Bottomonia evolution in QGP using OQS: Beyond Quantum Brownian regime

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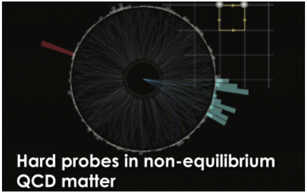

Introduction

- Heavy quarks are produced at the very initial stages of the heavy ion collisions.
- They experience the entirety of the QGP evolution.
- Quarkonia measurements can provide information about QGP medium.
- Matsui and Satz (1986) proposed sequential quarkonium suppression as a signal of QGP.
- There are number of experimental measurements available: R_{AA}, v_2...
- It is interesting to look at the quantum evolution of a quarkonium in the medium.
- Theoretically $b\bar{b}$ has comparatively lesser nuances than $c\bar{c}$.

Scales of the problem

- For system $Q\bar{Q}$: Mass M, Separation r, Binding energy E_b.
- $M \gg 1/r \gg E_b \rightarrow$ pNRQCD[1]to model the quarkonium

$$
\begin{aligned}
L_{\mathrm{pNRQCD}} = &\int d^3\mathbf{r}\ \mathrm{tr}\Big(\mathcal{S}(\mathbf{r},t)^\dagger[i\partial_0 - h_s]\mathcal{S}(\mathbf{r},t) + \mathcal{O}(\mathbf{r},t)^\dagger[iD_0 - h_o]\mathcal{O}(\mathbf{r},t) \\
&+ gV_A(r)[\mathcal{O}^\dagger(\mathbf{r},t)\mathbf{r}\cdot\mathbf{E}\mathcal{S}(\mathbf{r},t) + \mathcal{S}^\dagger(\mathbf{r},t)\mathbf{r}\cdot\mathbf{E}\mathcal{O}(\mathbf{r},t)] \\
&+ \frac{V_B(r)}{4}\{\mathcal{O}^\dagger(\mathbf{r},t),\ \{\mathbf{r}\cdot g\mathbf{E},\ \mathcal{O}(\mathbf{r},t)\}\}\Big) \\
&+ L_{\mathrm{light}}\ ,
\end{aligned}
\tag{1}
$$

- For the medium: Temperature T, m_D. $m_D \sim T$.
- Coulomb pot with $\alpha_s \sim 0.4$: $M \sim 5$ GeV, $1/r \sim C_F M\alpha_s/2 \sim 1.3$ GeV, $E_b = C_F M\alpha_s^2/4 \sim 500$ MeV

[1]Brambilla, Pineda, et al. 2005.

Density matrix approach

- System+Environment can be characterized by density matrix

$$i\frac{d\rho_{tot}}{dt} = [H_{tot}, \rho_{tot}] \quad \rho_{tot} = \sum_k p_k |\psi_k(t)\rangle\langle\psi_k(t)| \tag{2}$$

$$H_{tot} = H_{\mathrm{S}} + H_{\mathrm{E}} + H_I \tag{3}$$

- System evolution is described by the reduced density matrix ρ_S[2]

$$\rho_{\mathrm{S}} = \mathrm{Tr}_{\mathrm{E}}(\rho_{tot}) \tag{4}$$

- Evolution equation for $\rho_{\mathrm{S}}(t)$ is in general highly non trivial!

$$\frac{\partial \rho_{\mathrm{S}}(t)}{\partial t} = -i\mathrm{Tr}_{\mathrm{E}}\left([H_I(t), \rho_{\mathrm{tot}}(t)]\right)$$

[2] H.-P. Breuer and Francesco Petruccione 2007.

Density matrix approach

- Assume that the ρ_{S} doesn't influence ρ_{E}
- Expand to second order in g[3], accurate up to $O(H_I^3)$

$$\frac{\partial \rho_{\mathrm{S}}^I(t)}{\partial t} = -\int_0^t ds \mathrm{Tr}_{\mathrm{E}}\left[H_I(t), \left[H_I(t-s), \rho_{\mathrm{S}}^I(t) \cdot \rho_{\mathrm{E}}\right]\right]$$

  This is known in the literature as the Redfield equation.
- This is a non-Markovian equation as the operator $\mathrm{Tr}_{\mathrm{E}}[H_I(t)H_I(s)\rho_{\mathrm{E}}]$ depends on times s between 0 and t.
- Systematic formalism "TCL evolution"[4]-$>$ Can compute higher order corrections.

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[3] Akamatsu 2021.
[4] Chaturvedi and Shibata 1979.

