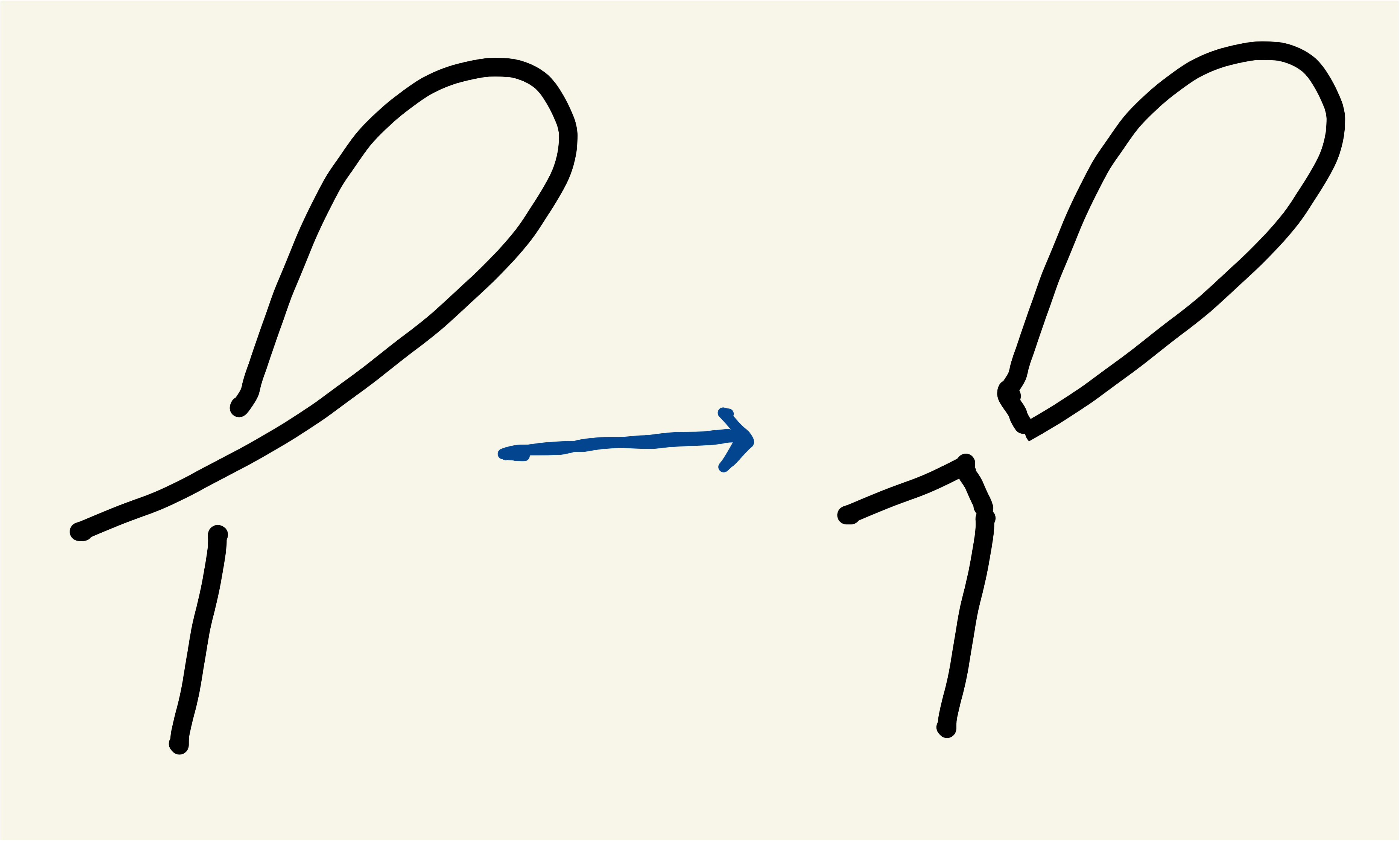
Suggests zero thickness limit.

$$S_{\mathrm{FT}} \to S_{\mathrm{NG}} = -\mu \int d^2\sigma \sqrt{-\gamma}$$

$$\sim \mathrm{area\ of\ world-sheet}$$

Reasonable approximation except when strings cross or in high curvature regions.

Intercommuting



Intercommuting probability =1 in field theory cosmic strings but not necessarily in string theory cosmic strings.

Strategy

Use Nambu-Goto dynamics while there is no intersection. Perform reconnection at intersection and then resume Nambu-Goto evolution.

Upon evolution

Albrecht & Turok;

Bennet & Bouchet;

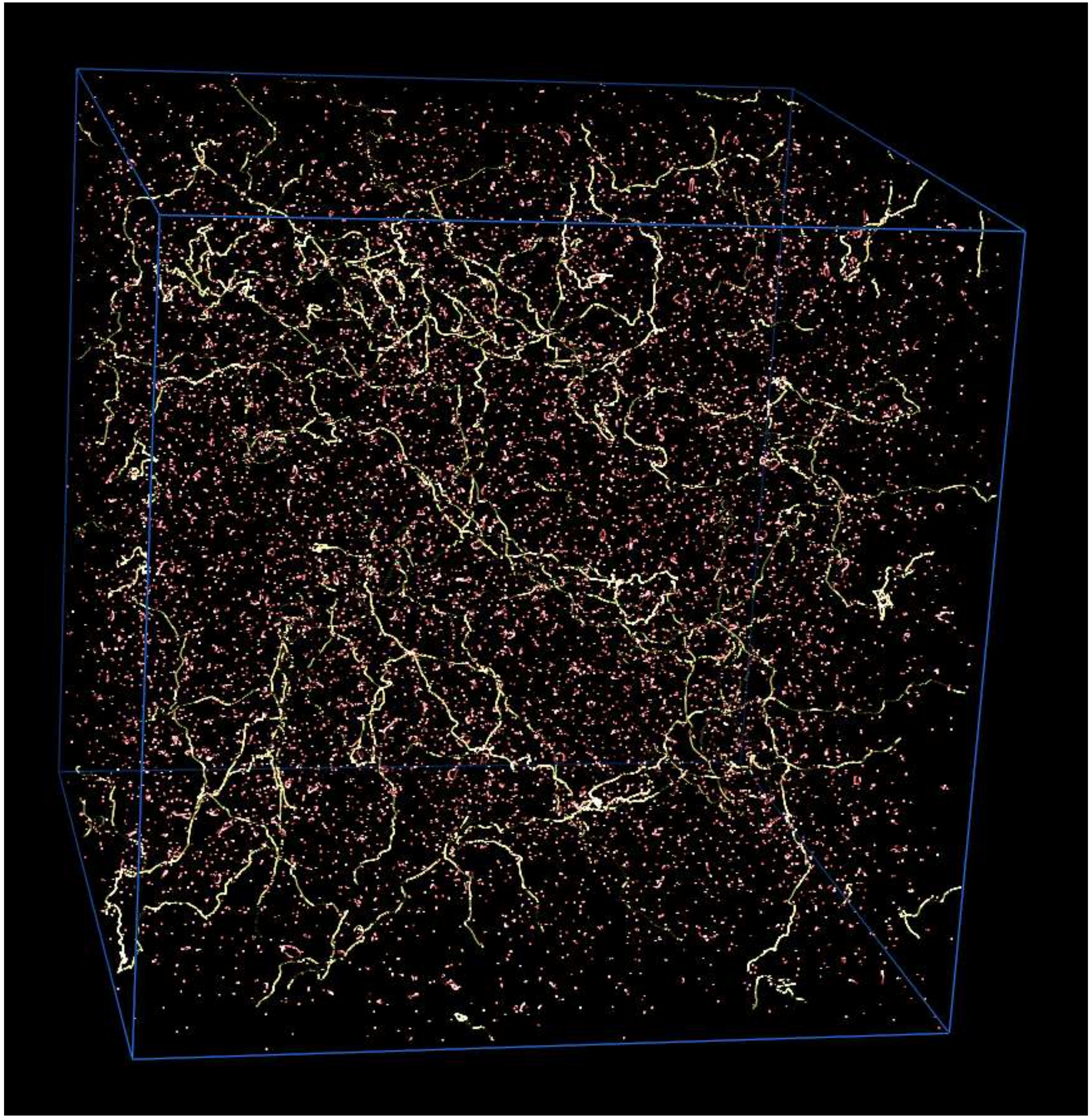
Allen & Shellard;

Blanco-Pillado & Olum;

Ringeval, Sakellariadou & Bouchet;

…

Hindmarsh & collaborators

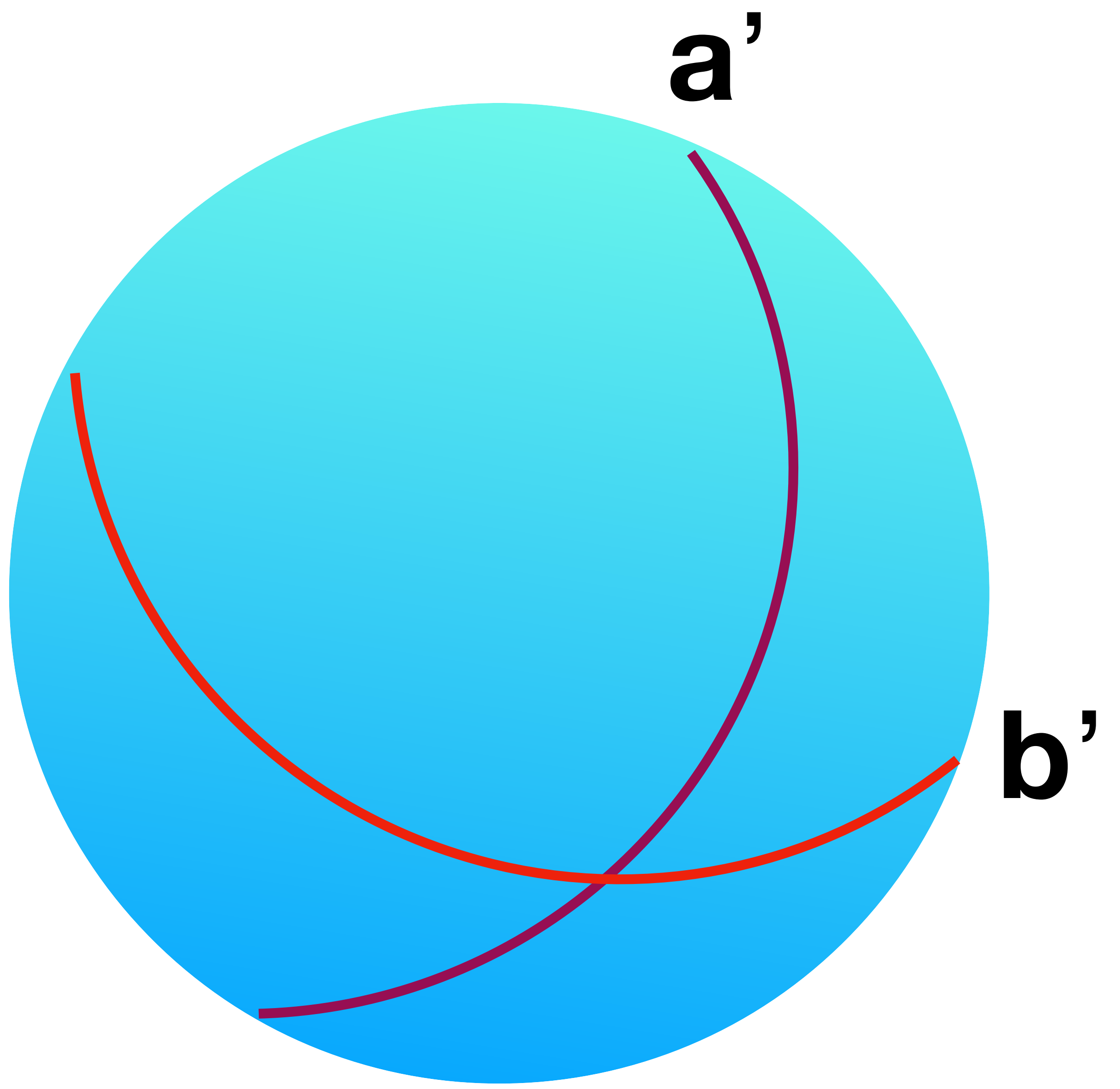


Nambu-Goto dynamics of a loop

Assume Minkowski background. Choose convenient world-sheet parametrization.

Then: $\mathbf{x}(t,\sigma) = \frac{1}{2}[\mathbf{a}(t-\sigma) + \mathbf{b}(t+\sigma)]$ “left- and right- movers”

with the constraint: $|\mathbf{a}'| = 1 = |\mathbf{b}'|$



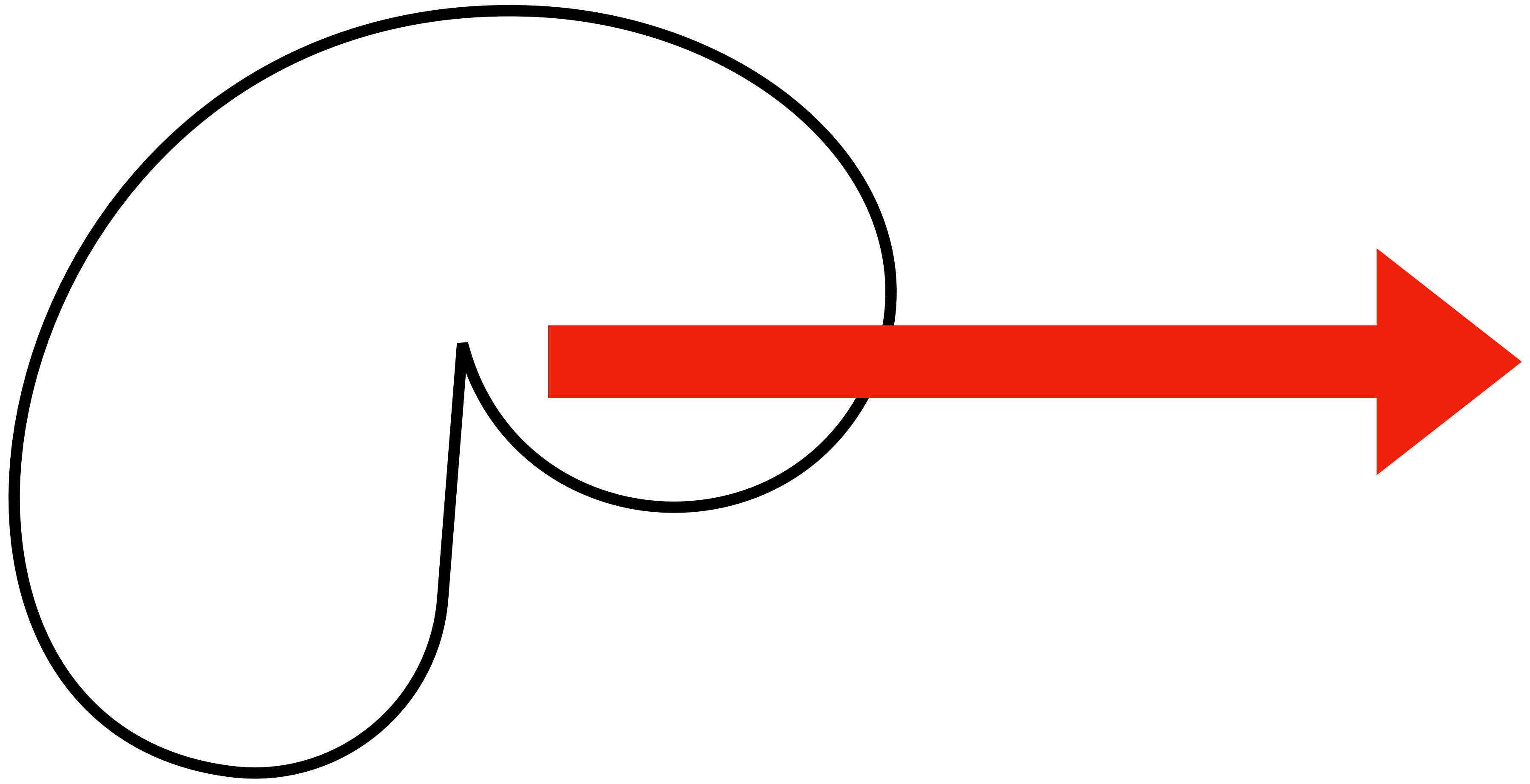
Intersection of **a**’ and **b**’ curves implies:

$$\dot{\mathbf{x}} = \frac{1}{2}[\mathbf{a}' + \mathbf{b}'] = \mathbf{a}'$$

and so, at this intersection point:

