Stability of Schwarzschild

–in honour of late Prof. C. V. Vishveshwara–

6th March 1938 - 16th January 2017

Amitabh Virmani

Institute of Physics, Bhubaneshwar, India

ICTS Discussion Meeting, Feb 23, 2017

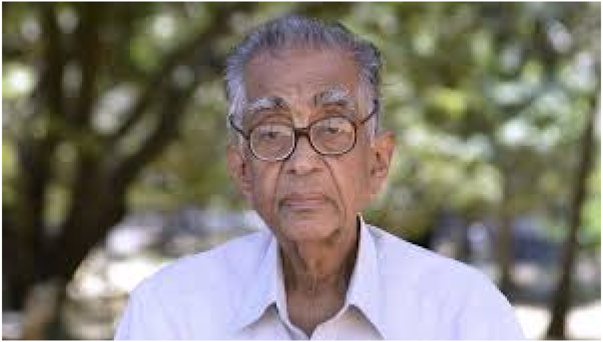

Figure: Prof. C. V. Vishveshwara: “the black hole man of India”

GRAVITATIONAL RADIATION

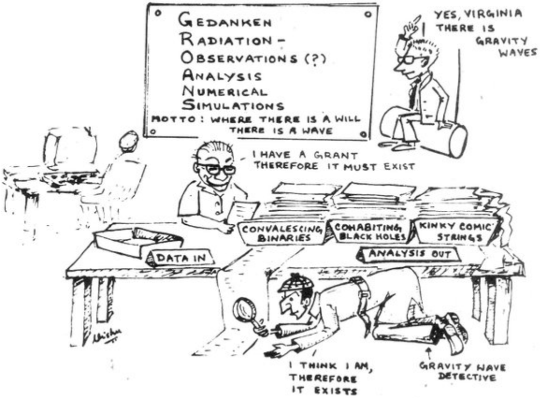

Figure: a cartoon by Prof. C. V. Vishveshwara

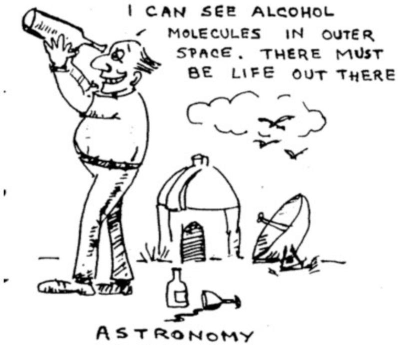

Figure: a cartoon by Prof. C. V. Vishveshwara

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- Afterwards I had a conversation with Roberto Emparan about my interactions with Prof. Vishveshwara. He told me that Prof. Vishveshwara had a visiting position in Spain and he has heard a colloquium on black holes from him.

Outline

Introduction

Historical context, BH perturbation primer

Quasi Normal Modes

Modern Developments

Legacy

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- bridge between classical and quantum gravity?

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- In 1970 he noticed that waveforms at later times consists of a damped sinusoid, with ringing frequency independent of the Gaussian wavepacket parameters. The subject of QNMs was born.
- Another group in 1971 also noticed similar phenomenon where they modelled radial infall of a particle in Schw in BH perturbation theory.

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- 1970: Zerilli extended the Regge-Wheeler analysis to general perturbation.
- 1970: Vishveshwara studies those perturbation equations numerically.
- 1975: Chandrasekhar and Detweiler computed numerically some QNMs. They show that Regge-Wheeler and Zerilli equations are iso-spectral.

Milestones

- 1983: Chandrasekhar's book of black hole perturbation theory appears.
- 1989: Ringdown based test of no-hair theorem of GR are shown to be possible.
- 1997: AdS/CFT era begins with Maldacena.
- 1998: Signal to noise ratio for ringdown waves is potentially larger than the signal to noise ratio for inspiral waves (both for LIGO and LISA).
- 1999: Horowitz and Hubeny computed QNMs of AdS BHs.

Milestones

- 2002: QNM spectra corresponds to poles of the retarded thermal CFT correlation functions
- 2005: Pretorius achieves numerical simulations of BH binary problem. Waveform indicates ringdown contribution is substantial in terms of radiated energy.
- 2006-2010: Low lying modes, fluid/gravity correspondence, BH membrane paradigm

Numerous Physical and Technical Result

BH perturbation theory has found numerous applications. Many mathematical physics results. It is a subject of startling complexity and richness.

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Variable Ψ satisfies Schrödinger equation

$$\frac{d^2\Psi}{dr_*^2} + \left(\omega^2 - V\right) \Psi_s = 0 , \tag{3}$$

where r_* is the tortoise coordinate $r_* \to -\infty$ at the horizon and $r_* \to +\infty$.

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- After appropriate gauge fixing (Regge Wheeler gauge) perturbations fall into two classes: odd parity and even parity.
- Perturbation problems reduce to Schrödinger equations.