- In COM of the quarkonia, system Hamiltonian is given by,

$$H_{\mathrm{S}} = h_s|s\rangle\langle s| + h_o|a\rangle\langle a|, \quad \text{where,} \quad h_{s,o}(r) = \frac{p^2}{M} + v_{s,o}(r),$$
$$H_I = -\, g\mathbf{E}^a \cdot \, \mathbf{r}\left[\frac{1}{\sqrt{2N_c}}|s\rangle\langle a| + \frac{1}{\sqrt{2N_c}}|a\rangle\langle s| + \frac{d_{abc}}{2}|b\rangle\langle c|\right] \tag{5}$$

- Define singlet and octet blocks of the density matrix.

$$\rho_s(t) = \langle s|\rho|s\rangle \qquad \rho_o(t) = \langle a|\rho|a\rangle$$

- Off diagonal elements decouple from $\rho_{s/o}(t)$ at all times, if initial conditions are block diagonal.

- We get for $\rho_{\rm S} = diag(\rho_s \quad \rho_o)$

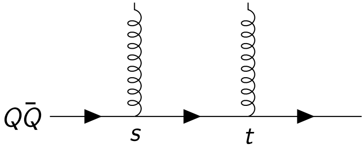
$$\frac{d\rho_{\rm S}}{dt} = -iH_{\rm eff}\rho_{\rm S} + \int_0^t ds \left\{ \Gamma(t,s) \sum_{n=1}^{3} \mathbf{V}_n(s)\rho_{\rm S}(t)\mathbf{V}_n^\dagger(0) \right\} + {\rm H.C} \quad (6)$$

where,

$$H_{\rm eff} = H_{\rm S} - i \int_0^t ds\, \Gamma(t,s) \sum_{n=1}^{3} \mathbf{V}_n^\dagger(0)\mathbf{V}_n(s) \,. \qquad (7)$$

- $\{\mathbf{V}_n(t)\}$ are operators for $s \to o$, $o \to s$ and $o \to o$ transitions.

$$\mathbf{V}_n(t) = \begin{cases} e^{ih_o t}\mathbf{r}e^{-ih_s t} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} & n = 1 \\ e^{ih_s t}\mathbf{r}e^{-ih_o t}\sqrt{\frac{1}{(N_c^2-1)}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} & n = 2 \\ e^{ih_o t}\mathbf{r}e^{-ih_o t}\sqrt{\frac{N_c^2-4}{2(N_c^2-1)}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} & n = 3 \,. \end{cases}$$



- All the information about the environment goes into the correlator

$$\Gamma(t,s) = \frac{g^2}{6N_c}\mathrm{Tr}_{\mathrm{E}}\left(E_i^a(t,\vec{0})E_i^a(s,\vec{0})\rho_{\mathrm{E}}\right) . \qquad (8)$$

- Relevant time scales: system time scale τ_S and environment relaxation scale τ_E

$$\tau_s \sim E_b^{-1} \quad \tau_E \sim \frac{1}{T}$$

- If one assumes $\tau_{\mathrm{E}} \ll \tau_{\mathrm{S}}$, one can make the following expansion:

$$\mathbf{V}_n(t) \sim e^{-ih_\alpha t}\mathbf{r}e^{ih_\beta t} \approx \mathbf{r} - it(h_\alpha\mathbf{r} - \mathbf{r}h_\beta) + \mathcal{O}\left[\left(\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{S}}}\right)^2\right] . \qquad (9)$$

- Truncation at the first term and second term gives leading order (LO)[5] & next-to-leading order (NLO)[6] equations.
- LO and NLO can be cast in the form Lindblad equation.

$$\frac{\partial \rho_{\rm S}(t)}{\partial t} = -i[H_{\rm S}, \rho_{\rm S}] + \sum_k \left(L_k \rho_{\rm S} L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_{\rm S}\} \right) , \qquad (10)$$

- For bottomonium $\tau_s^{-1} \sim 450 \; MeV \, (Coulomb \; 1S) \rightarrow$ hierarchy fails.
- Hence we need to solve the general non-Markovian equation.

[5] Brambilla, Escobedo, Strickland, et al. 2021.
[6] Brambilla, Escobedo, Islam, et al. 2022.