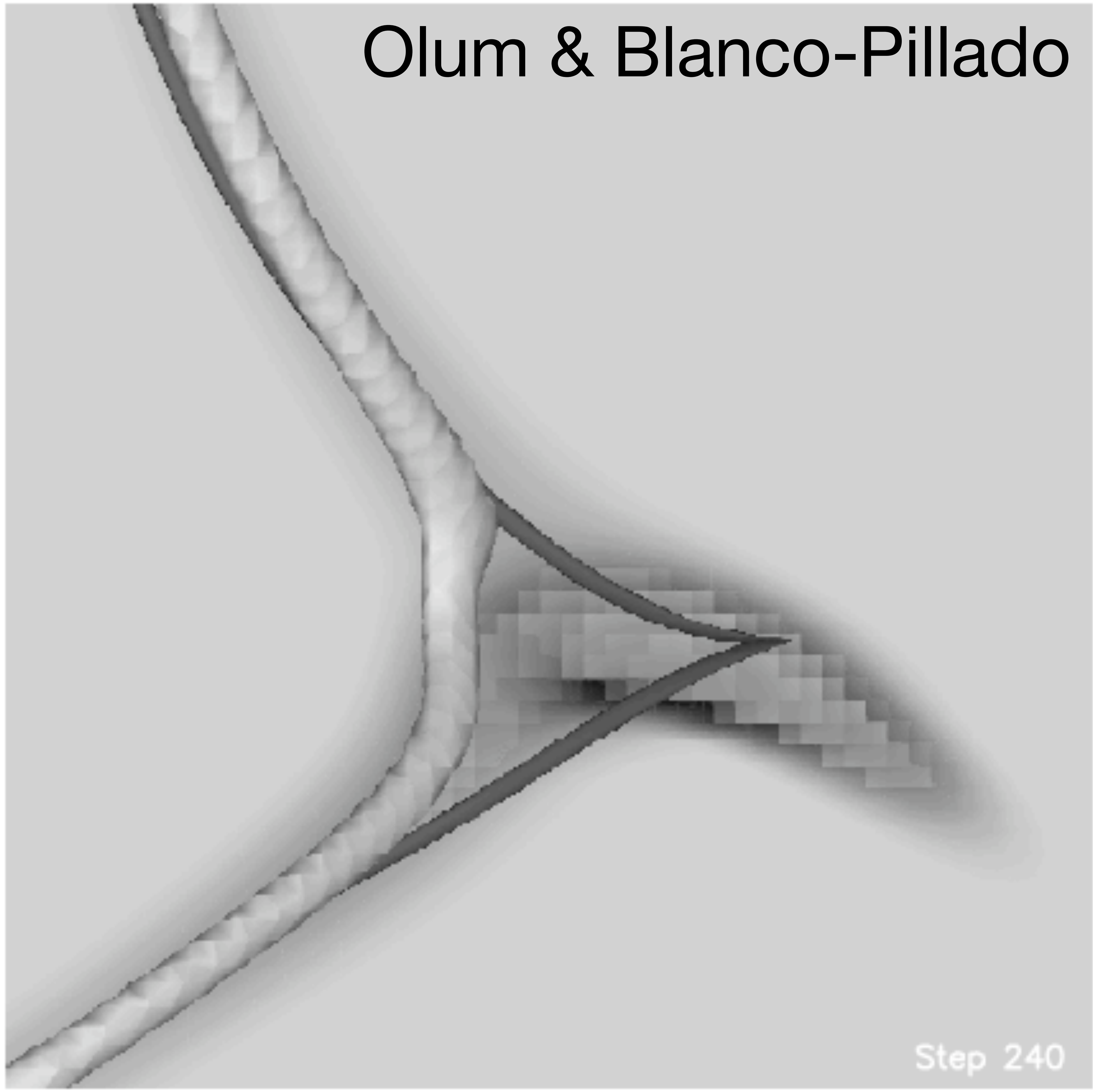
$|\dot{\mathbf{x}}| = 1$ “cusp”

Cusp



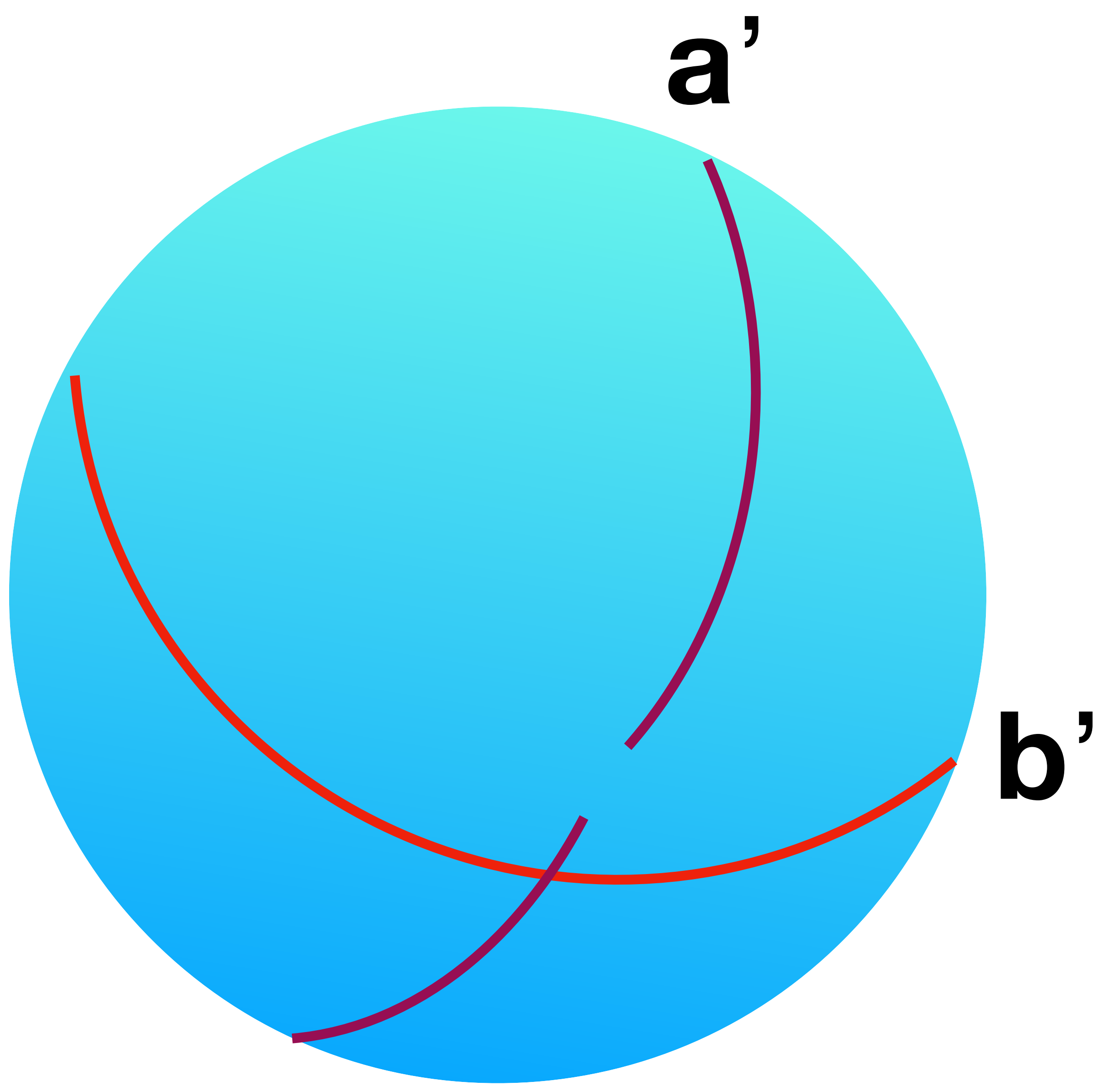
$v = 1$

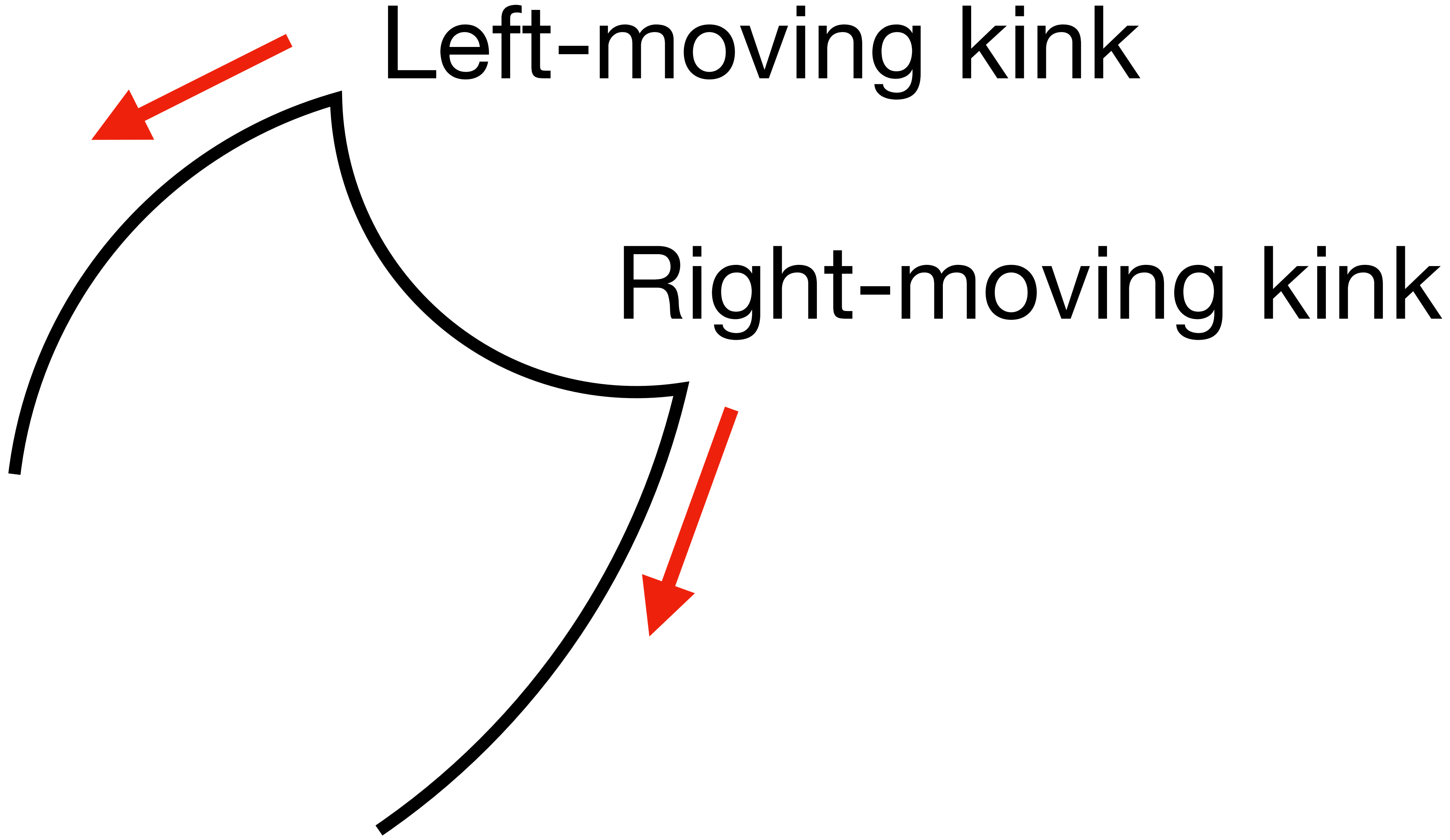
Field theory simulation of a cusp



Intercommutings and the dynamics of a loop

Intercommutings imply discontinuities in the **a**’ and **b**’ curves.





Fragmentation

Scherrer & Press, 1989

Casper & Allen, 1995

Copi & TV, 2011

Shape of loops

Copi & TV, 2011

Non-self-intersecting loops are planar, ~rectangular with ~4 kinks,

have center of mass velocities ~c/sqrt{2},

and angular momentum (that prevents collapse).

Gravitational radiation

Power emitted in gravitational waves is independent of the loop length L.

Quadrupole approximation:

$$P \sim G|\dddot{I}|^2 \sim GM^2L^4\omega^6 \sim G\mu^2$$

Numerically: $P \approx 50G\mu^2$ TV & Vilenkin, 1985;…

with only weak numerical dependence on the shape of the loop.

Gravitational radiation spectrum

Quadrupole approximation breaks down due to cusps.

Energy-momentum tensor: $T^{\mu\nu}(\mathbf{x},t) = \mu \int d\sigma [\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu] \delta^{(3)}(\mathbf{x} - \mathbf{X}(\sigma,t))$

Solve gravitational wave equation. Calculate energy emitted in each harmonic.

$$P_n \propto n^{-4/3} \text{ (cusp)}, \; n^{-5/3} \text{ (kink)}, \; n^{-2} \text{ (kink} - \text{kink)} \qquad \omega_n = 4\pi n/L$$

On average, total energy emitted from loop is consistent with:

$$P \approx 50 G\mu^2$$

Evolution

Gravitational waves or massive radiation?