Regge-Wheeler class: Easier: 1957: Odd Parity

Odd parity perturbation can be put in the form

$$\tilde{h}_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{bmatrix} \left(\sin\theta \frac{\partial}{\partial\theta}\right) Y_{l0}(\theta) \tag{6}$$

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Schrödinger equation

$$\frac{d^2\Psi^-}{dr_*^2} + \left(\omega^2 - V^- \right) \Psi^- = 0 . \tag{8}$$

Zerilli: Much harder: 1970: Even Parity

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- His key contribution was the idea of the Quasi Normal Modes.

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$$\Psi \sim e^{-i\omega(t-r_*)}, \quad r_* \to +\infty \ (r \to \infty). \tag{12}$$

Quasi Normal Modes: Stability

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- They are usually not so easy to calculate. A complete mathematical theory of such an eigenvalue problem is still not fully developed.
- Stability: The numerical criteria of stability could be the evidence that all QNMs are damped.

Quasi Normal Modes: Stability

Table: Black Hole Mode Stability

Schwarzschild	Vishveshwara, 1970
Reissner-Nordström	Moncrief, Alekseev, 1974
Extreme Reissner-Nordström	Aretakis, 2011
Kerr	Press, Teukolsky, 1973-74
Extreme Kerr	Lucietti, Reall, 2012
Kerr Newmann	???

Contributions of Prof. Vishveshwara

PHYSICAL REVIEW D VOLUME 1, NUMBER 10 15 MAY 1970

Stability of the Schwarzschild Metric

C. V. VISHVESHWARA*

Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

and

Institute for Space Studies, Goddard Space Flight Center, NASA, New York, New York 10025

(Received 11 August 1969)

The stability of the Schwarzschild exterior metric against small perturbations is investigated. The perturbations superposed on the Schwarzschild background metric are the same as those given by Regge and Wheeler, consisting of odd- and even-parity classes, and with the time dependence $\exp(-ikt)$, where k is the frequency. An analysis of the Einstein field equations computed to first order in the perturbations away from the Schwarzschild background metric shows that when the frequency is made purely imaginary, the solutions that vanish at large values of r, conforming to the requirement of asymptotic flatness, will diverge near the Schwarzschild surface in the Kruskal coordinates. Since the background metric itself is finite at this surface, the above behavior of the perturbation clearly contradicts the basic assumption that the perturbations are small compared to the background metric. Thus perturbations with imaginary frequencies that grow exponentially with time are physically unacceptable, and hence the metric is stable. Perturbations with real values of k representing gravitational waves are also examined. It is shown that the only nontrivial stationary perturbation that can exist is one that is due to the rotation of the source, which is given by the odd perturbation with the angular momentum $l=1$. The significance of solutions with complex frequencies is pointed out, as is the lack of a theorem (completeness of the eigenfunction) for the even-parity case to parallel the Sturm-Liouville theory, which is applicable to the odd-parity case. Such a theorem would be required to convert the computations indicating stability as given here into a fully rigorous stability theorem.

Figure: A famous paper of Prof. C. V. Vishveshwara

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- Stationary mode was identified to be deformation to the Kerr family.

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- A complete proof of linear stability has been reported in 2016 by Dafermos, Holzegel, and Rodnianski

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Contributions of Prof. Vishveshwara

- In 1970 CVV contributed in *Nature* a short letter titled Scattering of Gravitational Radiation by a Schwarzschild Black-hole.
- Infinite red-shift as well as one-way membrane nature of Schwarzschild surface is fully understood.
- CVV studied scattering of Gaussian wave-packets through the Schwarzschild potential.
- Ringdown waves were observed, and spacing of peaks was related to Schwarzschild mass.

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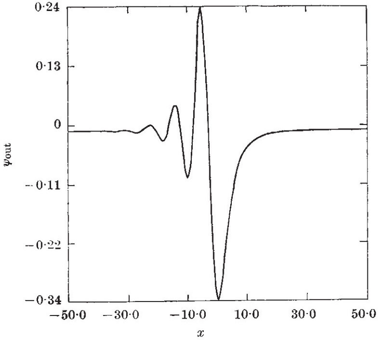

Figure: Ringdown figure from the 1970 *Nature* paper.

Outline

- Introduction
- Historical context, BH perturbation primer
- Quasi Normal Modes
- **Modern Developments**
- Legacy

Instability of extremal black holes

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- Lowest QNMs are closely related to hydrodynamic limit.

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- Lorentzian AdS/CFT, QNMs contain information about thermal correlation functions, and hence about spectrum and dispersion.
- Ringdown waves: late time behaviour of merging black holes.

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- Higher dimensional rotating black holes, Gregory-Lafflamme instability of black branes.
- Non-linear stability of black holes is a topic about which not much is known.

Legacy: AdS/CFT

- QNMs remain an important tool in AdS-CMT discussions, membrane paradigm discussions, hydrodynamics discussions.
- In global AdS it has been argued that there are very long lived QNMs, i.e., modes for which

$$\omega_I(l \to \infty) \to 0. \tag{13}$$

  The physics of these modes and there implications for classical and quantum is not at all explored. Literature is very confusing.

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- ...a lot remains to be understood. Thank You!