- $\rho_{\mathrm{S}}(t)$ can be reduced into block diagonal form in angular momentum basis.

$$\rho_{\mathrm{S}}^{l}(t) = \sum_{m} \langle l, m | \rho_{\mathrm{S}}(t) | l, m \rangle$$

- In angular momentum basis,

$$\frac{\partial \rho_{\mathrm{S}}^{l}}{\partial t} = -i h_{eff}^{l} \rho_{\mathrm{S}}(t) + \int_{0}^{t} ds \sum_{n} \sum_{l'} T_n(l \to l' | s) \rho_{\mathrm{S}}^{l'}(t) T_n^{\dagger}(l \to l' | 0) + h.c$$

- T_n's are transition operators that change $l \to l'$ and also the color states.
- As the interaction is $\sim r.E$, only transitions that take $l \to l \pm 1$ are allowed.

$$T_n(l \to l', t) \sim C_{l,l'} \times e^{ih_u^{l'} t} r e^{-ih_v^{l} t} \quad C_{l,l'} \propto \delta_{l',l\pm 1}$$

- Numerically the evolution eq can be solved thorough a stochastic method known as stochastic Unraveling.
- One defines a stochastic wavefn $|\psi\rangle$ and a corresponding evolution eq:
$$d|\psi(t)\rangle = -idt\, .h_{eff}(t)|\psi(t)\rangle + dN(t) \cdot J_i(t)|\psi(t)\rangle$$
- Computationally intensive due to averaging over large ensembles
- Shown in[7], for LO/NLO H_{eff} gives good estimates for $\Upsilon(1S)$.

$$\frac{\partial|\psi_l, t\rangle}{\partial t} = -\, ih_s^l\, |\psi_l, t\rangle - \int_0^t ds\, \Gamma(t, t-s) \left\{ \left(\frac{l+1}{2l+1}\right)\, r\, e^{-ih_o^{l+1} s} r\, e^{ih_s^l s} + \left(\frac{l}{2l+1}\right)\, r\, e^{-ih_o^{l-1} s} r\, e^{ih_s^l s} \right\} |\psi_l, t\rangle \tag{11}$$

[7] Brambilla, Escobedo, Islam, et al. 2022.

Results

- We want to illustrate the dependence of suppression on the hierarchy between τ_{S} and τ_{E}
- τ_{S} is fixed, so vary $\tau_{\mathrm{E}} = \xi_{\mathrm{E}}/T$.
- In this study the correlation function used is

$$\Gamma(t, s) = \frac{\kappa}{2\tau_{\mathrm{E}}} e^{-|t-s|/\tau_{\mathrm{E}}}$$

- Spectral function corresponding to this correlator is

$$\frac{\rho(\omega)}{\omega} \sim \frac{\kappa\Omega^2}{(\omega^2 + \Omega^2)T} \quad , \Omega = 1/\tau_{\mathrm{E}}$$

Results

- First, to understand the effect of varying τ_{E} in a simple setting, we will take the medium to be at a fixed temperature

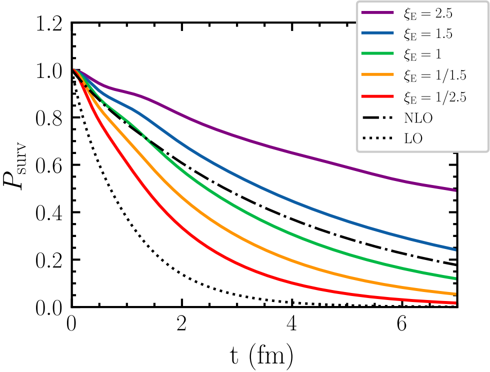


Figure: $\Upsilon(1S)$ evolution with a constant T medium: $T_0 = 300$ MeV, $\kappa/T^3 = 4.0$

- We have computed $\Upsilon(1S)$ survival probabilities at $\sqrt{s_{NN}} = 5.02$ TeV and 2.76 TeV energies.
- Background is modeled using AMPT[8] + (2+1) viscous hydrodynamics.

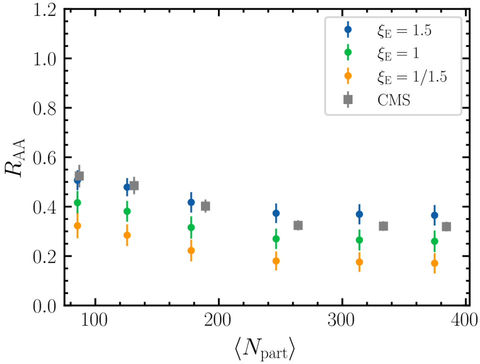


Figure: $\Upsilon(1S)$ survival probability for PbPb@ 5.02 TeV: Comparison of OQS with CMS(2017)[9]

[8] Lin et al. 2005.
[9] Sirunyan et al. 2019.