Nambu-Goto action: $S = -\mu \int d^2\sigma \sqrt{-g_2}$

Loops decay by gravitational radiation. **TV & A. Vilenkin, 1985; …**

Full field theory simulations:

Loops decay by particle radiation. **M. Hindmarsh et al, 2009; …**

Crucial to resolve for experiments (LIGO, NanoGrav,…) that are looking for gravitational wave signatures.

Evolution

Simulation equations

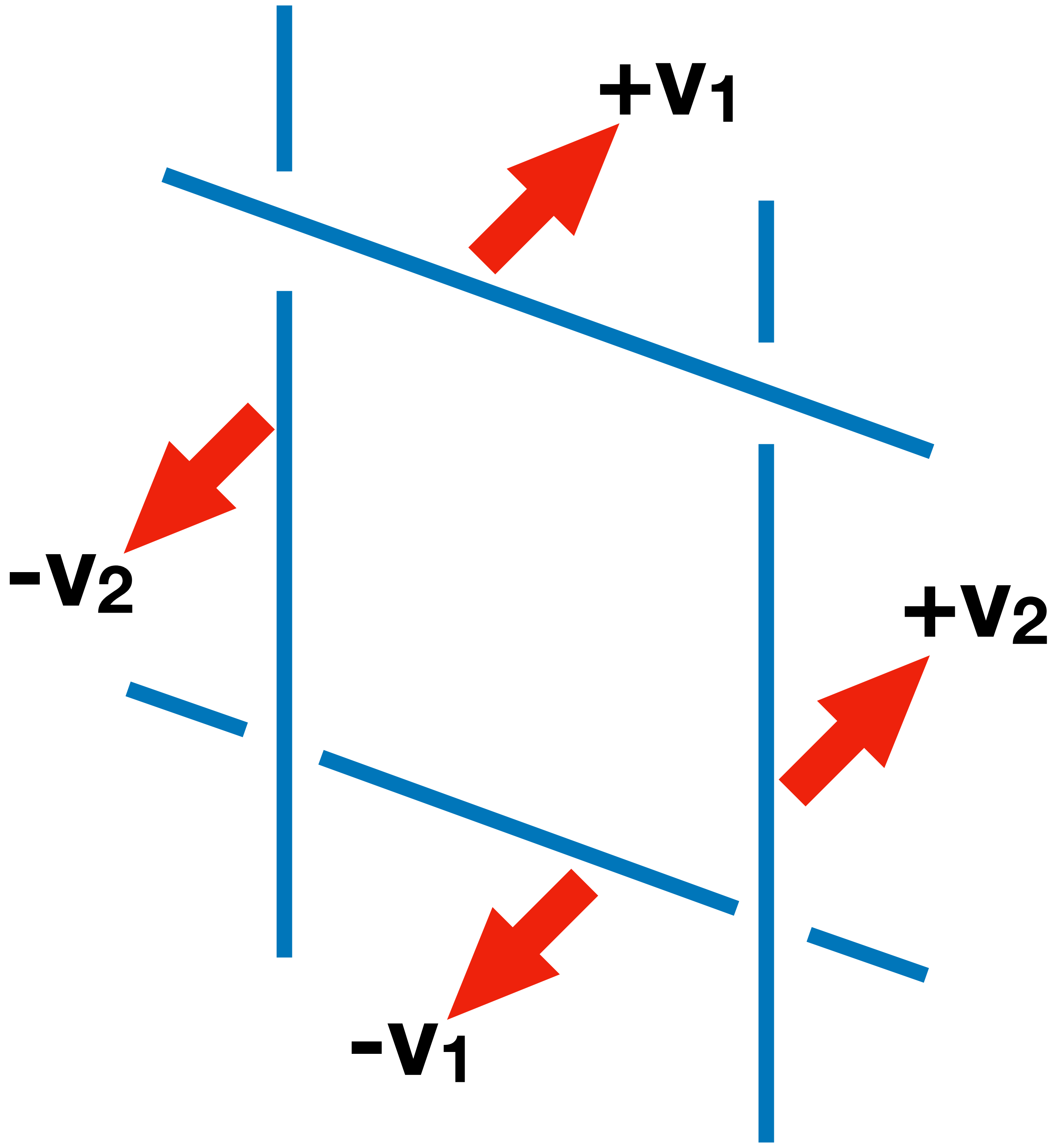
Technical note: Use Numerical Relativity technique for numerical stability.

$$
\begin{aligned}
\Gamma &= \partial_i A_i \\
\partial_t^2 \phi_a &= \nabla^2 \phi_a - e^2 A_i A_i \phi_a - 2e\epsilon_{ab}\partial_i \phi_b A_i - e\epsilon_{ab}\phi_b \Gamma - \lambda(\phi_b\phi_b - \eta^2)\phi_a \\
\partial_t F_{0i} &= \nabla^2 A_i - \partial_i \Gamma + e(\epsilon_{ab}\phi_a \partial_i \phi_b + eA_i \phi_a \phi_a) \\
\partial_t \Gamma &= \partial_i F_{0i} - \underbrace{g_p^2[\partial_i F_{0i} + e\epsilon_{ab}\phi_a \partial_t \phi_b]}_{\text{Gauss constraint}},
\end{aligned}
$$

(Code is available on request.)

Evolution

Initial conditions



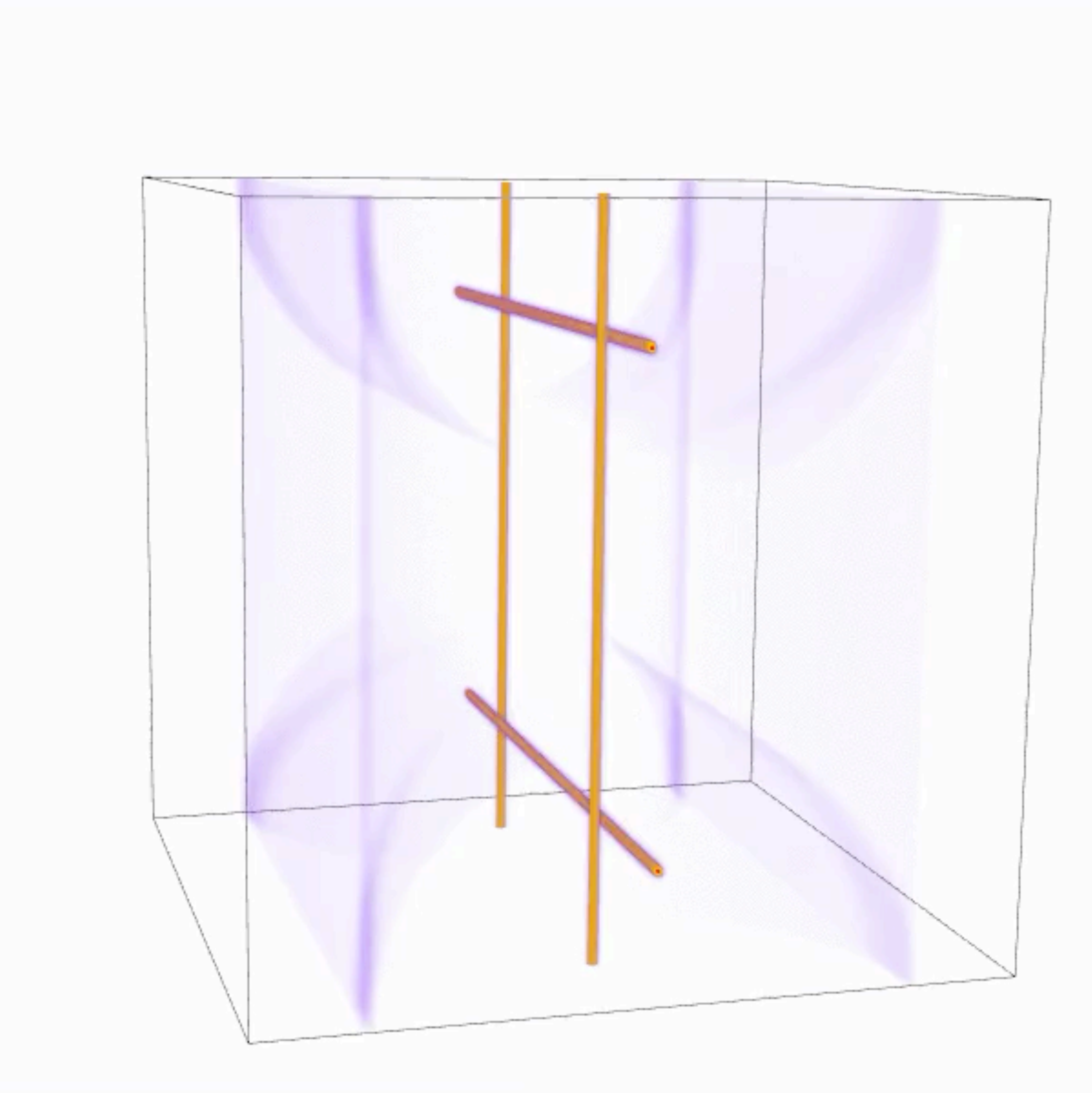
Technical notes

Boost takes the gauge field out of temporal gauge. Then one needs to perform a gauge transformation to go back to temporal gauge.

Periodic boundary conditions require some smoothing functions.

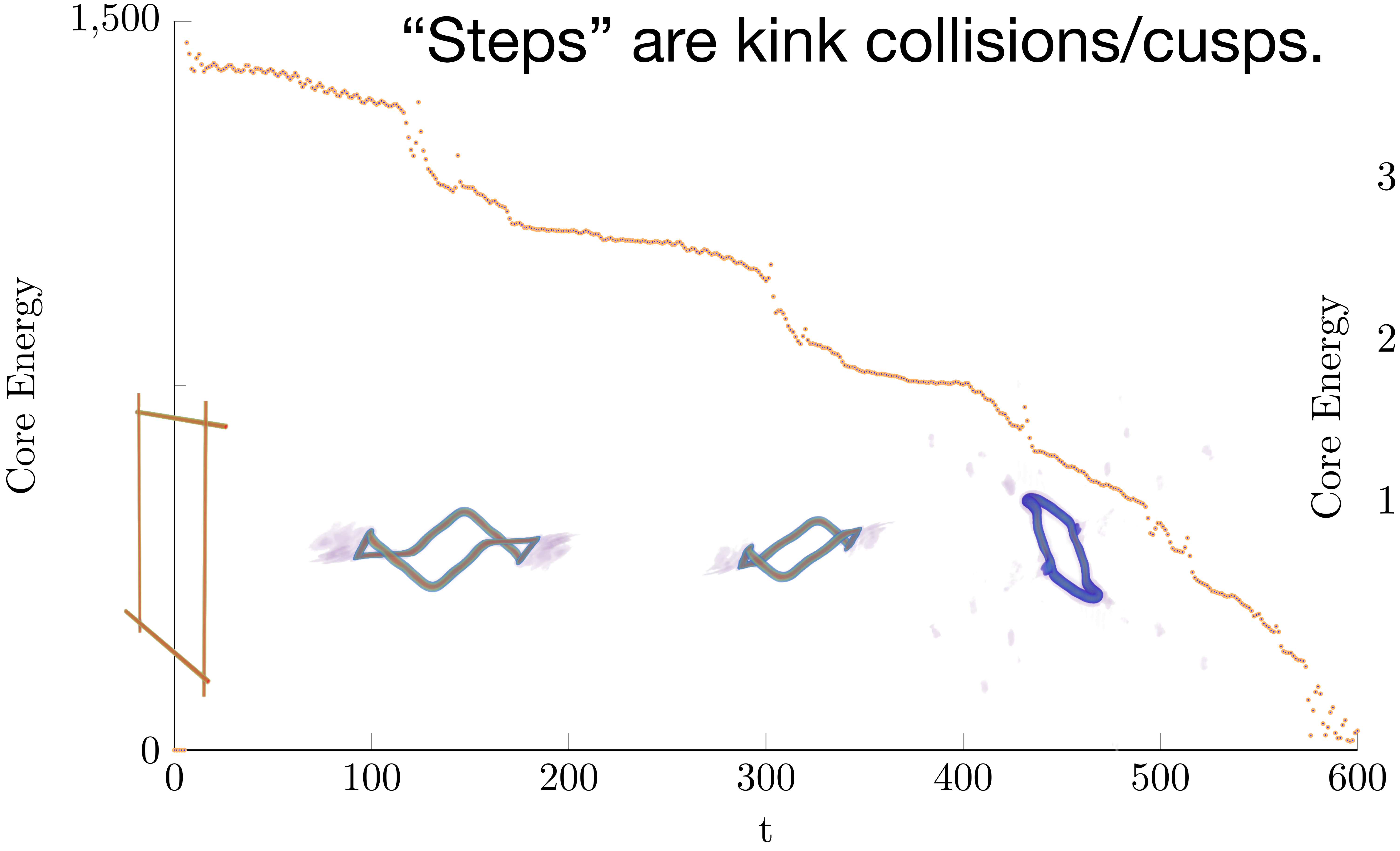
Evolution

Animation

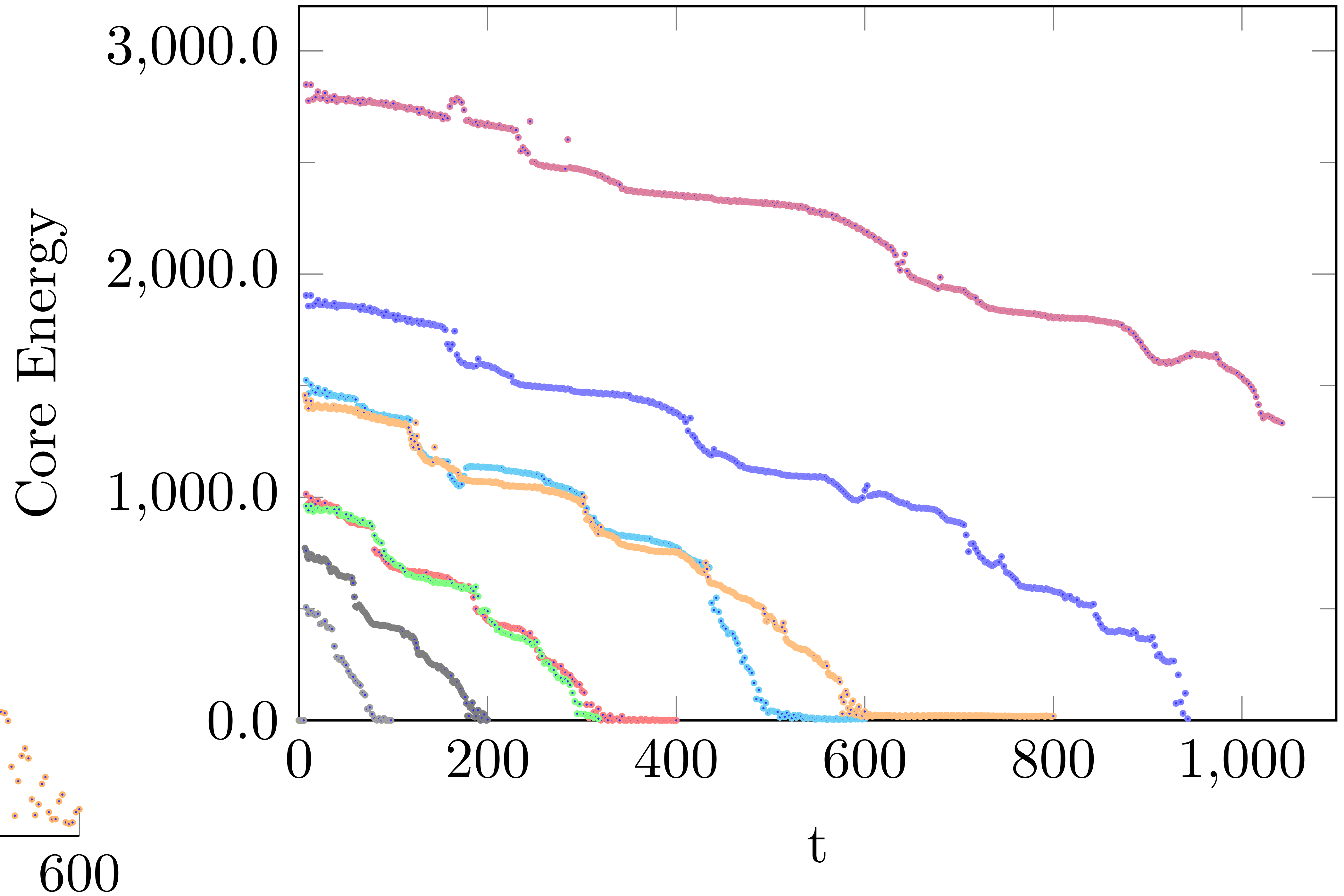


Evolution

Loop energy vs. time

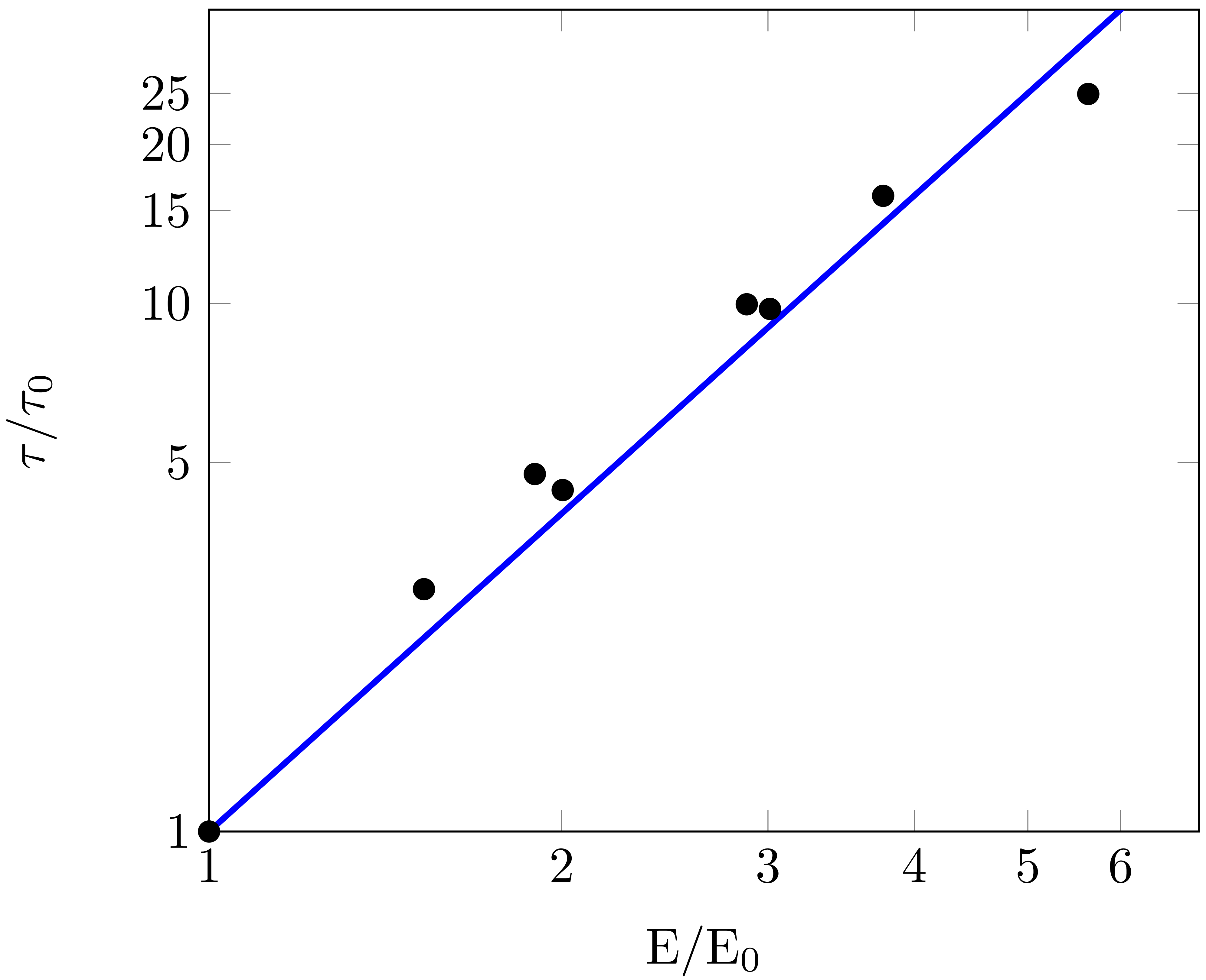


“Steps” are kink collisions/cusps.



Evolution

Lifetime vs. initial length



$$\tau_{\text{particle}} \propto L^2 \qquad \tau_{\text{grav}} \propto L$$

$$\tau_{\text{grav}} < \tau_{\text{particle}} \text{ for large } L$$

$$\boxed{L_{\text{crit}} \sim \frac{w}{G\mu}}$$

where w=width of the string, ***μ***=tension.

Strings with tension above the QCD scale primarily decay by gravitational radiation.

High frequency cutoff on gravitational wave spectrum due to particle radiation.

P. Auclair, D. Steer & TV, 2020

Stochastic gravitational background

Now sum over contribution of all loops at various cosmological epochs to get the stochastic background. (Long strings are sub-dominant.)

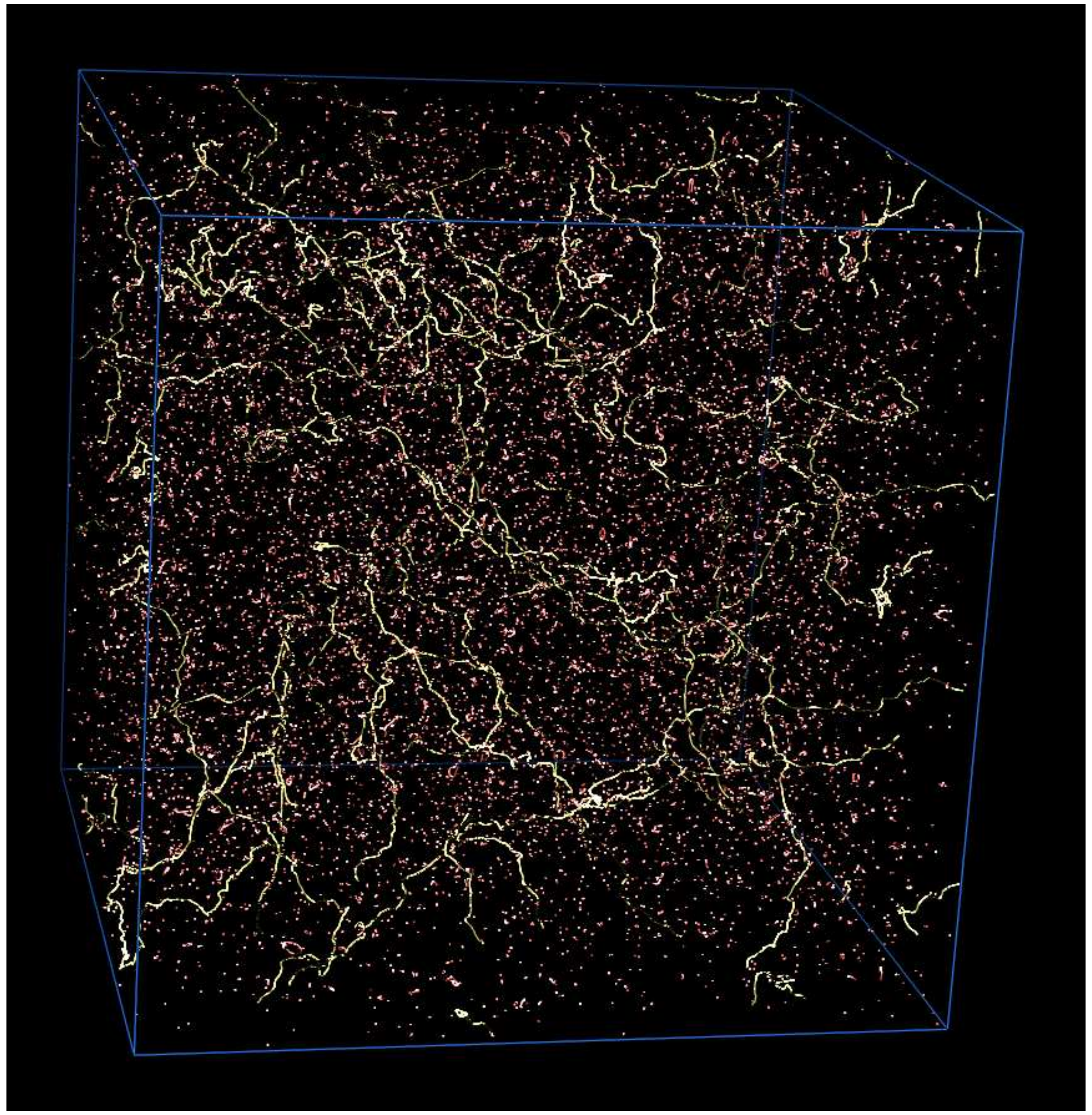
Result depends on the loop spectrum.

High frequency emission is from small loops, in the radiation era.

$$\Omega_g(f) = \frac{128\pi}{9}\alpha^{3/2}\beta(G\mu/\gamma)^{1/2}\Omega_\gamma$$

TV & Vilenkin, 1985;…

Independent of frequency!

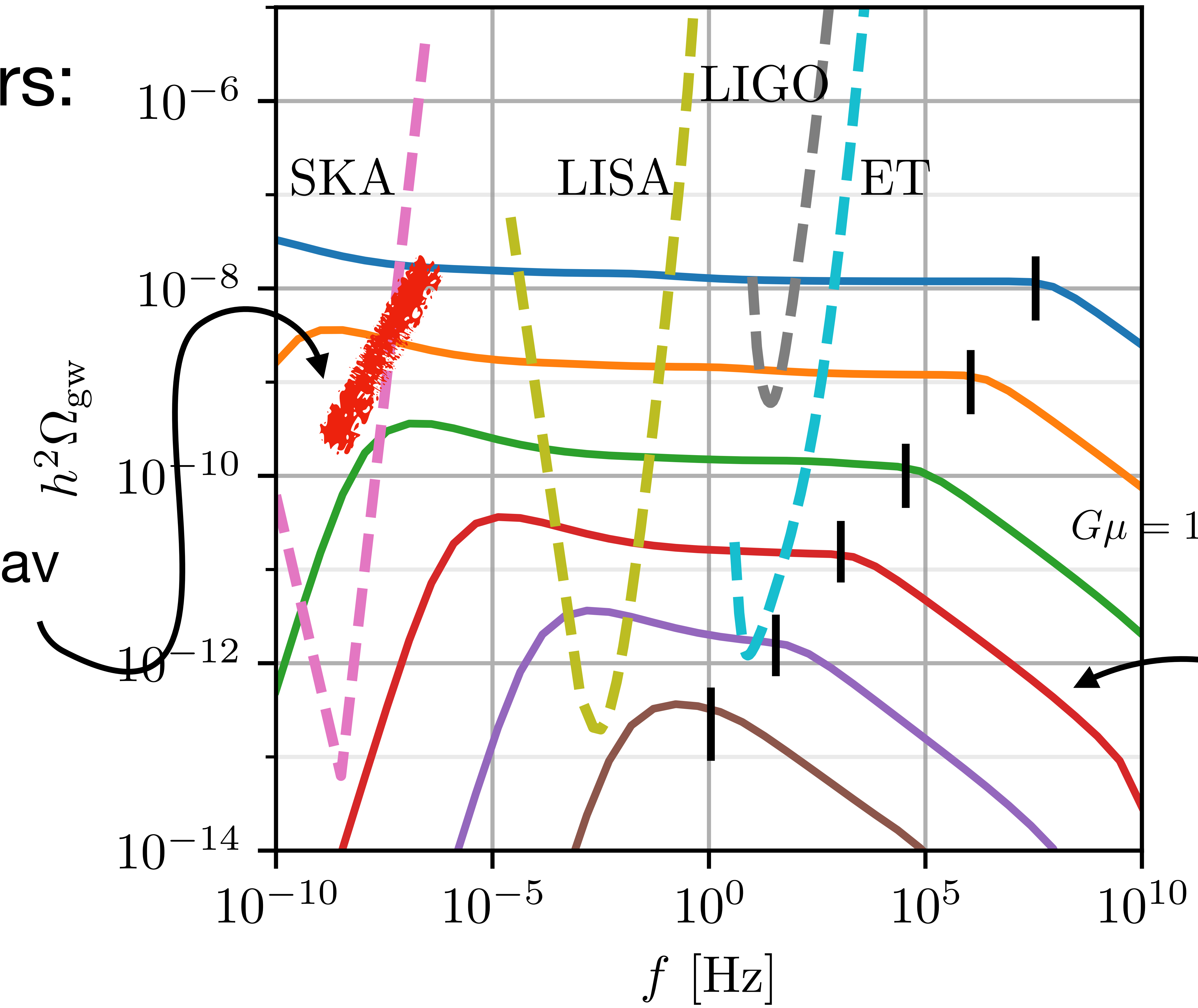


A more complete estimate

Several additional factors:

- Loops in matter era.
- Particle radiation.
- Loop spectrum.
- …

Allen & Caldwell;
….;
Auclair, Steer & TV;
(among many other groups)



$G\mu = 10^{-17}, 10^{-15}, 10^{-13}, 10^{-11}, 10^{-9}, 10^{-7}$

NANOGrav & EPTA

Effect of particle radiation.