Results

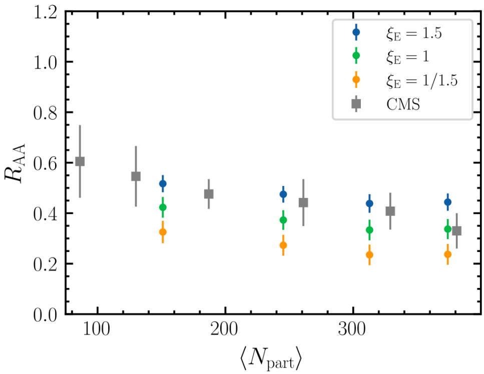


Figure: $\Upsilon(1S)$ survival probability PbPb@2.76 TeV: Comparison of OQS with CMS(2017)[10] data.

[10] Khachatryan et al. 2017.

Results

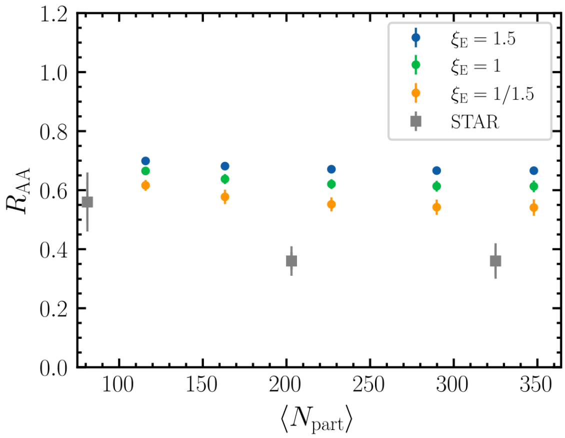


Figure: $\Upsilon(1S)$ survival probability AuAu@200 GeV: Comparison of OQS with STAR(2023)[11] data.

[11] Aboona et al. 2023.

- Within our model, suppression just from QGP is not enough to explain $\Upsilon(1S)$ data at RHIC energies.
- Cold nuclear matter effects on the parton density functions (PDFs) might affect the initial hard production of bottomonia.
- We haven't included the temperature dependence of κ/T^3.

Summary

- We solve non-Markovian master equations for the evolution of the $\Upsilon(1S)$ state in the QGP.
- We find a strong dependence of survival probabilities on the hierarchy between τ_{S} and τ_{E}.
- $\Upsilon(1S)$ phenomenology at 200GeV requires inclusion of additional effects.

Acknowledgment: We would like to thank Subrata Pal (TIFR), for providing the AMPT+ Hydro evolution data at LHC and RHIC energies.

Backup: Numerical simulation

- The Non-Markovian density matrix equation can be solved using quantum trajectories method.
- Consider a general master equation

$$\frac{\partial \rho_{sys}}{\partial t} = A(t)\rho_{sys} + \rho_{sys}B^{\dagger}(t) + \sum_i C_i(t)\rho_{sys}D_i^{\dagger}(t)$$

- The idea[12] is to define a two component wavefunction:

$$|\psi(t)\rangle = (|\phi_1(t)\rangle \;\; |\phi_2(t)\rangle)$$

- Then evolve the wavefunction using H_{eff} inter spaced with quantum jumps $J_i|\psi\rangle$

$$Heff = diag(A(t) \;\; B(t)) \quad J_i = diag(C_i(t) \;\; D_i(t))$$

- Averaging $|\phi_1\rangle\langle\phi_2|$ over different realisations gives the density matrix

[12] H. P. Breuer, Kappler, and F. Petruccione 1999.

Backup

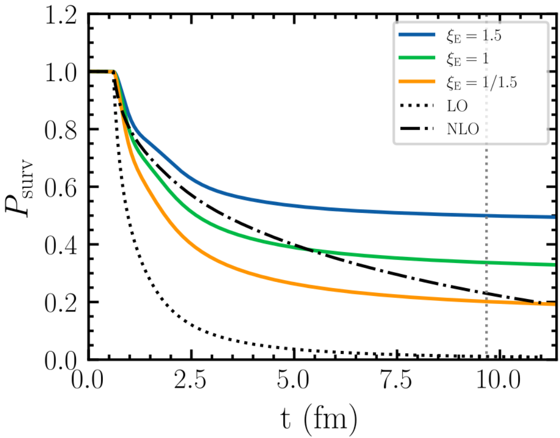


Figure: Bjorken background T=480MeV