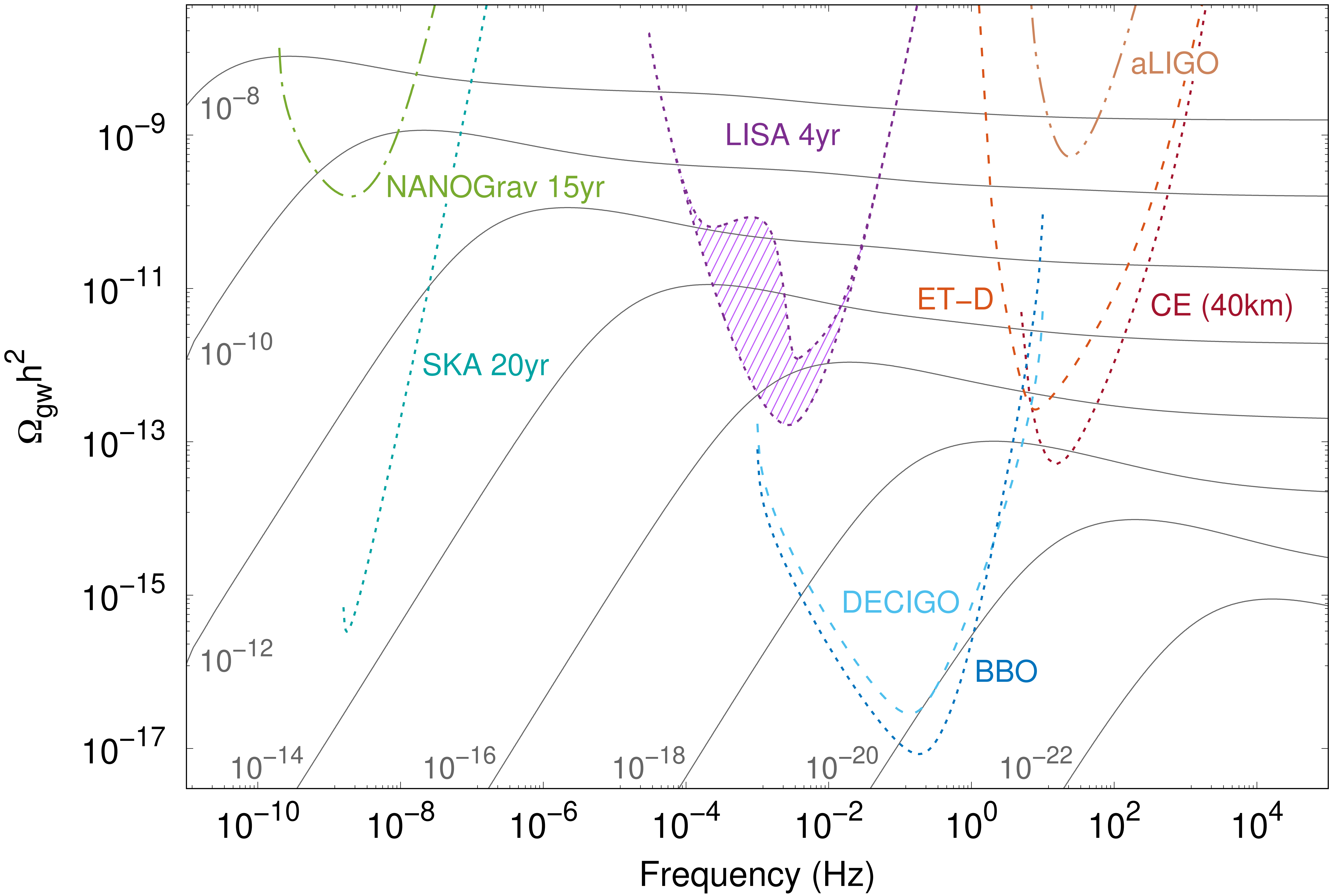
Matter era loops.

Radiation era loops.

String theory cosmic strings provide a better fit: smaller reconnection probability means more string, larger loops.

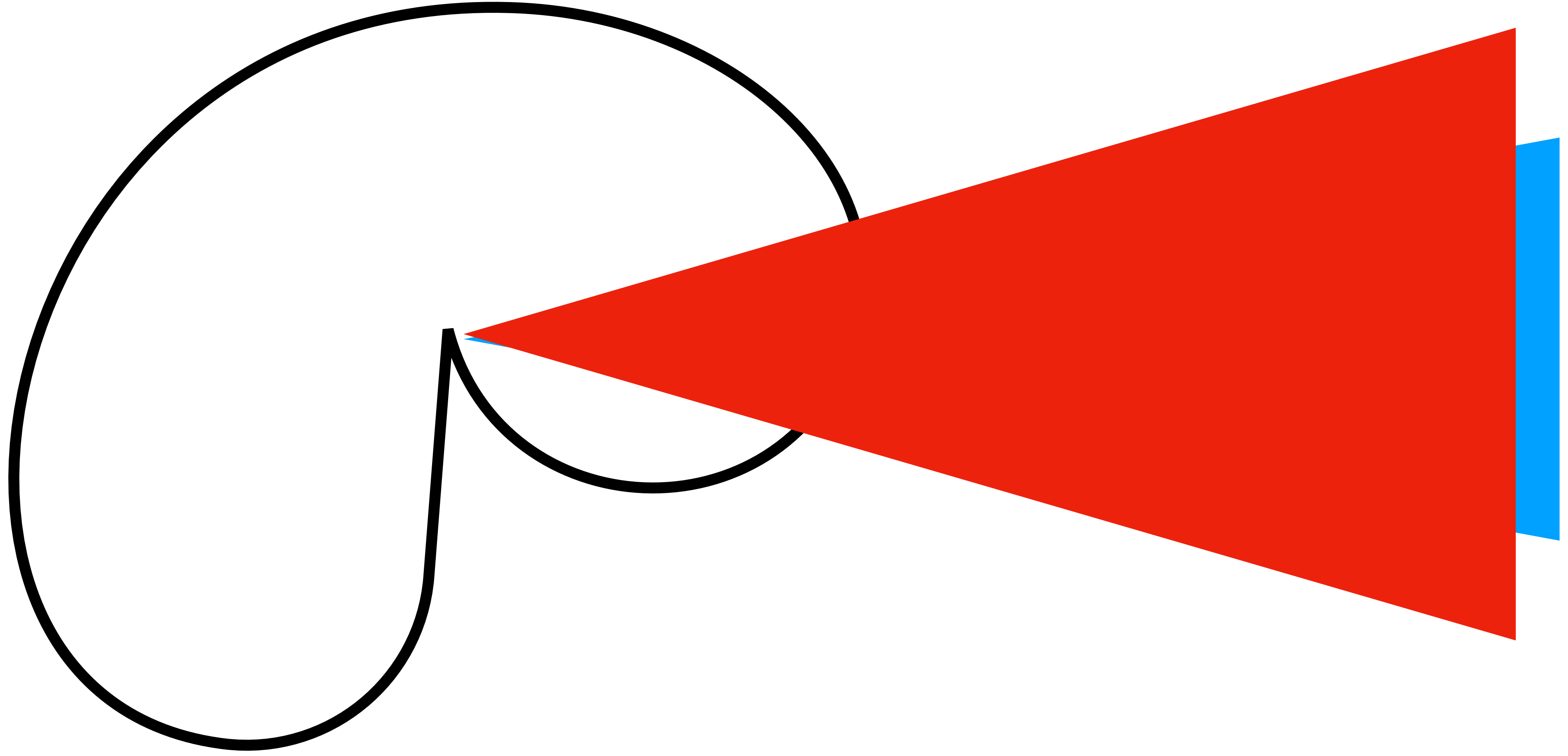
With gravitational backreaction (but no particle radiation)

Wachter, Olum, Blanco-Pillado, 2024



Gravitational wave bursts from cusps

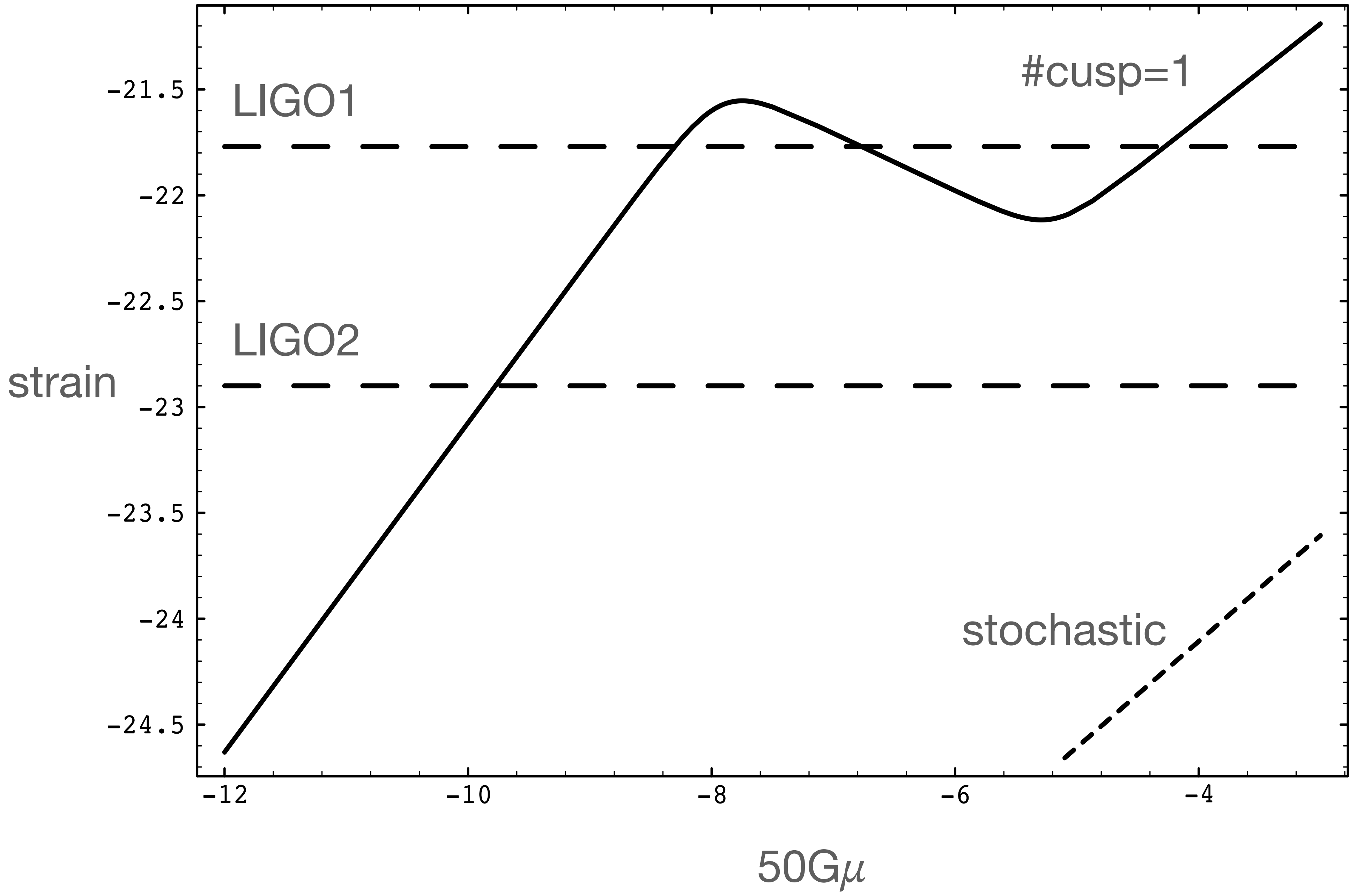
$$\theta(f) \sim [f(1+z)L]^{-1/3}$$



Damour & Vilenkin, 2000

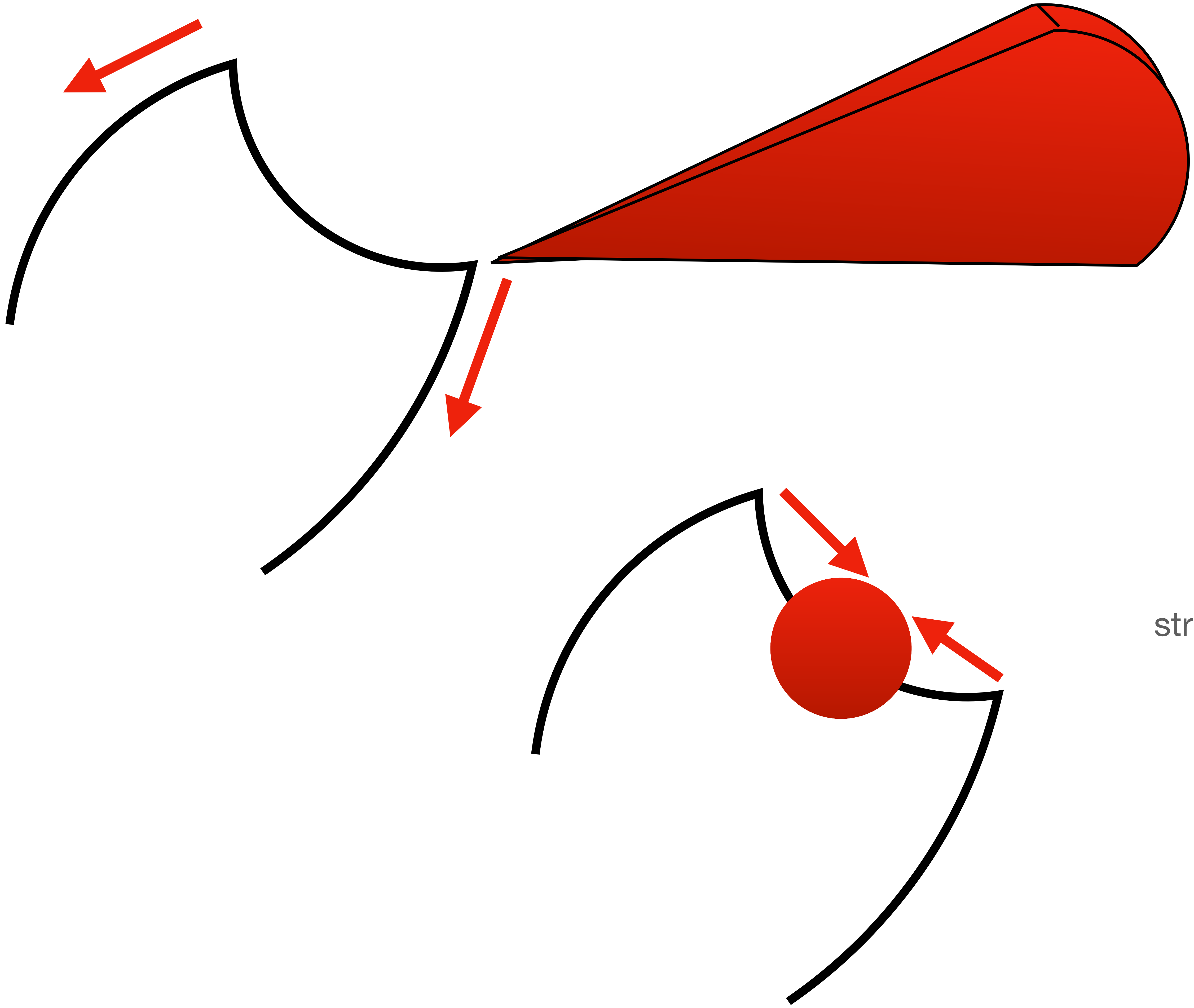
🌎

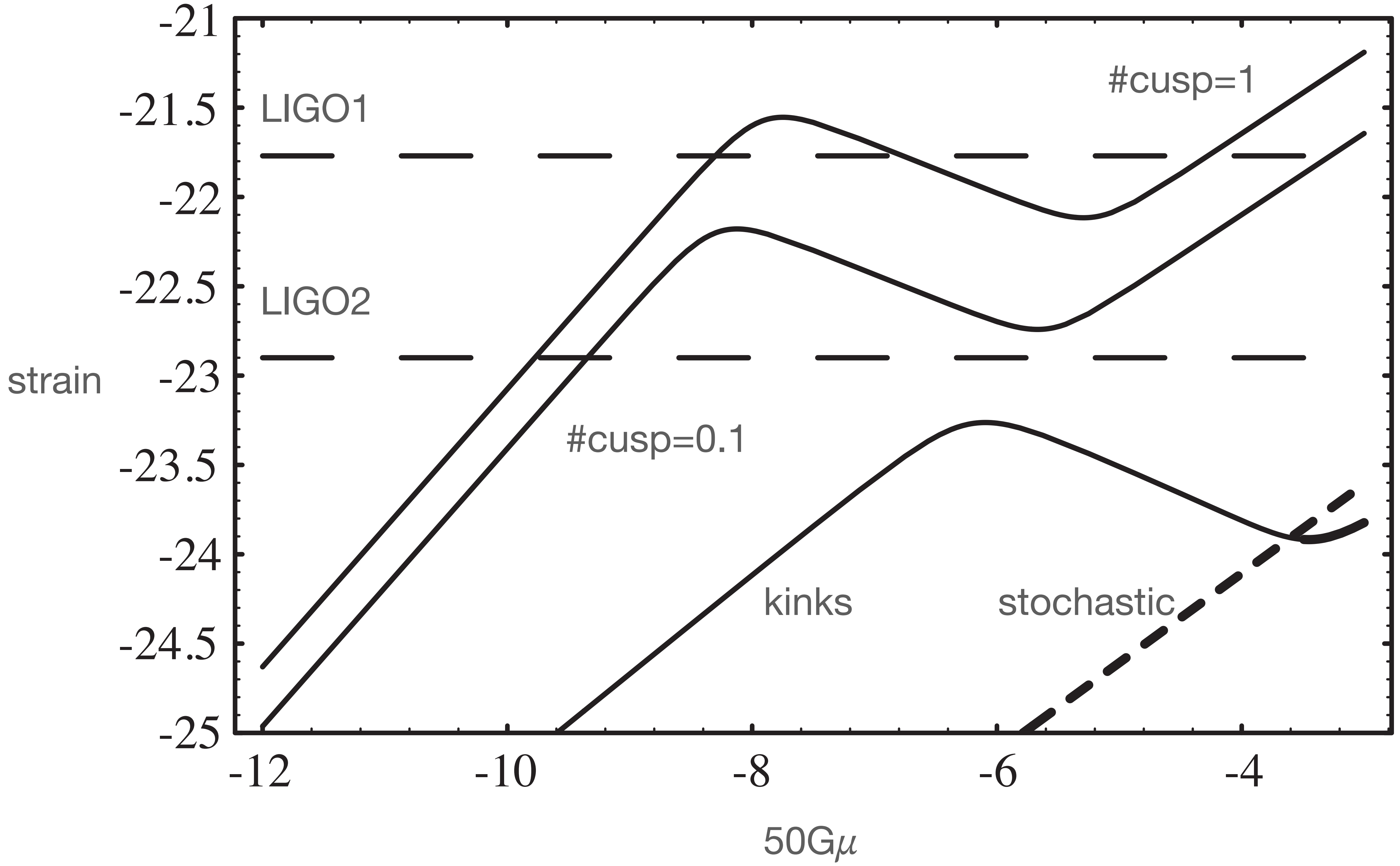
$$A \sim \frac{G\mu L^{2/3}}{(1+z)^{1/3} r(z)}$$



Gravitational wave bursts from kinks

Damour & Vilenkin, 2001





Repeated gravitational wave bursts

Auclair, Steer & TV, 2023

Loop dynamics is periodic with period L/2.

Cusp bursts will repeat unless gravitational backreaction changes the direction of the cusp velocity.

Estimate change in direction as: $I\dot{\theta} \approx \Delta J \approx G\mu^2 L$

$$I \approx \mu L^3$$

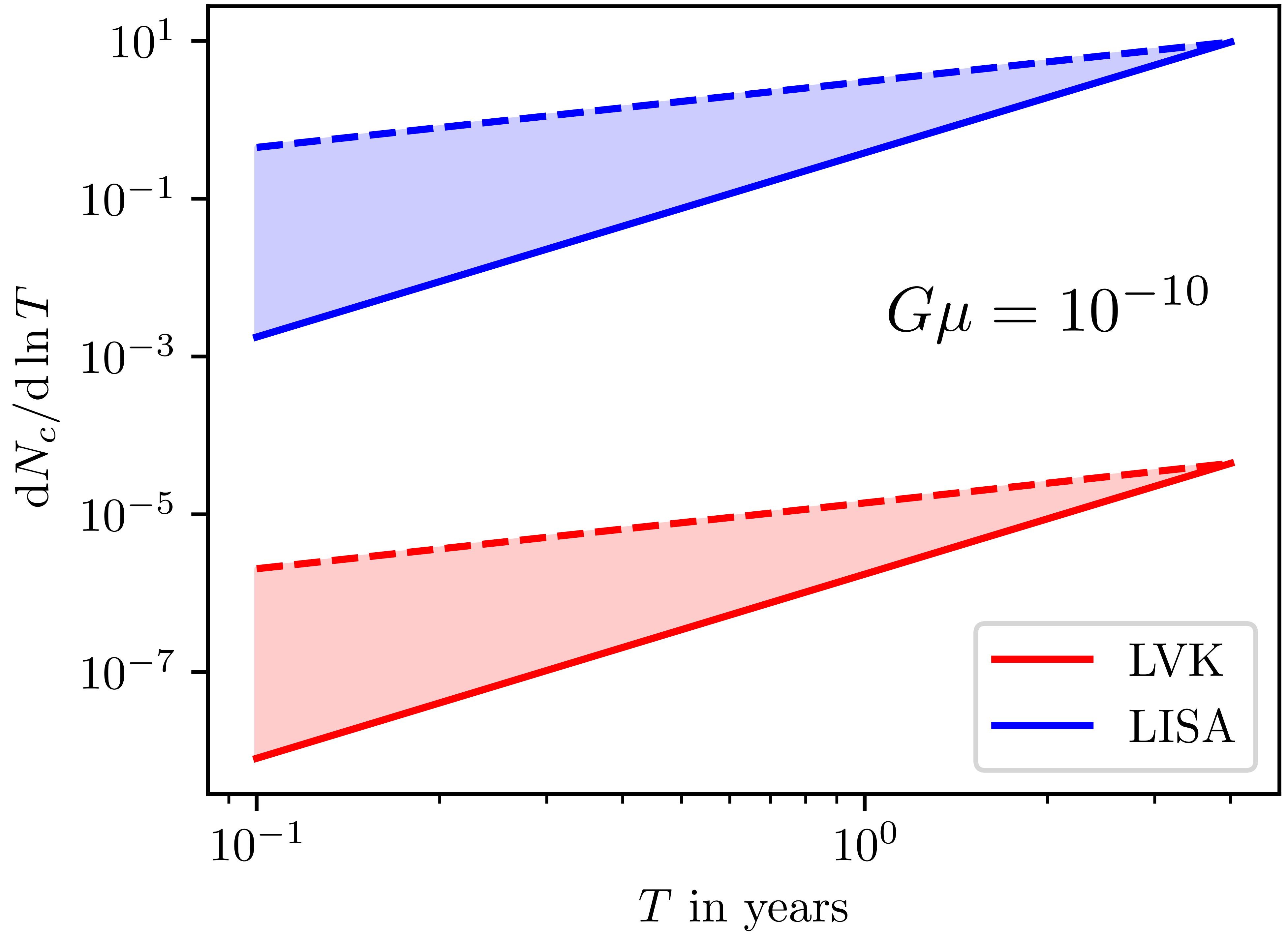
$$\Delta\theta \approx G\mu \ll \theta_{\mathrm{beam}}(f) \sim [f(1+z)L]^{-1/3}$$

LISA will be sensitive to repeated bursts within its lifetime if $G\mu \gtrsim 10^{-10}$

Repeated gravitational wave bursts

First ensure that single burst is observable,

$$A \sim \frac{G\mu L^{2/3}}{(1+z)^{1/3} r(z)} > A_{\text{threshold}}$$



Then, fold in observable loops with loop distribution to estimate the number of repeated bursts.

(Result most sensitive to detector threshold.)

LISA will be sensitive to repeated bursts within its lifetime if $G\mu \gtrsim 10^{-10}$

Summary of gravitational wave signatures of cosmic strings

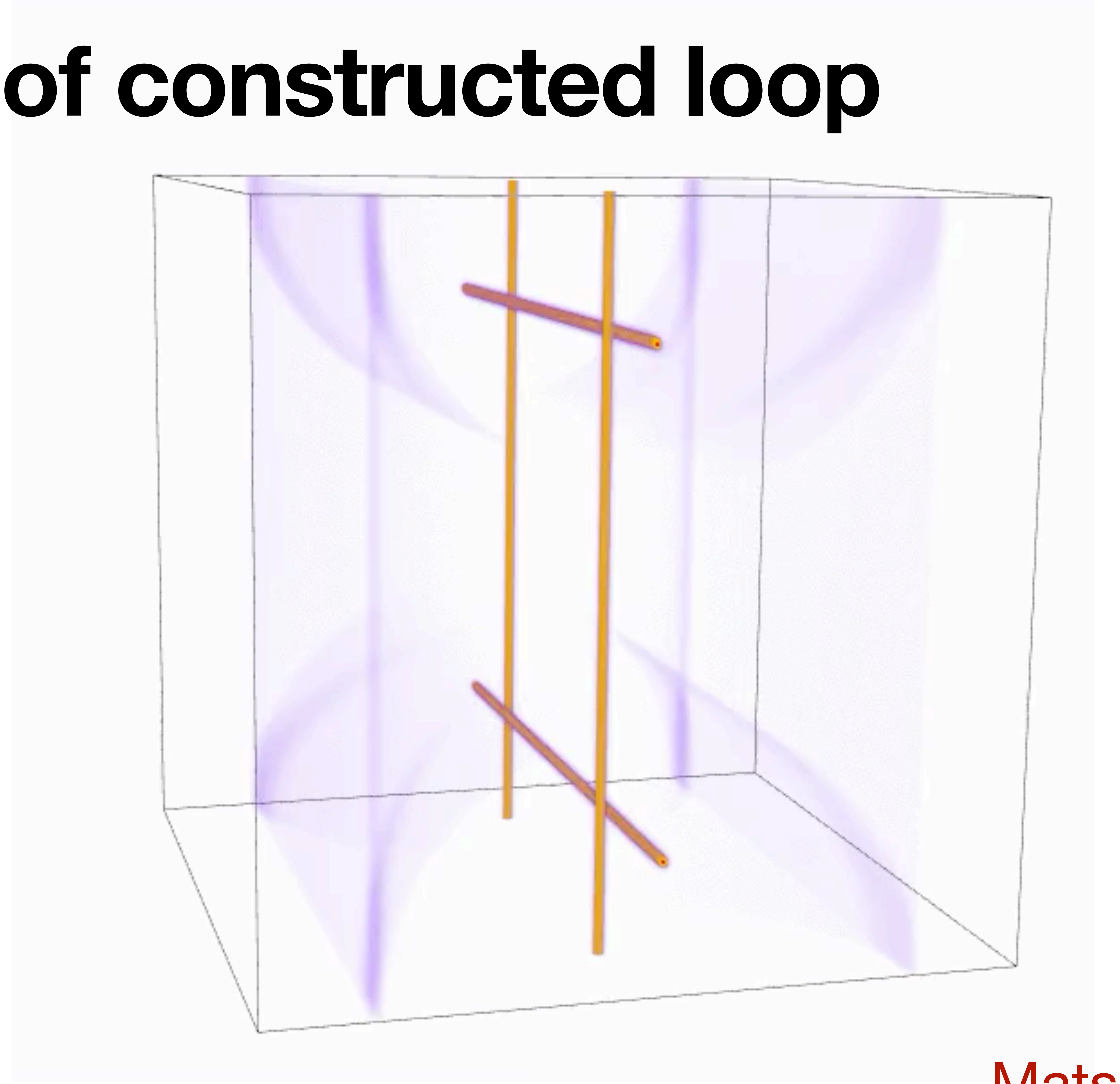
1. Stochastic background. (PTAs)
2. Gravitational wave bursts from cusps, kinks and kink-kink collisions. (LIGO/LISA)
3. Repeated gravitational wave bursts. (LISA)

Key uncertainty: how many loops are there?

Three simulations, three results.

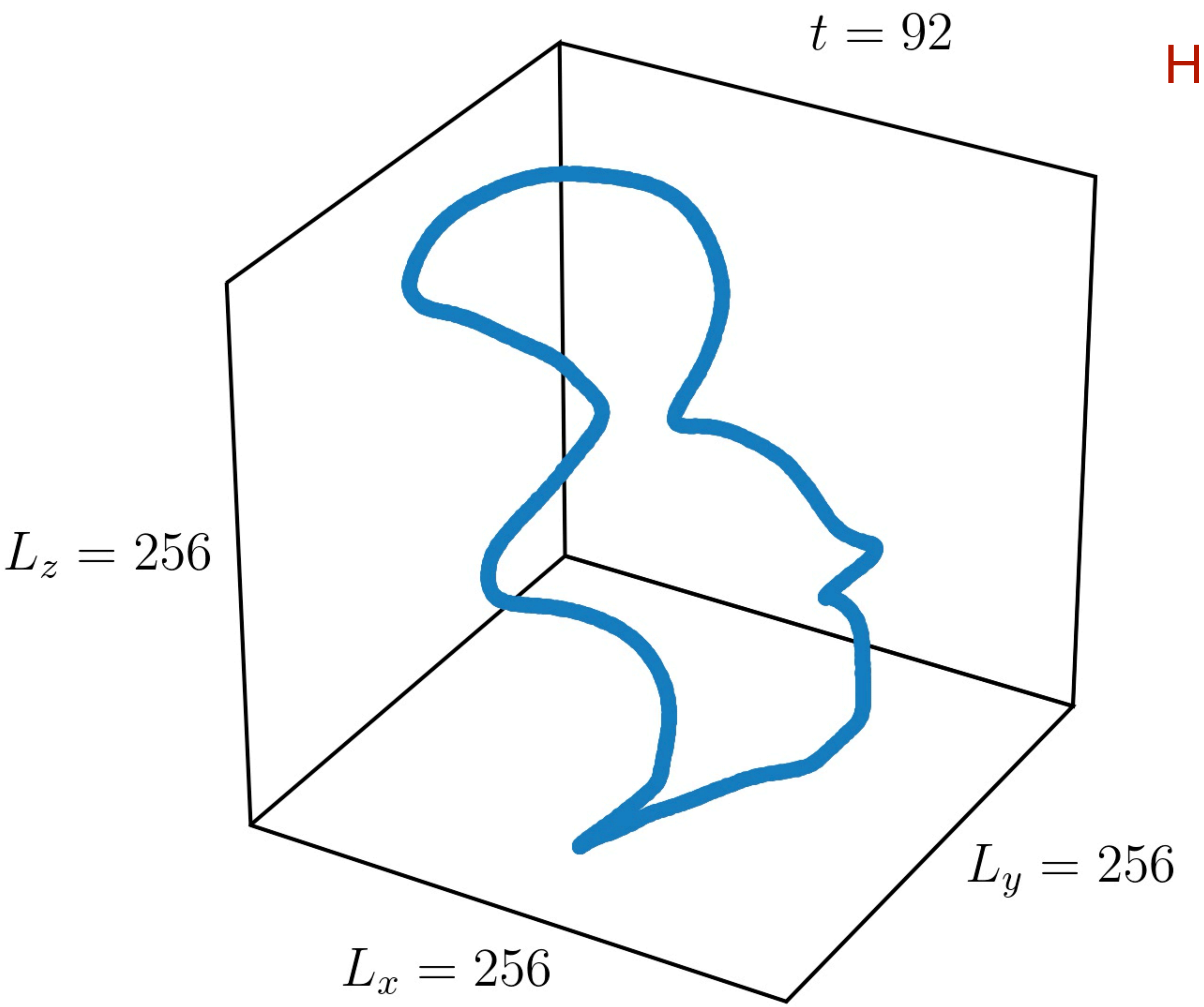
1. Nambu-Goto simulations — scaling loops. Blanco-Pillado & Olum; ….
2. Nambu-Goto simulations — small loops. Ringeval, Sakellariadou & Bouchet;…
3. Field theory simulations — no loops. Hindmarsh & collaborators

Evolution of constructed loop



Matsunami et al

Loop extracted from field theory simulation



Hindmarsh & collaborators

Field theory simulation conclusions

Hindmarsh & collaborators

Loops decay by *particle emission*, not gravitational waves.

They find that the long strings in their simulations have lower velocities than expected and so only produce loops with small angular momentum.

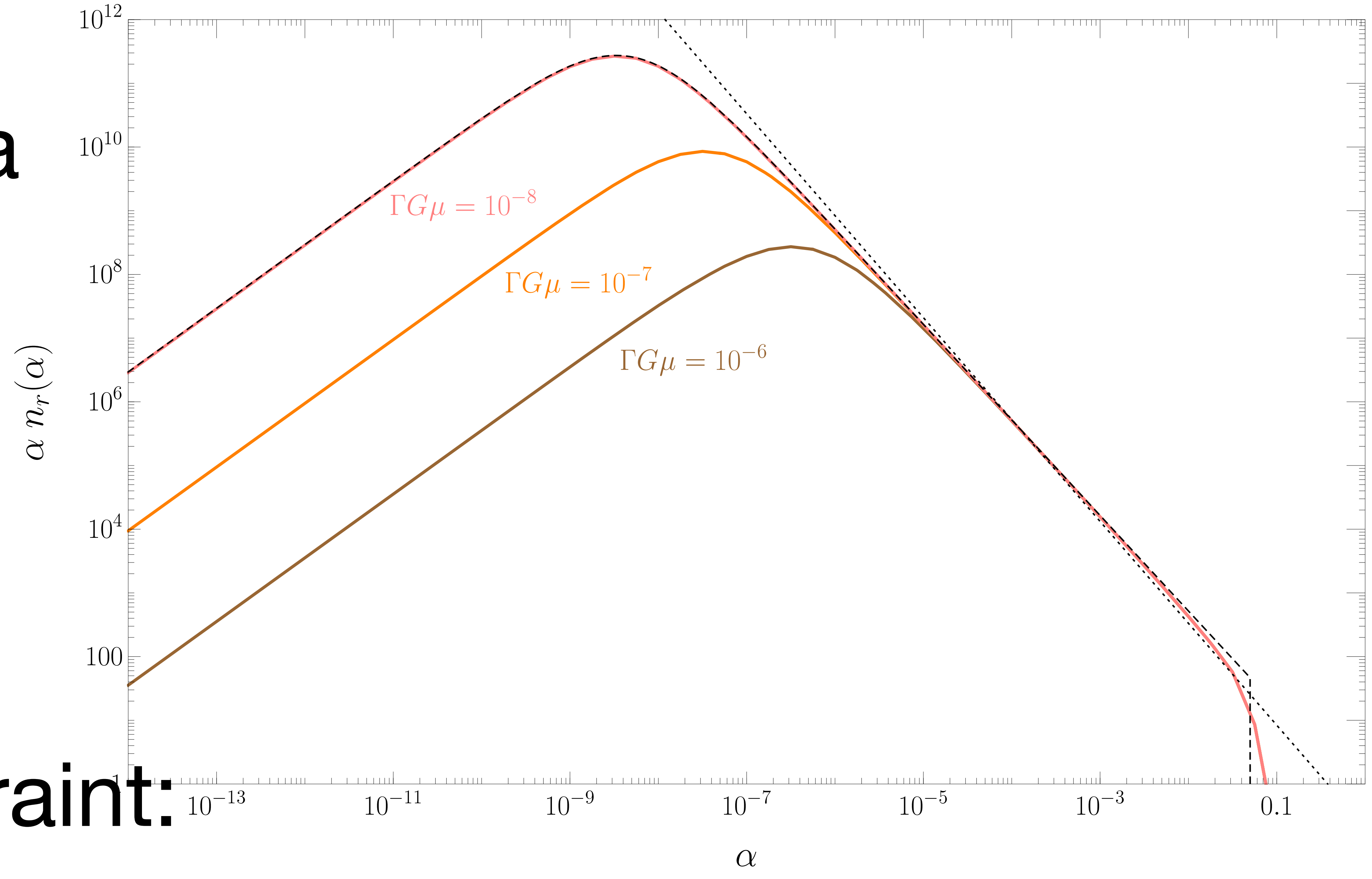
Constraints arise from cosmic ray observations and depend on the underlying particle physics model.

Nambu-Goto simulations

Blanco-Pillado et al, 2023

Confirm Nambu-Goto evolution very accurately except briefly in regions of high string curvature. (δ-function production of loops at ~0.1 t)

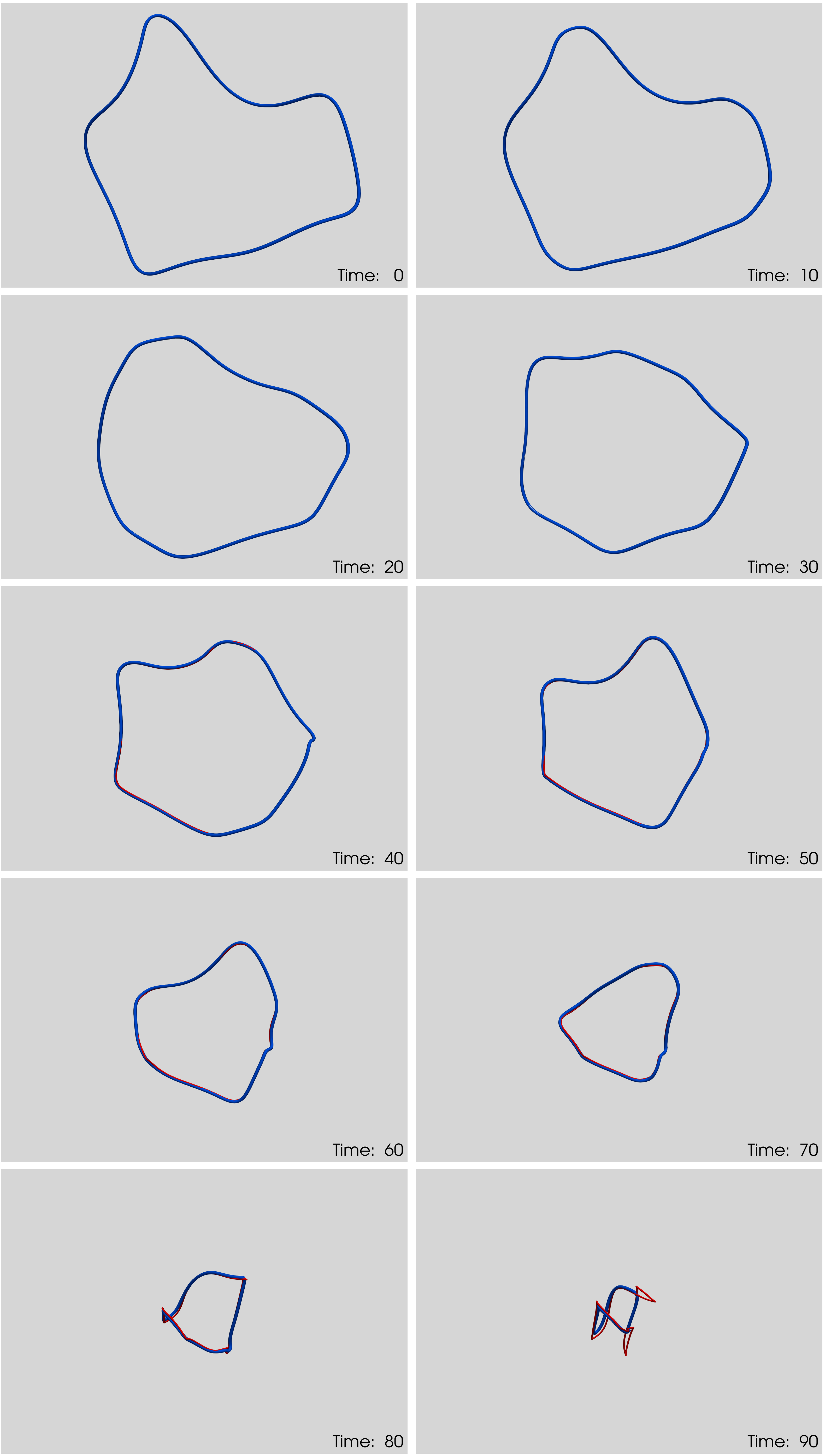
Radiation era number density of loops



Loop size/t

Obtain constraint:

$$G\mu \lesssim 10^{-10}$$

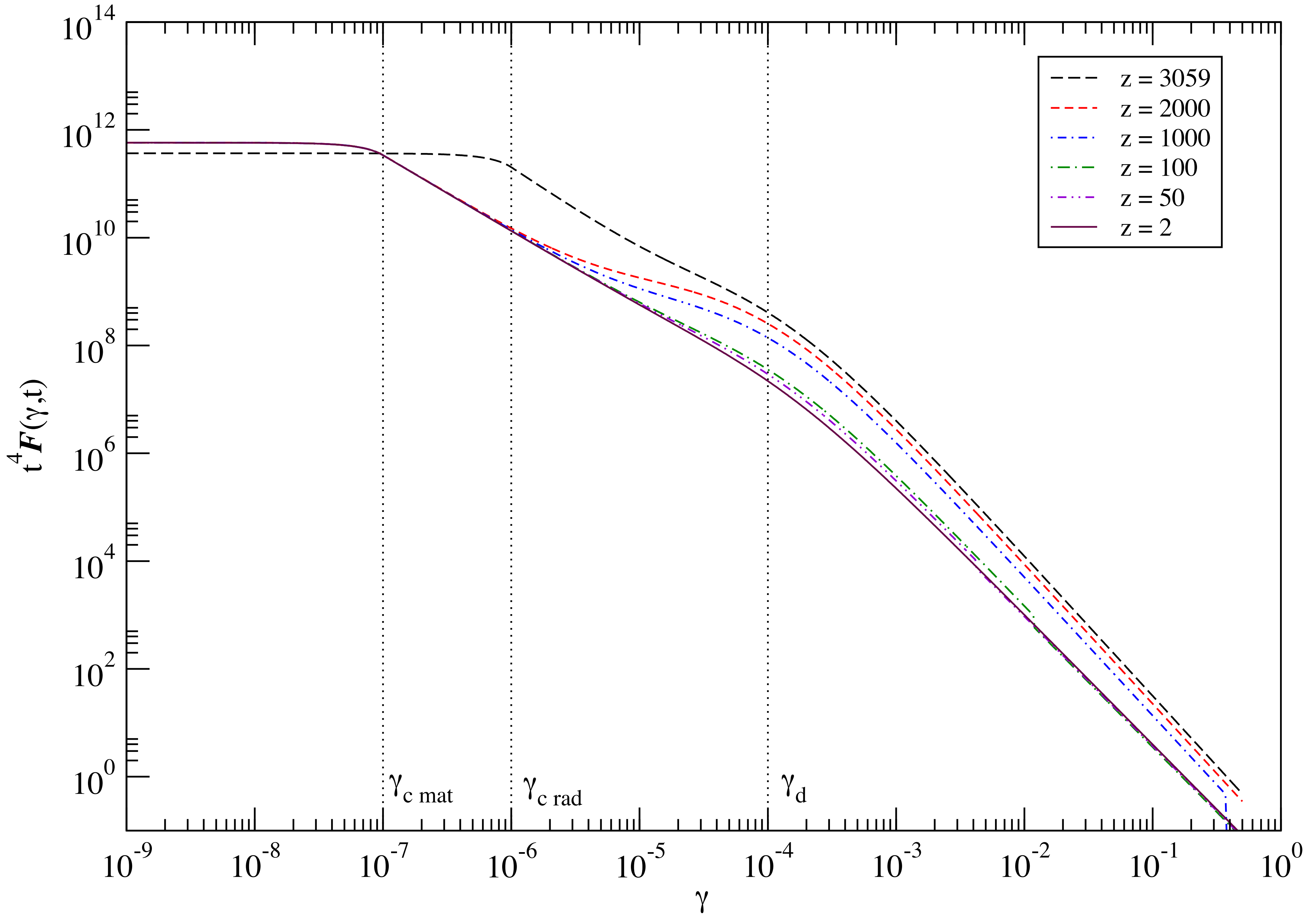


Nambu-Goto simulations

Ringeval, Sakellariadou & Bouchet, 2007;

Ringeval & Suyama, 2017

Number density of loops (radiation & matter)



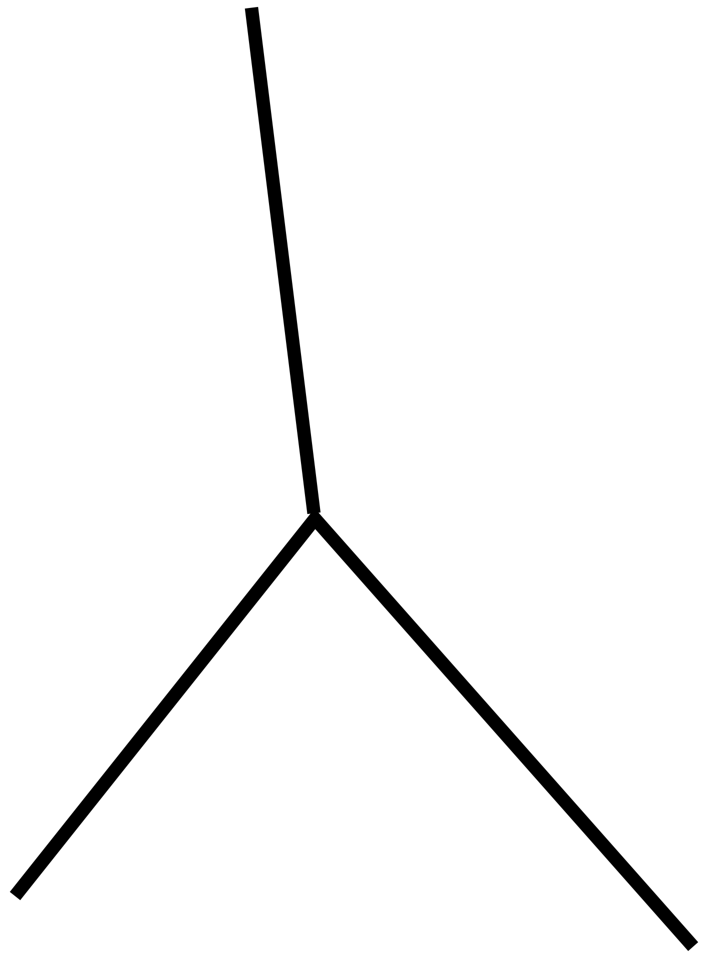
Loop size/t

They find an overproduction of small loops and use the results of Polchinski & Rocha to extrapolate to smallest loop sizes.

$$G\mu \lesssim 10^{-10} - 10^{-15}$$

depending on some assumptions.

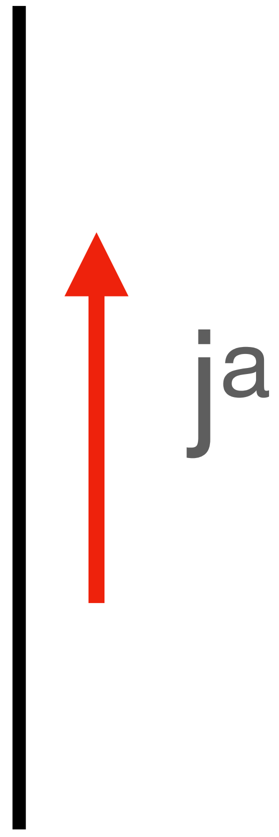
Diversity of strings



Strings with junctions

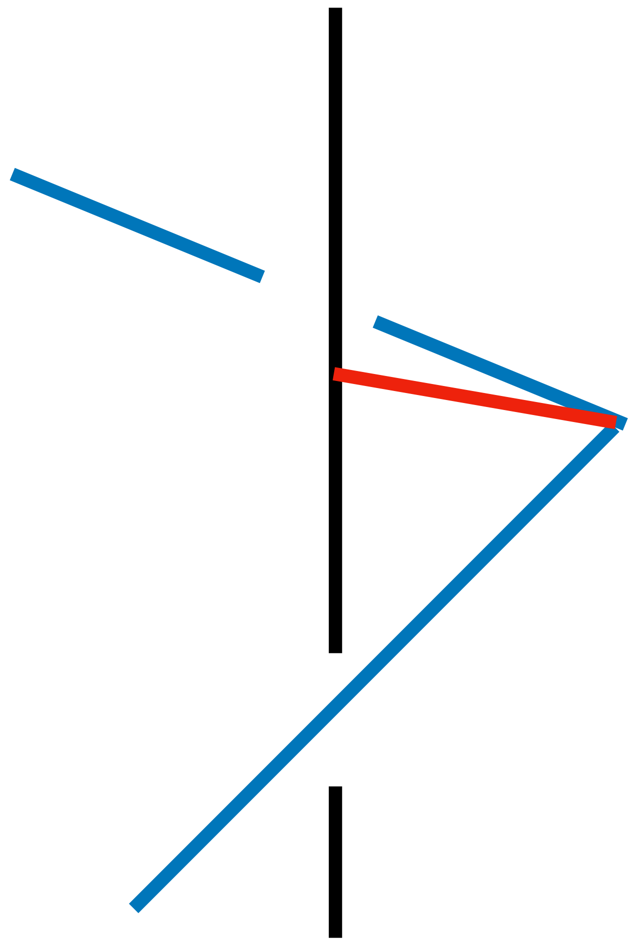
$\pi_1 = Z_N$

T. Kibble



Superconducting strings

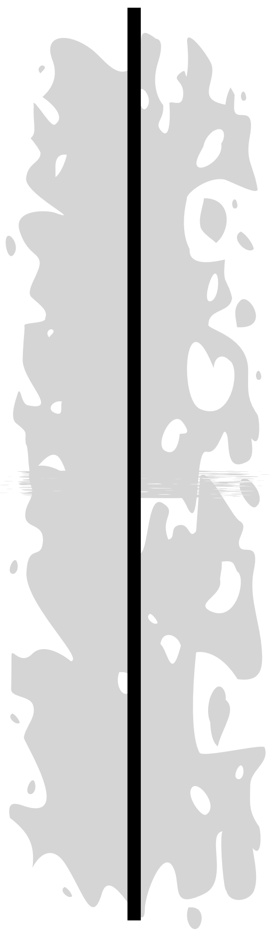
E. Witten



Non-abelian strings

$\pi_1 = \mathrm{non-Abelian}$

M. Kleman



Global strings

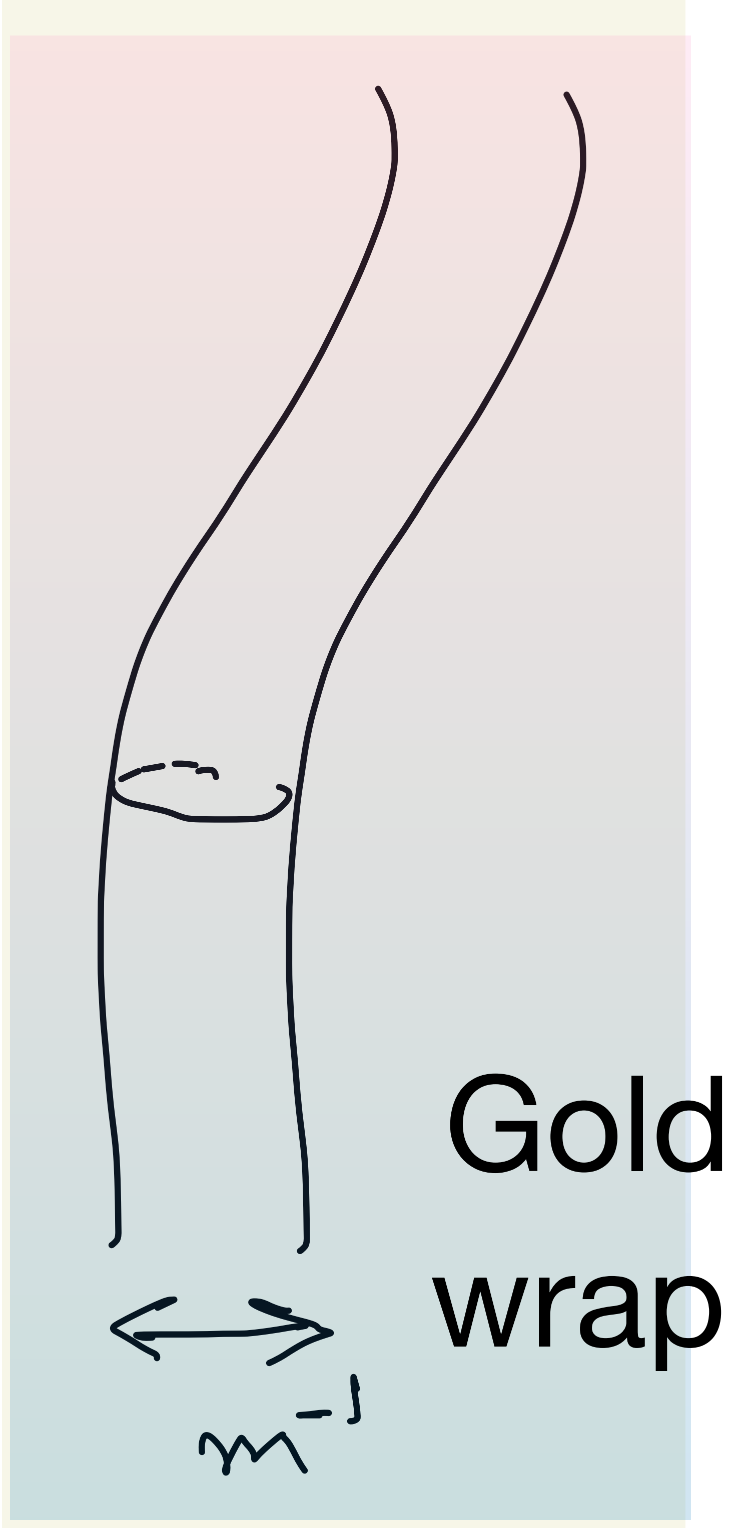
(Fast Radio Burst signatures)

Global/axion strings

Vilenkin & TV, 1987

$$S_{\text{global}} = \int d^4x \left[|\partial_\mu \phi|^2 - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 \right]$$

U(1) symmetry breaks. Global string solutions.



Zero core thickness approximation

$$S_{\text{Kalb-Ramond}} = -\mu \int d^2\sigma \sqrt{-\gamma} + 2\pi\eta \int d\sigma^{\mu\nu} B_{\mu\nu} + \int d^4x \frac{1}{6} B_{\mu\nu\lambda} B^{\mu\nu\lambda}$$

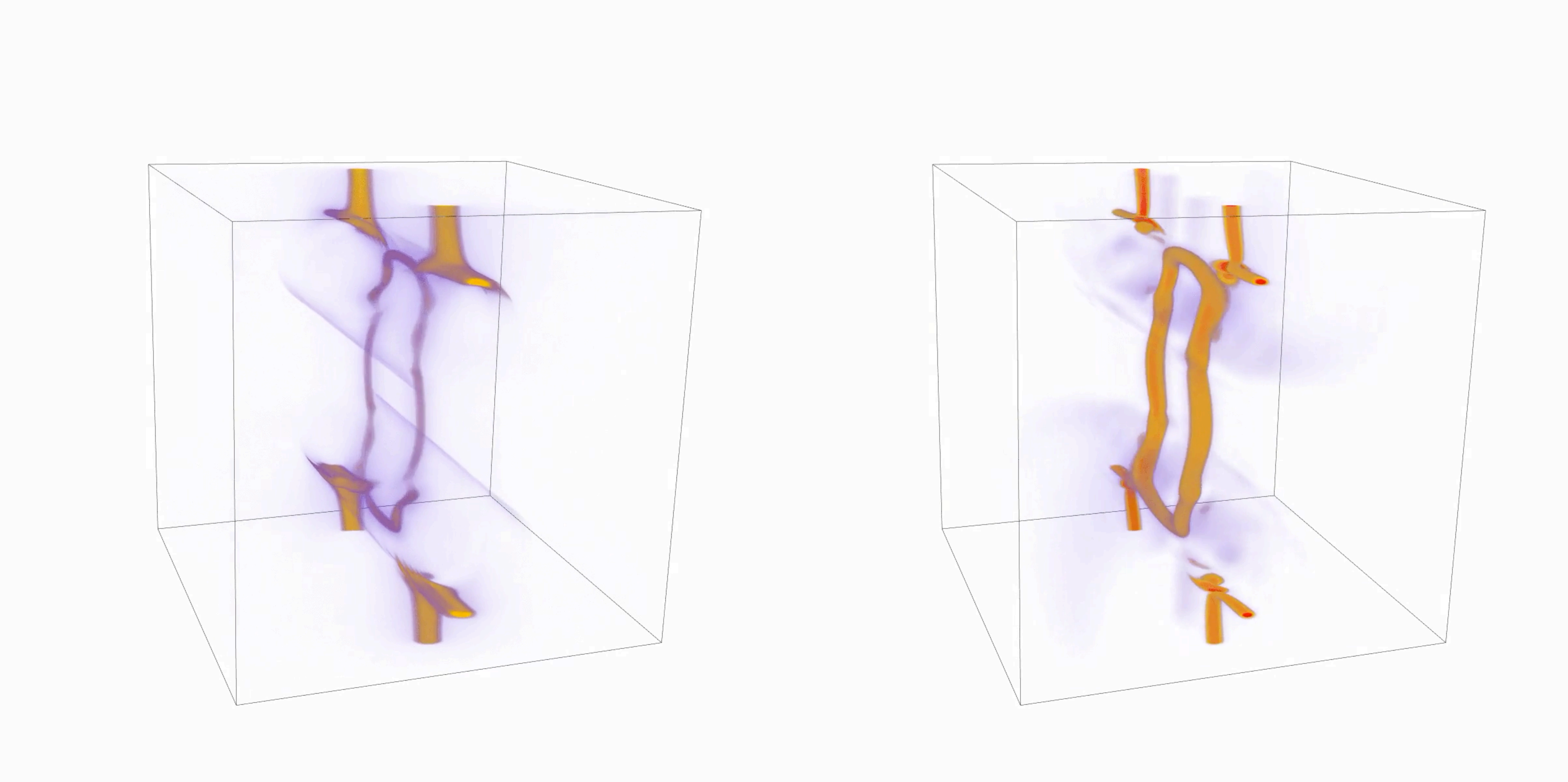
Can calculate spectrum of Goldstone radiation.

Goldstone boson cloud
wrapped around a core.

Global string loop evolution

Total energy; potential energy

$$|\mathbf{v}| = 0.6$$



Axion string debate

Recent simulations:
Benabou et al, 2024
Blasi et al, 2022

Spectrum of radiated axions feeds into their abundance which affects their detectability.

Kalb-Ramond action gives a sharp spectrum peaked at k~1/L; numerical evolution gives a 1/k spectrum.

- Kalb-Ramond action assumes the dynamics is governed entirely by the core;
- Numerical results are limited by dynamic range and need to be extrapolated.

Conclusions

- Cosmic strings occur in various high energy theories and have rich dynamics and interactions.
- Cosmic strings decay by gravitational radiation and particle emission.
- Observations of a stochastic background of gravitational radiation may be sourced by cosmic strings.
- Observations of gravitational wave bursts can lead to a direct detection of cosmic strings.
- Repeated gravitational wave bursts are a unique signature of cosmic strings.
- Diversity of strings — junctions; superconducting; non-Abelian; global.
- Evolution of global strings important for axion cosmology.